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MHD natural convection in saturated porous media with heat generation/absorption and thermal radiation: closed-form solutions

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The present work is devoted to the free convection flow occurring about a heated vertically stretching permeable surface placed in a porous medium under the influence of a temperature dependent internal heat generation or absorption. There are volume radiative heat sources in the fluid and the system is permeated by a uniform magnetic field. It is shown that the governing equations are reducible to a self-similar nonlinear ordinary differential equation of third order whose solutions are constructed analytically in the purely exponential series form. Under special circumstances, closed-form solutions are available which clearly indicate the existence of dual natural convection solutions. Otherwise, analytical solutions are still possible which are shown to be computed from an elegant algorithm without a need to invoke any numerical means. Exact solutions demonstrate, in physical insight that, in the presence of a heat sink absorbing the temperature from the porous medium increases the rate of heat transfer from the wall, whereas a heat source mechanism will surely overheat the system during the wall heating process, resulting in poor heat transfer rates. The presented exact solutions are beneficial for investigation of free convection phenomena in different geometries taking into account more complex physical features in higher dimensions.

Key words: porous vertical wall, magnetic field, radiative flux, permeable surface, heat generation/absorption, rate of heat transfer, analytical solutions.

1. Introduction

It is well-known that the density variations due to the temperature gradients in a fluid medium lead to a physical mechanism called free (natural) convection. Such a physical operation is encountered in many real-life applications as far as the porous media is concerned, for example while cooling the electronic equipment in computers, forming the clouds in atmosphere, interaction of spices in chemical engineering, processes in biological systems, removing heat from nuclear fuel debris, insulating the constructions, storing the energy and foods and so on, refer to the comprehensive reviews [1, 2] and [3] for more applications. The current investigation is also about the phenomenon of magnetohydrodynamics (MHD) natural convection in saturated porous media with heat generation/absorption and thermal radiation with an objective of gaining closed-form solutions.

The heated vertical plate placed in a medium of porous feature received considerable attention lately in order to explain the process of free convection. In the absence of a source to cause a heat generation, several temperature conditions, from a variable surface temperature to a variable heat flux were successfully analyzed in [4]. Effects of porosity were numerically simulated in [5]. On the other hand, the internal heat generation has an active role in the natural convection heat transfer in many porous media problems. Due to its significance, [6] first considered the internal heat generation term in the governing energy equation as a fixed variable term for the problem of free convection from a vertical wall embedded in a porous medium. Many researchers adhered to this plausible work, by taking into account such an internal heat generation term. For instance, [7] extended the work of [6] to the non-Newtonian fluid case; [8] incorporated the effects of mass injection/removal through the wall boundary; [9] discussed the convective boundary condition associated with the temperature; [10] took into account the double diffusive Soret and Dufour effects with a power-law fluid. The radiation effects were included in the numerical investigation of [11]; [12] studied the effects of viscous dissipation and magnetic field. The free convection problem affected by the presence of a saturated nanofluid medium was the focus of the study in [13]. The conditions of variable thermal diffusivity and mass diffusivity in a non-Newtonian fluid were examined in [14], followed by the concentration effects in [15].

The internal heat generation term introduced and employed in [6] to the heated vertical wall was also considered in the following applications, such as the natural convection phenomena in vertical cones, see [16] and [17], and in the horizontal plates [18] and [19]. Very interesting applications of porous media correlated with natural convection can be found in the recent papers [20–24].

The prime motivation for the current research is to substitute the fixed internal heat generation term as used in the above papers with that of a temperature dependent heat flux term representing more realistic situations. Within this perspective, the effects of such a term are to be investigated on the free convection taking place along a vertical plate embedded in a porous medium. The plate is assumed to be heated permeable and radiative subjected to a uniform magnetic field. The governing equations are reduced to a self-similar form whose solutions, unlike the existing numerical literature, are presented in an elegant analytic form which can be expressible in either closed-form or infinite series. The solutions perfectly conform to those numerical ones for the particular parameters. In the case of series, an algorithm is further introduced necessitating no numerical computations. The engineering interest of the heat transfer rate can be easily derived from the presented formulas.

2. Physical problem and mathematical formulation

As depicted in Fig. 1, we consider the free convection phenomenon about a flat plate heated with the wall temperature $T_w(x) = T_\infty + ax^{\Lambda}$, with T_∞ being the ambient temperature and (a, Λ) being positive temperature-related constants. The flat body is embedded in a porous medium, taking into account a heating/cooling heat flux term (heat generation rate) dependent on the temperature in the manner $q''' = \tilde{Q}(T - T_w)$ in place of a preassigned exponentially decaying heat generation as introduced in [6], where $\tilde{Q}$ is the volume heating generation/absorption rate. The advantage of the present thermal flux term is that it enables us to gain the unknown internal heat generation as a solution of the full energy equation, rather than prescribing it in the form of an exponential function. The wall is assumed to be permeable with a wall transpiration velocity $V_w(x) = v_w x^{\frac{\Lambda-1}{2}}$, such that the wall permeability parameter v_w will be apparent as a consequence of the following analysis. The thermal radiation effect is accounted for with the Rosseland based radiation flux $q_r = -c(T^4)_y$ (c is constant), but linearized such that $q_r = dT_y$, where the constant d is owing to the Taylor expansion of the quadruple temperature. A uniform magnetic field further acts against the surface in the positive y direction to retard the motion with a uniform magnetic field strength B_0, refer to Fig. 1.

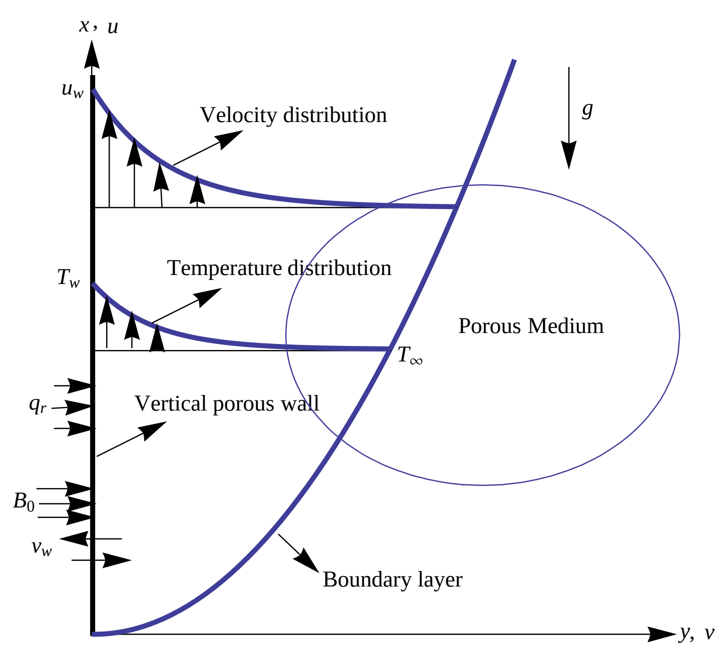

Fig. 1. The schematic of the natural convection about a heated vertical flat surface subjected to various physical phenomena.

Having the practical engineering applications as aforementioned in the Introduction, in the process of free convection about a vertical surface, the specific task of flow through the porous medium along the heated plate under the imposed physical mechanisms is to assess the temperature distribution and estimate the resultant heat transfer rate from the surface of the wall, facilitating the cooling process. The particular aim is to use a more realistic radiation term than it was being done up to now in the literature on the topic. Therefore, in the subsequent analysis it is targeted to solve the governing nonlinear equations of porous media and obtain exact representative formulae for the flow, temperature and the Nusselt number. Following [6] and in view of the Boussinesq and boundary layer approximations together with the Darcy porous model, the natural convection takes place according to the following governing equations and boundary conditions [25]

$$\begin{aligned} u_x + v_y &= 0, \\ \left(1 + \frac{\sigma B_0^2 K}{\nu}\right) u &= \frac{gK\beta_m}{\nu}(T - T_\infty), \\ uT_x + vT_y &= \frac{k}{\rho c_p} T_{yy} + \frac{\tilde{Q}}{\rho c_p}(T - T_\infty) + \frac{d}{\rho c_p} T_{yy}, \\ v &= V_w(x), \quad T = T_w(x) \quad \text{on} \quad y = 0, \\ u &\to 0, \quad T \to T_\infty \quad \text{as} \quad y \to \infty. \end{aligned} \tag{2.1}$$

In (2.1), (u, v) represents the two-dimensional velocity field along the coordinates (x, y), T is the temperature field, σ is the electrical conductivity, K is the porosity factor, ν is the kinematic viscosity, g is the gravitational acceleration, β_m is the thermal expansion parameter, k is the thermal conductivity and c_p is the specific heat. Provided that the magnetic Reynolds number is assumed small, the magnetic term in the momentum equation in (2.1) can be represented as a consequence of the magnetohydrodynamic (Lorentz) resistive force evaluated from

$$\sigma\left((u\overrightarrow{i} + v\overrightarrow{j}) \times B_0\overrightarrow{j} \times B_0\overrightarrow{j}\right) = -\sigma B_0^2 u\overrightarrow{i}. \tag{2.2}$$

With the help of the change of units

$$e = \frac{gK\beta_m}{\alpha_m \nu (1 + M)}, \qquad \alpha_m = \frac{k}{\rho c_p}, \tag{2.3}$$

the scaled boundary layer coordinate

$$\eta = \sqrt{ae}\, x^{\frac{\Lambda - 1}{2}} y, \tag{2.4}$$

and the similarity transformations f and θ

$$
\begin{aligned}
u &= \alpha_m a e x^{\Lambda} f'(\eta),\\
v &= -\alpha_m \sqrt{\frac{ae}{4}} x^{\frac{\Lambda-1}{2}} [(1+\Lambda) f(\eta) + (\Lambda-1)\eta f'(\eta)],\\
T &= T_\infty + a x^{\Lambda}\theta, \qquad (2.5)
\end{aligned}
$$

system (2.1) is expressed in the reduced similarity form

$$
\begin{gathered}
f''' + \alpha f f'' - \beta f'^2 + Q f' = 0,\\
f(0) = s, \quad f'(0) = 1, \quad f'(\infty) = 0 \qquad (2.6)
\end{gathered}
$$

with $f' = \theta$ and $\alpha_m a e x^{\Lambda} = u_w$ corresponding to the surface deformation. In (2.6), defining the wall permeability in the form $v_w = -\alpha_m \sqrt{\frac{ae}{4}}(\Lambda + 1)s$ leads to the wall suction parameter $s = f(0) > 0$, and the wall injection when $s < 0$. Moreover, the appearing parameters are

$$
\alpha = \frac{1+\Lambda}{2(1+Nr)}, \quad \beta = \frac{\Lambda}{(1+Nr)},
$$

with $Nr = d/k$ being the thermal radiation parameter, $Q = \tilde{\tilde{Q}}/(kae(1+Nr))$ being the heat generation (> 0) or absorption (< 0) parameter ($\tilde{\tilde{Q}} = \tilde{Q}x^{1-\Lambda}$) and $M = \sigma B_0^2 K/\nu$ being the magnetic interaction parameter. We should note that replacing Qf' in (2.6) by the exponential function $e^{-\eta}$ and also accounting for $M = s = Nr = 0$, (2.6) turns into the mathematical model given in [6].

In terms of an engineering viewpoint, we are mainly interested in the scaled local rate of heat transfer, or the scaled local Nusselt number defined by

$$
Nu = \frac{Nu_{local}}{\sqrt{ea}x^{\frac{\Lambda+1}{2}}} = -\theta'(0) = -f''(0), \qquad (2.7)
$$

where Nu_{local} is as a result of the local surface heat flux. Integrating (2.6) once from the semi-infinite physical domain, we have

$$
f'' + \alpha f f' + Q f + (\alpha+\beta)\int_{\eta}^{\infty} f'^2(\eta)\, d\eta = Q f_\infty, \qquad (2.8)
$$

where $f_\infty = f(\infty)$ and so, from (2.7),

$$
Nu = (\alpha + Q)s - Q f_\infty + (\alpha+\beta)\int_{0}^{\infty} f'^2(\eta)\, d\eta. \qquad (2.9)
$$

The result presented in (2.9) alone is successfully able to explain the effects of s and Q, for preassigned values of α and β (preserving the effects of magnetic field, temperature and radiation) on the heat transfer analysis of the considered problem, having identified f from the system in (2.6).

3. Analytical solution method

Dissimilar to the available numerical solutions in the literature obtained from various numerical schemes, we plan to find out exact solutions representing the flow and temperature fields. The following solution method stems from the study published in [26], particularly valid when $Q < 0$ (in place of M in [26]). Taking into account the asymptotic far field condition of the system (2.6), it is realistic to search for solutions of the purely exponential serial form

$$f(\eta) = \sum_{n=0}^{\infty} A_n e^{-n\lambda\eta}, \tag{3.1}$$

such that the exponent λ in (3.1) is a positive constant to match to the infinity boundary condition and $A_0 = \lim_{\eta\to\infty} f(\eta) = f(\infty) = f_\infty$. Both A_0 and λ are to be found.

Injecting (3.1) into (2.6) results in

$$\begin{aligned} & -n\lambda(n^2\lambda^2 + Q)A_n + \alpha n^2\lambda^2 A_0 A_n \\ & \qquad + \sum_{k=0}^{n-1} k\lambda^2[(\alpha-\beta)k + \beta(2k-n)]A_k A_{n-k} = 0, \quad n \geq 0. \end{aligned} \tag{3.2}$$

There are two subcases to be considered now, in accordance with the far field behavior of the stream function f.

3.1. $A_0 = f(\infty) = 0$, $A_1 \neq 0$

In this case the parameter Q cannot be zero and we have

$$\lambda = \sqrt{-Q}$$

from (3.2) implying that (3.1) type solutions are available only when $Q < 0$ corresponding to the presence of heat absorption, and hence to a system cooling, to be determined from the system

$$\begin{aligned} A_n &= \frac{1}{\lambda n(n^2-1)} \sum_{k=0}^{n-1} k[(\alpha-\beta)k + \beta(2k-n)]A_k A_{n-k}, \quad n \geq 2, \\ s &= \sum_{n=1}^{\infty} A_n, \qquad 1 = \sum_{n=1}^{\infty} (-n\lambda)A_n. \end{aligned} \tag{3.3}$$

Making use of (3.3), for a given s, we can numerically determine the pair of unknowns (λ, A_1). On the other hand, to avoid the numerics, we propose the use of new coefficients a_n from

$$A_n = \lambda\kappa^n a_n, \tag{3.4}$$

with $\kappa = A_1/\lambda$ such that $a_1 = 1$.

With the help of (3.4), (3.3) can be rewritten as

$$
\begin{aligned}
a_0 &= 0,\\
a_1 &= 1,\\
a_n &= \frac{1}{n(n^2-1)} \sum_{k=0}^{n-1} k[(\alpha-\beta)k + \beta(2k-n)]a_k a_{n-k}, \quad n \geq 2,\\
\frac{s}{\lambda} &= \sum_{n=0}^{\infty} \kappa^n a_n,\\
\frac{1}{\lambda^2} &= -\sum_{n=0}^{\infty} n\kappa^n a_n. \qquad (3.5)
\end{aligned}
$$

Thus, for fixed values of α and β, since a_n's are all known, prescribing a value of κ so that the sum's in (3.5) are both convergent will produce λ and s from the last two equations in (3.5) as

$$
\begin{aligned}
\lambda &= \left(-\frac{1}{\sum_{n=0}^{\infty} n\kappa^n a_n}\right)^{1/2},\\
s &= \lambda \sum_{n=0}^{\infty} \kappa^n a_n. \qquad (3.6)
\end{aligned}
$$

It should be alerted that, if s is fixed in (3.5), then a numerical scheme must be employed to get κ and λ, which is not what we want here. We emphasize that expressions in (3.6) are exact formulas for the physical parameters as opposed to the numerical values existing in the literature. The valid region of κ may be obtained from a map by forcing the sums in (3.6) are convergent with simple exercising. This can be achieved up to any required degree of accuracy by taking as many terms as possible in the sum. In the more special values of $\alpha = \beta$ corresponding to $\Lambda = 1$, a closed-form solution can be found from (3.5) and (3.6) resulting in

$$
f(\eta) = s e^{\frac{\eta}{s}}, \qquad (3.7)
$$

valid for $s < 0, \quad \lambda = -1/s, \quad Q = -1/s^2$, which indicates that $Nu = -\theta'(0) = -f''(0) = -1/s$ is positive and hence an increased heat transfer from the wall is attained for the stream function of the form (3.7). Obviously, as the injection gets stronger, the rate of heat transfer will degrade.

3.2. $A_0 = f(\infty) \neq 0,\ A_1 \neq 0$

In this case, from Eq. (3.2) we anticipate that

$$A_0 = \frac{\lambda^2 + Q}{\alpha\lambda}, \tag{3.8}$$

and the rest of the coefficients can be found from

$$A_n = \frac{\lambda}{n(n-1)(n\lambda^2 - Q)} \times \sum_{k=0}^{n-1} k[(\alpha-\beta)k + \beta(2k-n)]A_k A_{n-k}, \quad n \geq 2. \tag{3.9}$$

With the help of the new scaled parameters $Q = \lambda^2\bar{Q}$, $s = \lambda\bar{s}$, $A_n = A_0\kappa^n a_n$ with $\kappa = A_1/A_0$, (3.9) and the boundary conditions are no longer explicitly relying upon the unknown coefficients λ and A_1, and hence we have

$$\begin{aligned} a_0 &= 1, \\ a_1 &= 1, \\ a_n &= \frac{1+\bar{Q}}{\alpha n(n-1)(n-\bar{Q})} \sum_{k=0}^{n-1} k[(\alpha-\beta)k + \beta(2k-n)]a_k a_{n-k}, \quad n \geq 2, \\ \bar{s} &= \frac{1+\bar{Q}}{\alpha} \sum_{n=0}^{\infty} \kappa^n a_n, \\ 1 &= -\lambda^2\left(\frac{1+\bar{Q}}{\alpha}\right) \sum_{n=0}^{\infty} n\kappa^n a_n. \end{aligned} \tag{3.10}$$

From the last two equations in (3.10), λ and $\bar{s}$ can be evaluated as

$$\begin{aligned} \lambda &= \left(-\frac{\alpha}{(1+\bar{Q})\sum_{n=0}^{\infty} n\kappa^n a_n}\right)^{1/2}, \\ \bar{s} &= \frac{1+\bar{Q}}{\alpha} \sum_{n=0}^{\infty} \kappa^n a_n. \end{aligned} \tag{3.11}$$

Therefore, for preassigned values of κ (and fixed $\bar{Q}$, α and β), $\bar{s}$ and λ from (3.11) can be found which are later on used to get s and Q and hence all the physical quantities are known. It is also noteworthy to draw attention that when $\alpha = \beta$, the exponent λ can be exactly worked out from (3.10) and (3.11)

$$\lambda = \frac{\alpha s \pm \sqrt{\alpha^2 s^2 + 4(\alpha - Q)}}{2}, \tag{3.12}$$

implying the existence of dual solutions for the suction case and unique solution for the injection, both depending on the heat generation/absorption parameter Q and α.

4. Results and discussions

We should in prior mention that our model collapse onto the well-documented one of [27], when $\alpha = \frac{\Lambda+1}{2}$ and $\beta = \Lambda$ in the absence of M, Nr, s and Q. To further validate the extracted model in the present investigation, and to justify the correctness of the exponential type solutions, Tables 1 and 2 are listed comparing the present outcomes (15 terms in the series) with those available in the open literature. Excellent agreement is anticipated in the Tables. We notice that the present model also covers the physical situation valid for the natural convection about a vertical cone formulated in [16], as inferred from Table 2.

Table 1. The heat transfer rates Nu at $M = Nr = s = Q = 0$ over a vertical plate.

	[28]	[6]	[7]	Present
$(\alpha = 1/2,\ \beta = 0.0)$	0.4437	0.4440	0.443885	0.443833
$(\alpha = 2/3,\ \beta = 1/3)$	0.6776	0.6788	0.677707	0.677648

Table 2. The heat transfer rates Nu at $M = Nr = s = Q = 0$ over a vertical cone.

	[29]	[16]	Present
$(\alpha = 3/2,\ \beta = 0.0)$	0.7685	0.7686	0.768742
$(\alpha = 7/4,\ \beta = 1/2)$	0.9896	0.9897	0.989621

Present solutions are discussed next separately for $A_0 \neq 0$ and $A_0 = 0$.

4.1. $A_0 \neq 0$

A few of the coefficients are displayed below from (3.10)

$$\begin{aligned}
a_0 &= 1,\\
a_1 &= 1,\\
a_2 &= -\frac{(\alpha-\beta)(1+\bar{Q})}{2\alpha(-2+\bar{Q})},\\
a_3 &= \frac{(5\alpha-4\beta)(\alpha-\beta)(1+\bar{Q})^2}{12\alpha^2(-3+\bar{Q})(-2+\bar{Q})},\\
a_4 &= -\frac{(\alpha-\beta)(1+\bar{Q})^3(-68\alpha^2+106\alpha\beta-42\beta^2+(31\alpha^2-47\alpha\beta+18\beta^2)\bar{Q})}{72\alpha^3(-2+\bar{Q})^2(12-7\bar{Q}+\bar{Q}^2)}.
\end{aligned} \tag{4.1}$$

When the infinity boundary condition is not vanishing, under the special circumstance $\alpha = \beta$, from (3.12) exponential solutions are restricted to

$$Q \leq \frac{1}{4}(4\alpha + s^2\alpha^2).$$

Moreover, there exists a critical pair (Q, λ) for the appearance of dual solutions which are computed as

$$(Q, \lambda) = \left(\frac{1}{4}(4\alpha + s^2\alpha^2), \frac{s\alpha}{2}\right), \tag{4.2}$$

which clearly points that in the presence of wall suction together with a positive value of α, dual solutions exist as also evident from Fig. 2. Such solutions are likely to appear only for the heat generation case. This scenario, of course may change if negative values of α are taken into account, which is also possible by the negative values of Λ.

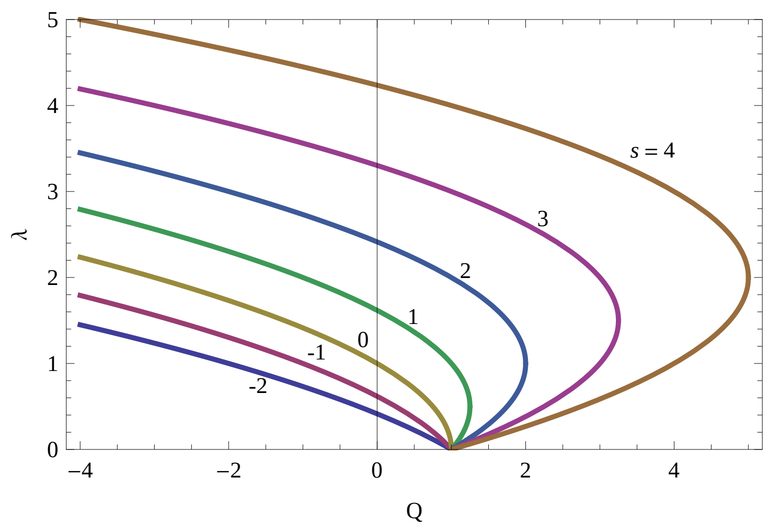

FIG. 2. Domain of the existence of purely exponential solutions when $\alpha = \beta = 1$.

Figures 3(a–c) reveal the effects of heat generation/absorption parameter $\bar{Q}$ on the domains of λ, $\bar{s}$ and κ as well as on the heat transfer rate Nu, when $\alpha = 2/3$ and $\beta = 1/3$. Hence, we can see that preassigning κ can produce all other physical parameters as demonstrated in the figures. We can also see the domain of the parameter κ yielding convergent series. A common observation from the figures is that when κ is larger, the exponent λ, the suction/injection parameter $\bar{s}$ and the Nusselt number Nu are also larger, in compliance with the

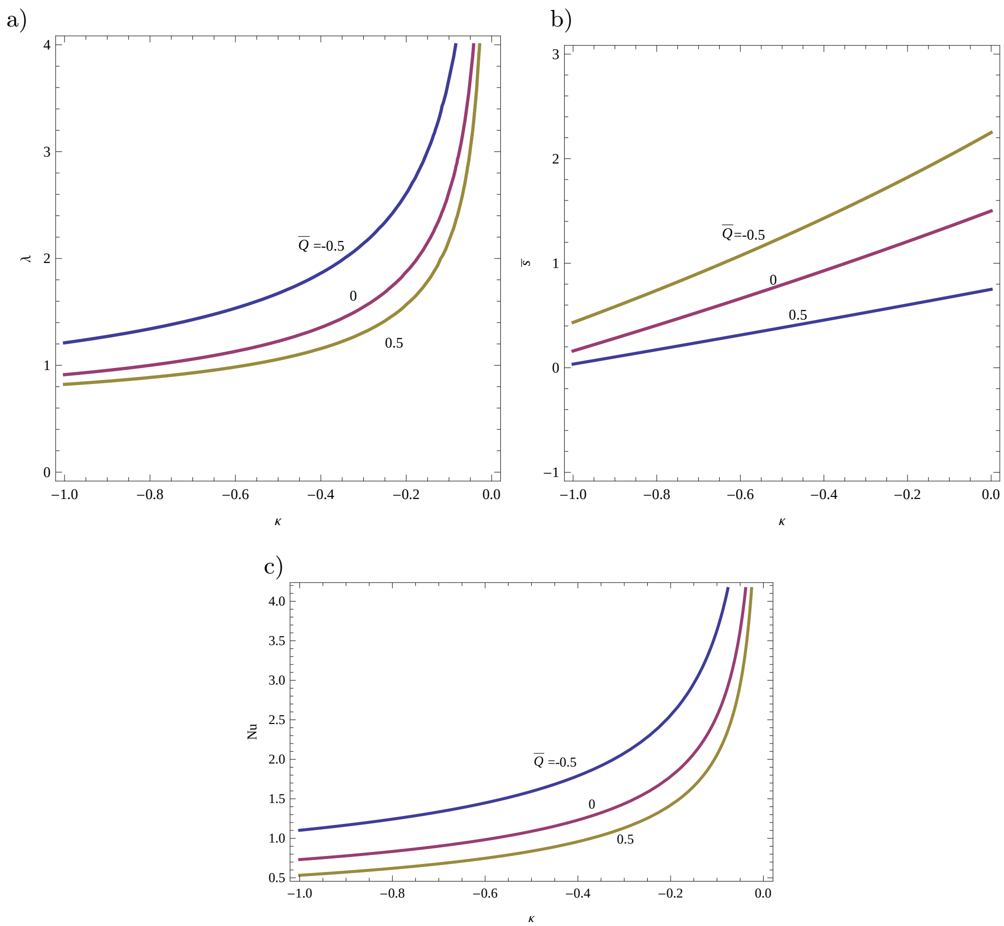

FIG. 3. The effects of $\bar{Q}$ on the physical parameters λ, $\bar{s}$, κ and Nusselt number.

physical expectations. From the figures it is easy to deduce the well-known result that the suction cools down the porous medium whereas the injection heats it up leading to poorer heat transfer rates. All these physical outcomes are connected with the exponent λ. In addition to this, absorbing the heat from the system with a negative $\bar{Q}$ results in better heat transfer rates as compared to the positive

Table 3. The values of λ, $\bar{s}$ and Nu at $\alpha = 2/3$, $\beta = 1/3$ and $\kappa = -1/2$.

$\bar{Q} = -1/2$	$\bar{Q} = 0$	$\bar{Q} = 1/2$
1.6731748403	1.2246910238	1.0563777443
0.3840527014	0.7932861455	1.2473565111
1.5940722585	1.0889155503	0.8375230026

values of $\bar{Q}$, as approved from the figures. For future reference, Table 3 tabulates some values of λ, $\bar{s}$ and Nu for three values of $\bar{Q}$ at the selected parameters $\alpha = 2/3$, $\beta = 1/3$ and $\kappa = -1/2$.

4.2. $A_0 = 0$

A few of the coefficients are displayed below from (3.5)

$$a_0 = 0,$$
$$a_1 = 1,$$
$$a_2 = \frac{\alpha-\beta}{6},$$

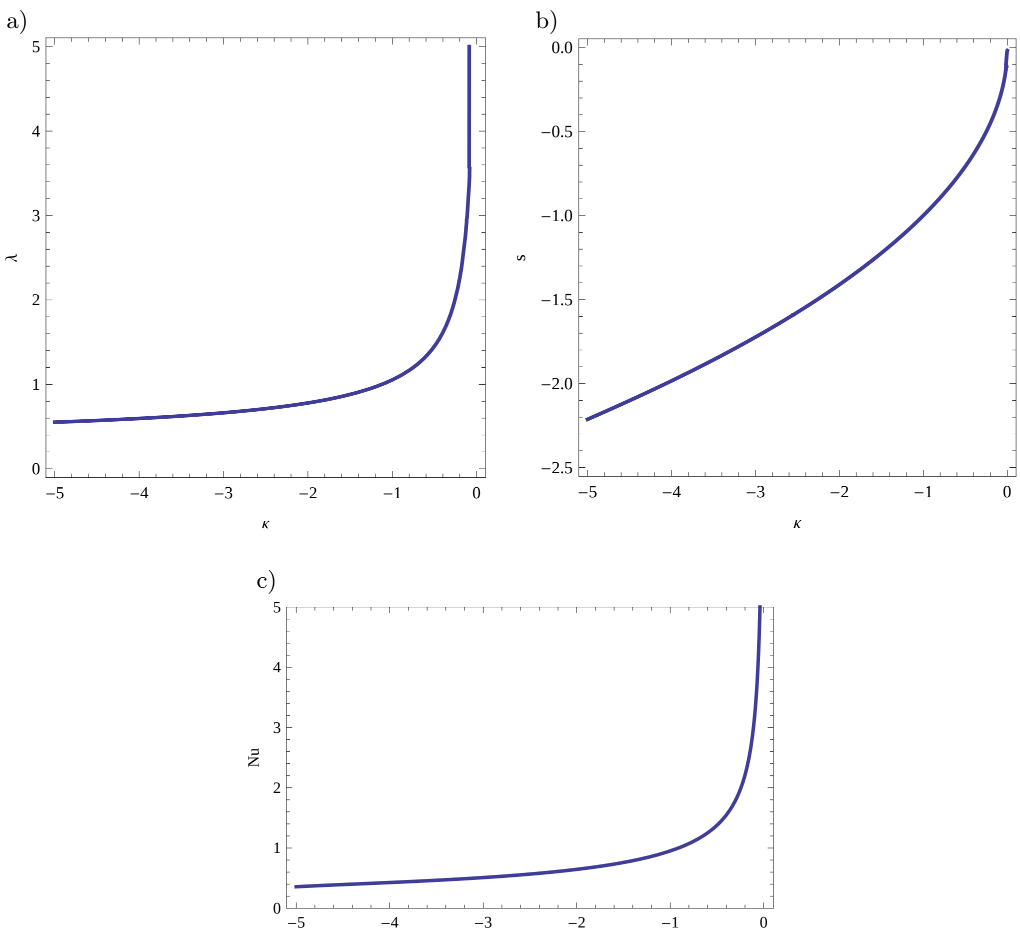

Fig. 4. The effects of κ on the physical parameters λ, s and Nusselt number.

$$a_3 = \frac{1}{144}(5\alpha-4\beta)(\alpha-\beta),$$
$$a_4 = \frac{(\alpha-\beta)(33\alpha^2-51\alpha\beta+20\beta^2)}{4320},$$
$$a_5 = \frac{(\alpha-\beta)(443\alpha^3-1008\alpha^2\beta+774\alpha\beta^2-200\beta^3)}{259200},$$
$$a_6 = \frac{(\alpha-\beta)(41861\alpha^4-125686\alpha^3\beta+142913\alpha^2\beta^2-72800\alpha\beta^3+14000\beta^4)}{108864000}. \tag{4.3}$$

Figures 4(a-c) reveal the effects of κ on the domains of λ, s as well as on the heat transfer rate Nu, when $\alpha = 2/3$ and $\beta = 1/3$. Similar behaviors are exhibited in Figs. 4(a-c). We should remark that $Q = -\lambda^2$ in this case. For future reference, Table 4 tabulates values of λ, s, Q and Nu at the selected parameters $\alpha = 2/3$, $\beta = 1/3$ for some κ.

Table 4. The values of λ, s and Nu at $\alpha = 2/3$, $\beta = 1/3$ for some κ.

	λ	s	Q	Nu
$\kappa = -2$	0.7797878688	−1.4106793076	−0.6080691204	0.6450007908
$\kappa = -1$	1.0533632712	−0.9993075844	−1.1095741812	0.9507880027
$\kappa = -1/2$	1.4527010507	−0.7069776589	−2.1103403428	1.3772752129
$\kappa = -1/4$	2.0274923962	−0.4999765437	−4.1107254167	1.9730705227

Finally, certain velocity and temperature profiles corresponding to Table 4 are displayed in Fig. 5. The graphs in the figure clearly exhibit the influences of physical parameters on the flow and temperature fields, in parallel to the physical intuition.

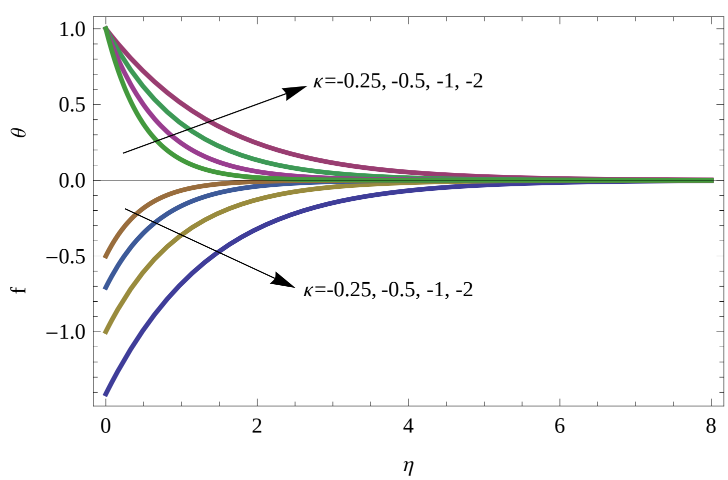

FIG. 5. Velocity and temperature distributions for some prescribed parameters, see Table 4 for the corresponding parameters.

5. Concluding remarks

The free convection flow occurring about a heated permeable vertically stretching surface placed in a porous medium is the main concern of the present work. The originality stems from the use of a temperature dependent internal heat generation or absorption flux, in place of the one which is preassigned in the previous studies in the literature. The analysis is conducted in the presence of volume radiative heat sources in the fluid while the system is permeated by a uniform magnetic field.

The present work is also differentiated from the literature in that the reduced nonlinear ordinary differential equation governing the physical phenomenon is treated analytically here. As a result, either closed-form exact solutions are obtained for some specific values of the physical parameters, otherwise solutions are sought in the purely exponential series form. In this case, an elegant algorithm is also proposed to determine the temperature distribution with an accuracy up to the desired decimal place without resorting to any numerical schemes.

Exact solutions are found, which are shown to be either dual or unique, depending on the governing parameters. To conclude, in the presence of a heat sink absorbing the temperature from the medium increases the rate of heat transfer from the wall, whereas a heat source will surely heat up the system under consideration, resulting in poorer heat transfer rates.

Since the present work generalizes the internal heat generation term suggested in [6], in future works, many publications pursuing that article may be reconsidered within the context of the present model. Finally, the presented data has the potentiality to be considered as a verification tool for the natural convection processes about more complex surfaces in higher dimensions.

References

1. D.B. Ingham, I. Pop, *Transfer Phenomena in Porous Media*, Pergamon, Oxford, 1998.

2. K. Vafai, *Handbook of Porous Media*, CCR Press Taylor & Francis Group LLC, Boca Raton, 2005.

3. D. Nield, A. Bejan, *Convection in Porous Media*, Springer Science, New York, 2006.

4. T.Y. Na, I. Pop, *Free convection flow past a vertical flat plate embedded in a saturated porous medium*, International Journal of Engineering Science, **21**, 517–526, 1983.

5. N. Beithou, K. Albayrak, A. Abdulmajeed, *Effects of porosity on the free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium*, Turkish Journal of Engineering and Environmental Science, **22**, 203–209, 1998.

6. A. Postelnicu, I. Pop, *Similarity solutions of free convection boundary layers over vertical and horizontal surfaces in porous media with internal heat generation*, International Communications in Heat and Mass Transfer, **26**, 8, 1183–1191, 1999.

7. T. Grosan, I. Pop, *Free convection over a vertical plate with variable wall temperature and internal heat generation in porous medium saturated with a non-Newtonian fluid*, Technische Mechanik, **21**, 313–318, 2001.

8. M.E. Ali, *The effect of lateral mass flux on the natural convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with internal heat generation*, International Journal of Thermal Sciences, **46**, 157–163, 2007.

9. J.H. Merkin, I. Pop, *Natural convection boundary-layer flow in a porous medium with temperature-dependent boundary conditions*, Transport in Porous Media, **85**, 397–414, 2010.

10. D. Srinivasacharya, G. Swamy Reddy, *Double diffusive natural convection in power-law fluid saturated porous medium with Soret and Dufour effects*, Journal of the Brazilian Society of Mechanical Sciences and Engineering, **4**, 5250–530, 2012.

11. D. Achemlal, M. Sriti, M.E. Haroui, M. Guedda, *Free convection modeling over a vertical flat plate embedded in saturated porous medium with a variable heat source and radiation flux*, World Journal of Modelling and Simulation, **9**, 163–172, 2013.

12. C. RamReddy, P.V.S.N. Murthy, A.J. Chamkha, A.M. Rashad, *Influence of viscous dissipation on free convection in a non-Darcy porous medium saturated with nanofluid in the presence of magnetic field*, The Open Transport Phenomena Journal, **5**, 20–29, 2013.

13. N.A.H. Haroun, *Double-diffusive convection flow in a porous medium saturated with a nanofluid*, Master Thesis, School of Mathematics, Statistics and Computer Science, Pietermaritzburg, 2014.

14. Y. Cao, X. Cui, *Natural convection of power law fluids in porous media with variable thermal and mass diffusivity*, International Journal of Heat and Technology, **33**, 85–90, 2015.

15. K. Javaherdeh, M.M. Nejad, M. Moslemi, *Natural convection heat and mass transfer in MHD fluid flow past a moving vertical plate with variable surface temperature and concentration in a porous medium*, Engineering Science and Technology, an International Journal, **18**, 423–431, 2015.

16. T. Grosan, A. Postelnicu, I. Pop, *Free convection boundary layer over a vertical cone in a non-Newtonian fluid saturated porous medium with internal heat generation*, Technische Mechanik, **24**, 91–104, 2004.

17. C.Y. Cheng, *Free convection heat transfer from a non-isothermal permeable cone with suction and temperature-dependent viscosity*, Journal of Applied Science and Engineering, **18**, 17–24, 2015.

18. S. Bagai, C. Nishad, *Free convection in a non-Newtonian fluid along a horizontal plate embedded in porous media with internal heat generation*, International Communications in Heat and Mass Transfer, **39**, 537–540, 2012.

19. M. Rabeti, *Free convection heat transfer of nanofluids from a horizontal plate embedded in a porous medium*, Advanced Energy: An International Journal, **1**, 23–32, 2014.

20. X. You, H. Xu, I. Pop, *Free convection along a convectively heated vertical flat sheet embedded in a saturated porous medium*, International Communications in Heat and Mass Transfer, **55**, 102–108, 2014.

21. M.A. SHEREMET, I. POP, R. NAZAR, *Natural convection in a square cavity filled with a porous medium saturated with a nanofluid using the thermal nonequilibrium model with a Tiwari and Das nanofluid model*, International Journal of Mechanical Sciences, **100**, 312–321, 2015.

22. H.W. WU, I.H. LIN, M.L. CHENG, *Heat transfer with natural convection of varying viscosity fluids inside porous media between vertically eccentric annuli*, International Journal of Heat and Mass Transfer, **94**, 145–155, 2016.

23. W. CHOI, R. OOKA, *Effect of natural convection on thermal response test conducted in saturated porous formation: Comparison of gravel-backfilled and cement-grouted borehole heat exchangers*, Renewable Energy, **96**, 891–903, 2016.

24. A.I. ALSABERY, H. SALEH, I. HASHIM, P. G. Siddheshwar, *Transient natural convection heat transfer in nanoliquid-saturated porous oblique cavity using thermal non-equilibrium model*, International Journal of Mechanical Sciences, **114**, 233–245, 2016.

25. D. NIELD, A. BEJAN, *Convection in Porous Media*, [2nd ed.], Springer, New York, 1998.

26. M. TURKYILMAZOGLU, *An analytical treatment for the exact solutions of MHD flow and heat over two-three dimensional deforming bodies*, International Journal of Heat and Mass Transfer, **90**, 781–789, 2015.

27. P. CHENG, W.J. MINKOWYCZ, *Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike*, Journal of Geophysical Research, **82**, 2040–2044, 1977.

28. H.T. CHEN, C.K. CHEN, *Free convection flows of non-Newtonian fluids along a vertical plated embedded in a porous medium*, Americal Society of Mechanical Engineers Journal of Heat Transfer, **110**, 257–259, 1988.

29. P. CHENG, T.T. LE, I. POP, *Natural convection of a Darcian fluid about a cone*, International Communications in Heat and Mass Transfer, **12**, 705–717, 1985.

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