Arch. Mech., **70**, 6, pp. 535–550, Warszawa 2018, DOI: 10.24423/aom.3009
SEVENTY YEARS OF THE ARCHIVES OF MECHANICS

Subtle structure of streamers under conditions resembling those of Transient Luminous Events

J. S. BŁĘCKI[1)], K. A. MIZERSKI[2)]

[1)] *Plasma Physics Department*
Space Research Centre
Polish Academy of Sciences
Bartycka 18A
00-716 Warszawa, Poland
e-mail: jblecki@cbk.waw.pl

[2)] *Department of Magnetism*
Institute of Geophysics
Polish Academy of Sciences
Księcia Janusza 64
01-452, Warszawa, Poland
e-mail: kamiz@igf.edu.pl

The paper presents a brief review of the observational facts related to plasma filamentation in astrophysics and the subtle structures of plasma in Transient Luminous Events (TLE's) and an analysis of the physical mechanism that could contribute to formation of filaments in plasma inside streamers. The values of physical parameters are assumed such as to resemble the physical conditions in streamers of the TLE's. Estimates of the typical spatial scales of these structures and temporal characteristics of filament formation are given. The analysis concerns a non-magnetic mechanism based on a form of non-relativistic dissipative instability and the electron-nitrogen collisional $^2\Pi_g$ resonance. It is argued that the influence of the magnetic field is negligible at the leading order at least up to the altitudes of about 65–70 km. Under the conditions related to those in plasma inside the TLE's, derived based on the current knowledge of physical parameters within the electric discharges, the identified dissipative-resonant instability is demonstrated to be the only/most vigorous linear instability developing in the system. It results in periodic plasma density distribution in the direction transverse to the electric field. The obtained time scales of the instability development are quick and proportional to the inverse of the ion-neutral collision frequency, $1/\nu_i$, whereas the proposed spatial scale of the density stripes/filaments is proportional to the electron temperature and inversely proportional to the speed of the discharge.

Key words: plasma instability, plasma filamentation, electric discharges, Transient Luminous Events.

1. Introduction

Plasma turbulence is one of the most significant features of space plasma. Furthermore, plasma also often exhibits a tendency to create filamentary structures. This is e.g. an evident general feature of magnetized plasma, when field aligned currents are formed (cf. [1]). Such filamentation can be seen by optical and X-ray observations of the solar chromosphere and corona, in the cometary tails and on the Earth in the auroras (cf. [1–6]). Magnetospheric plasma also exhibits a tendency to form subtle structures in the current flowing along magnetic field lines. To study this subtle, filamentary structure in situ, sensitive tools with high time resolution are necessary.

In the upper Earth's atmosphere some plasma structures are also present. These are formed through different mechanisms than those of magnetized plasma, within the so-called Transient Luminous Events (TLE's), i.e. electric discharges propagating between clouds and the ionosphere. The magnetic field shows only a weak influence on the dynamics of TLE's and hence magnetic mechanisms of filamentation, valid in the case of astrophysical plasma, cannot be directly applied to TLE's. However, the ground base telescopic observations also indicate a subtle structure of plasma there. We discuss the origin and dynamics of these structures within the scope of the unmagnetized plasma linear stability theory.

As described by Słomińska *et al.* [40]: "The Transient Luminous Events are bright, short-lived, atmospheric phenomena occurring above the clouds level, some of them reaching the ionosphere. The first suggestion of existence of that kind of phenomena was given by the Scottish physicist C.T.R. Wilson in 1924 [7], who observed that at a certain altitude above the clouds level the value of the electric field associated with the thunder cloud discharge equals the critical value of the electric field allowing for the conventional breakdown mechanism. This, in turn, leads to emission of light. First confirmed observations of the luminous phenomena above the clouds level date back to 1989. The experimental discoveries of one type of these events were made in 1989 by Otha Vaughan Jr. and Bernard Vonnegut and scientists from the University of Minnesota Robert Franz, Robert Nemzek and John Winckler (cf. [8] and [9]). These first detections were related to a phenomenon, which is now called the red sprite. The observation was performed during night time, and only in 1998 Mark Stanley and his collaborators from New Mexico Tech reported electromagnetic signatures indicative of a day-time sprite ([10, 11]). Since then, thousands of observations of different types of TLEs have been carried out from aircrafts, balloons, space stations and satellites (cf. e.g. [12]). Apart from red sprites, other types of TLE's – blue jets, elves, halos and trolls – were observed. Different types of TLE's are schematically shown in Fig. 1.

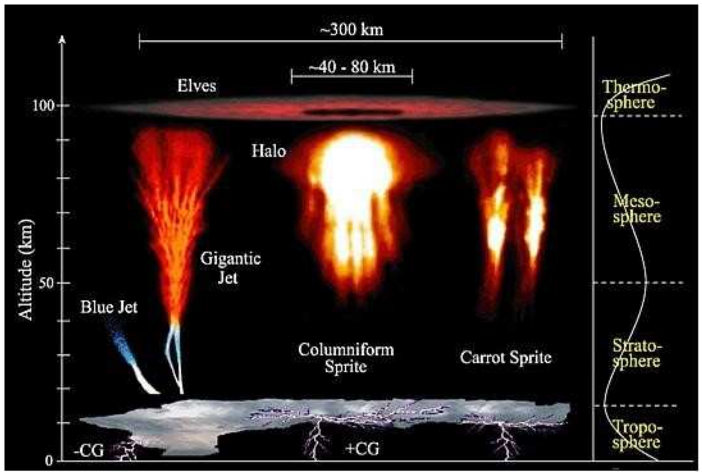

FIG. 1. A schematic overview of different types of the Transient Luminous Events [14].

According to all collected up to date observations, the TLE's are triggered by a significant electric discharge either inside a cloud or between the cloud and the Earth, which causes a strong electromagnetic field above the cloud system. The electromagnetic field is a source of different light phenomena, some of them classified as TLE's. It is estimated that the energy produced in the atmosphere by all types of TLE's is about 700 MJ per minute (cf. [13]). In the case of sprites and elves the positive or negative cloud to Earth discharges are a necessary condition for this type of TLE to occur. Therefore, a strong correlation between the frequency of the thunderstorms and the sprites and elves is observed but the mechanism responsible for triggering the TLE's is still not fully understood", e.g. the role of a runaway electron mechanism for air breakdown or the detailed relation between the TLE and its causative lightning is not clear (see review by [14]).

Since the first registration of TLE in 1989 regular observations of TLE's are performed using ground base and satellite cameras and telescopes. To resolve details high speed cameras with framing rates as high as tens or even hundred thousands frames per second are necessary. In high-speed images, sprite events typically contain a number of distinctly identifiable features, such as moving streamers, glow, and beads (cf. [12, 16–18]). Streamers are very dynamic features and only so high speed registration can give the details of their temporal development. Streamers are relatively small, typically less than a few hundreds of meters in diameter. GERKEN and INAN followed by MARSHALL and INAN have made sprite observations using a telescope and reported streamer widths as small as a few tens of meters in [19] and [20]. High-speed observations of sprite streamers show that they are frequently split into several smaller streamers. Overall, streamers appear to split into smaller and smaller streamers. Summarizing, the telescopic images have shown that the fine structure of sprites covers a wide range of spatial scales between about 15 and 1000 meters. Similar discharge/streamers development has been studied numerically and experimentally by [21–25] and theoretically e.g. by [26]. Here we study the dynamics of plasma within streamers and their subtle structure. We start from fundamental Boltzmann equations for ions and electrons and perform a stability analysis within the parameter range relevant to TLE's. Although nonlinear processes are definitely important for sprite propagation, as pointed out in the aforementioned works, we are not concerned here with the process of propagation of the streamer head. Instead we look at the inside of a streamer and find a type of simple linear instability, which may cause plasma filamentation inside it. The advantage is the simplicity of this mechanism, corresponding in this sense to the mechanisms of magnetic filamentation in auroras or Solar corona, which are also based on linear, although magnetic and thus entirely different instabilities. The aim of this paper is simply to check, whether such simple mechanisms of filament formation are allowed in plasma within streamers.

In the next section we study the physical conditions within the Transient Luminous Events and on neglecting the effects of the magnetic field, through analysis of the plasma Boltzmann equations we find a dominant instability mechanism of the dissipative-resonant type, which leads to plasma filamentation.

2. Dynamics of plasma within streamers in TLE's

2.1. Physical conditions within streamers

To study the stability of plasma in Transient Luminous Events (TLE's) we first identify the conditions, i.e. the parameters of the plasma state at which the electric discharges between clouds and the ionosphere take place. The numerical values of the electric field triggering the discharge E and the number densities of neutrals N_n, ions n_i and electrons n_e used below were taken from [14], whereas the propagation velocity u of the electric discharge (velocity of the leader) was provided by numerous authors, e.g. by [27, 28] and [14]. The electron temperature within the streamers, T_e is well-known to be of the order of several tens of thousands degrees Kelvin (cf. [14, 27, 29]), however, the ion temperature T_i is a poorly known quantity. Sentman *et al.* in [29] argue, that under conditions of sprite streamers involving strongly heated electrons up to energies of 1–10 eV the neutral molecules can reach temperatures of a few thousands K with the temperature of ions possibly even somewhat higher due to the presence of the electric field. Moreover, Sharma *et al.* in [30] provide experimental evidence of enhancement of an average ion temperature up to 1000-2000 K after a few sprite events. In accordance with the common knowledge about the ionospheric

D-region, as confirmed by many authors, e.g. [31, 32], the Nitrosonium NO^+ were assumed the dominant ions and nitrogen N_2 the dominant neutrals. Therefore, at the heights between about 40 km and about 80 km, where sprites and gigantic jets take place the most important parameters can be estimated as follows

$$T_i \approx 5 \times 10^3 \text{ [K]}, \quad T_e \approx 5 \times 10^4 \text{ [K]}, \tag{2.1a}$$

$$u \approx 10^2 \left[\frac{\text{km}}{\text{s}}\right], \quad v_{Ti} \approx 1.5 \left[\frac{\text{km}}{\text{s}}\right], \quad v_{Te} \approx 8 \times 10^2 \left[\frac{\text{km}}{\text{s}}\right], \tag{2.1b}$$

$$E \approx 10^3 - 10 \left[\frac{\text{V}}{\text{m}}\right], \quad B \approx 0.5 \times 10^{-4} \text{ [T]}, \tag{2.1c}$$

$$\Omega_i \approx 1.6 \times 10^2 \left[\frac{1}{\text{s}}\right], \quad \Omega_e \approx 8.8 \times 10^6 \left[\frac{1}{\text{s}}\right], \tag{2.1d}$$

$$\omega_{pi} \approx 1.5 \times 10^7 - 1.1 \times 10^5 \left[\frac{1}{\text{s}}\right], \quad \omega_{pe} \approx 3.9 \times 10^9 - 2.5 \times 10^7 \left[\frac{1}{\text{s}}\right], \tag{2.1e}$$

$$N_n \approx 2.7 \times 10^{25} \times \mathrm{e}^{-h/\delta} \left[\frac{1}{\text{m}^3}\right], \quad n_i = n_e \approx 10^{20} \times \mathrm{e}^{-2h/\delta} \left[\frac{1}{\text{m}^3}\right], \tag{2.1f}$$

$$\nu_i \approx 7.3 \times 10^7 - 3.7 \times 10^5 \left[\frac{1}{\text{s}}\right], \quad \nu_e \approx 5.4 \times 10^{10} - 2.7 \times 10^8 \left[\frac{1}{\text{s}}\right], \tag{2.1g}$$

where $v_{T\alpha} = \sqrt{k_B T_\alpha / m_\alpha}$ with $\alpha = i, e$ are the ion and electron thermal velocities, B is the Earth's magnetic field, $\Omega_\alpha = eB/m_\alpha$ are the ion and electron gyrofrequencies, $\omega_{p\alpha} = \sqrt{n_\alpha e^2/\epsilon_0 m_\alpha}$ are the ion and electron plasma frequencies, h is the height above the Earth's surface and the parameter $\delta = 8$ km. The following convention is used throughout the paper for height dependent parameters – for a range of values of a given parameter, the first value always corresponds to low altitudes, i.e. 40 km and the second to high altitudes, i.e. 80 km. The duration time of TLE's is $\tau \approx 10$–100 ms. The measured width of streamers varies between $l \approx 100 - 1$ m (see e.g. [16], [19]).

The TLE's typically take place in the height range 40 km–80 km where atmosphere is only weakly ionized, therefore we can neglect the effects of collisions between ions and electrons and include only the collisions with neutrals. In such a case the collisional frequency is $\nu_\alpha = N_n \int_0^\infty \sigma_\alpha(v) v f_\alpha \mathrm{d}^3 p$, where f_α is the α-species distribution function. The collisional frequency, very roughly, by the order of magnitude can be estimated with $\nu_\alpha = v_{T\alpha} \sigma_\alpha N_n$, where σ_α is the cross section for collisions between species of type α and neutral, mainly nitrogen, molecules (which typically depends on the collision energy); this estimate was used in (2.1g). In the case of ions-neutrals collisions the cross section σ_i is provided by $\sigma_i \approx \pi(a_{\mathrm{N}_2} + a_{\mathrm{NO}^+})^2 \approx 24.45 \times 10^{-20}$ m^2, with $a_{\mathrm{N}_2} \approx 1.42 \times 10^{-10}$ m being the radius of the nitrogen, N_2, molecule (sum of two covalent radii of nitrogen atoms) and $a_{\mathrm{NO}^+} \approx 1.37 \times 10^{-10}$ m being the radius of the nitrosonium,

NO^+, molecule (sum of two covalent radii of nitrogen and oxygen atoms). For electrons-neutrals collisions we may use a little more precise model. Itikawa in [33] provides a compilation of numerous experimental results for cross section for collisions between electrons and nitrogen molecules. Hence based on the data, for TLE conditions, i.e. electrons with temperatures $T_e \approx 10^4$–10^5 K, thus energies $k_B T_e \approx 1 - 10$ eV the cross section falls in the region of the ${}^2\Pi_g$ resonance (cf. [33–36]) and takes values $\sigma_e \approx 10 - 34 \times 10^{-20}$ m^2.

The above parameter values allow us to formulate a first crude approximation

$$\omega_{pe} \gg \Omega_e, \quad \omega_{pi} \gg \Omega_i, \tag{2.2}$$

and also

$$\frac{uB}{E} \approx 5 \times 10^{-3} - 0.5 < 1. \tag{2.3}$$

In anticipation of the results we can also write, that the growth rate of the dissipative-resonant instability leading to formation of the filamented subtle structure of TLE's satisfies $\omega/\Omega_e \sim \nu_i/\Omega_e \sim 10 - 10^{-1}$ and $\omega/\Omega_i \sim \nu_i/\Omega_i \sim 10^6$-$10^3 \gg 1$. This allows us to assume, that the plasma is weakly magnetised and proceed with neglection of the magnetic fields influence on the stability properties. We note, however, that at high altitudes (say $h \gtrsim 70$ km) the ratios $uB/E \sim 0.5$, $\omega/\Omega_e \sim 10^{-1}$ indicate, that the effect of the inclined magnetic field may, in fact, play an important role in the stability of plasma in TLE's; moreover the rather small ratio $k^2 v_{Te}^2/\Omega_e^2 \sim 3 \times (10^{-1} - 10^{-5})$ seems to confirm that. Nevertheless, for simplicity and clarity of the analysis it seems reasonable at the first stage, to neglect the magnetic field, thus in the present analysis the magnetic effect is entirely neglected. Moreover, since $N_n \gg n_i$ we assume the plasma to be weakly ionised.

Hence we study the stability of electrons propagating at a constant velocity u in non-magnetised, weakly ionised plasma in a constant external electric field E. As it will be useful in the following analysis, based on the values given in (2.1a)–(2.1g) we calculate the following non-dimensional parameters

$$\frac{\omega_{pe}}{kv_{Te}} \approx 4.5 \times 10^4 - 3.1, \quad \frac{\omega_{pi}}{kv_{Ti}} \approx 4.8 \times 10^5 - 35, \tag{2.4a}$$

$$\frac{\nu_e}{kv_{Te}} \approx 1.1 \times 10^5 - 5.4 \times 10^2, \quad \frac{\nu_i}{kv_{Ti}} \approx 7.7 \times 10^4 - 3.9 \times 10^2, \tag{2.4b}$$

where $k = 2\pi/l$ and we assume $l \approx 100 - 1$ m. Therefore in the majority of the analysed regions all the above non-dimensional parameters can be assumed large, i.e.

$$\frac{\omega_{pe}}{kv_{Te}} \gg 1, \quad \frac{\omega_{pi}}{kv_{Ti}} \gg 1, \quad \frac{\nu_e}{kv_{Te}} \gg 1, \quad \frac{\nu_i}{kv_{Ti}} \gg 1. \tag{2.5}$$

Furthermore, we calculate

$$(2.6)\quad \frac{\nu_i}{\nu_e} \approx 1.35 \times 10^{-3} \ll 1, \quad \frac{\omega_{pi}}{\omega_{pe}} \approx 4 \times 10^{-3} \ll 1, \quad \frac{v_{Ti}}{v_{Te}} \approx 1.9 \times 10^{-3} \ll 1.$$

2.2. Boltzmann equations

The Boltzmann equations for the electron and ion probability distribution functions, under the standard Bhatnagar–Gross–Krook (BGK) approximation for elastic particle collisions take the form

$$(2.7a)\quad \frac{\partial f_e}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} f_e - \frac{e}{m_e}(\mathbf{E} + \mathbf{E}' + \mathbf{v} \times \mathbf{B}') \cdot \nabla_{\mathbf{v}} f_e = -\nu_e (f_e - n_e \varPhi_e),$$

$$(2.7b)\quad \frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} f_i + \frac{e}{m_i}(\mathbf{E} + \mathbf{E}' + \mathbf{v} \times \mathbf{B}') \cdot \nabla_{\mathbf{v}} f_i = -\nu_i (f_i - n_i \varPhi_i),$$

where the subscripts 'e' and 'i' correspond to electron and ion variables respectively, $\mathbf{E} = E\hat{\mathbf{e}}_z$ is the external electric field, $\mathbf{E}'$ and $\mathbf{B}'$ are the small perturbation electric and magnetic fields respectively and $f_\alpha(\mathbf{v}, \mathbf{r}, t)\mathrm{d}\mathbf{v}\mathrm{d}\mathbf{r}$ denotes the number of particles of type α at time t in the phase space infinitesimal volume $\mathrm{d}\mathbf{v}\mathrm{d}\mathbf{r}$, so that $n_\alpha(\mathbf{r}, t) = \int f_\alpha(\mathbf{v}, \mathbf{r}, t)\mathrm{d}\mathbf{v}$ and $n_\alpha(\mathbf{r}, t)\langle\mathbf{v}\rangle_\alpha(\mathbf{r}, t) = \int \mathbf{v} f_\alpha(\mathbf{v}, \mathbf{r}, t)\mathrm{d}\mathbf{v}$. The functions $\varPhi_e$ and $\varPhi_i$ are the local Maxwellians for both types of species,

$$(2.8)\quad \varPhi_\alpha = \frac{1}{(2\pi m_\alpha T_{\alpha n})^{3/2}} \exp\left[-\frac{m_\alpha(\mathbf{v} - \langle\mathbf{v}\rangle_\alpha)^2}{2T_{\alpha n}}\right],$$

and since $m_e \ll m_n$ and we assume $T_n \approx T_i$,

$$(2.9)\quad T_{\alpha n} = \frac{m_\alpha T_n + m_n T_\alpha}{m_\alpha + m_n} \approx T_\alpha,$$

where the subscript 'n' refers to neutrals. The introduction of local Maxwellians corresponds to the so-called conventional approach with neglection of the runaway electron process, often undertaken in the description of TLE's as pointed out e.g. by [14] however, as the authors also point out the runaway electron process cannot be ruled out here, but its role is uncertain as of yet.

2.3. Linear stability of streamers

We briefly report the results of linear stability analysis, which allowed to demonstrate, that the plasma in streamers is likely to be linearly stable and its dynamics due to nonlinear effects, although not all possible limits of parameter values could be analytically resolved. Assuming small perturbations of the form $\delta f_\alpha \sim \exp[\mathrm{i}(\mathbf{k} \cdot \mathbf{x} - \omega t)]$ and including the effect of the constant electric field through applying the Gallilean transform to a system moving with uniform velocity $\mathbf{u} = u\hat{\mathbf{e}}_z$ (cf. the seminal book by Alexandrov, [p. 159, 37]) we get the following expression for the dielectric tensor ϵ_{ij} in the moving frame

$$\epsilon_{ij} = \delta_{ij} + \left(\delta_{i\mu} + \frac{k_\mu u_i}{\omega}\right)(\epsilon'^{e}_{\mu\nu} - \delta_{\mu\nu})\left(\delta_{\nu j} + \frac{k_\nu u_j}{\omega}\right) + \delta_{i\mu}(\epsilon'^{i}_{\mu\nu} - \delta_{\mu\nu})\delta_{\nu j}. \tag{2.10}$$

In the above $\epsilon'^{\alpha}_{\mu\nu}$ denotes the contribution from the particles of species α to the dielectric tensor $\epsilon'_{\mu\nu}$ of the weakly ionised collisional plasma in a laboratory frame provided in [Eq. (4.5.8), p. 92, 37], i.e.

$$\epsilon'_{ij} = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\epsilon'^{tr} + \frac{k_i k_j}{k^2}\epsilon'^{lo}, \tag{2.11a}$$

$$\epsilon'^{lo} = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{k^2 v_{T\alpha}^2} \frac{1 - I_+\left(\frac{\omega + \mathrm{i}\nu_\alpha}{k v_{T\alpha}}\right)}{1 - \frac{\mathrm{i}\nu_\alpha}{\omega + \mathrm{i}\nu_\alpha} I_+\left(\frac{\omega + \mathrm{i}\nu_\alpha}{k v_{T\alpha}}\right)}, \tag{2.11b}$$

$$\epsilon'^{tr} = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega(\omega + \mathrm{i}\nu_\alpha)} I_+\left(\frac{\omega + \mathrm{i}\nu_\alpha}{k v_{T\alpha}}\right), \tag{2.11c}$$

where $I_+(x) = x\mathrm{e}^{-x^2/2}\int_{\mathrm{i}\infty}^{x} \mathrm{e}^{s^2/2}\mathrm{d}s$ is the plasma dispersion function.

First we consider the longitudinal oscillations, with the phasor $\exp[\mathrm{i}(\mathbf{k}\cdot\mathbf{x} - \omega t)]$. Therefore the general dispersion relation takes the form

$$1 + \frac{\omega_{pe}^2}{k^2 v_{Te}^2} \frac{1 - I_+\left(\frac{\omega - \mathbf{k}\cdot\mathbf{u} + \mathrm{i}\nu_e}{k v_{Te}}\right)}{1 - \frac{\mathrm{i}\nu_e}{\omega - \mathbf{k}\cdot\mathbf{u} + \mathrm{i}\nu_e} I_+\left(\frac{\omega - \mathbf{k}\cdot\mathbf{u} + \mathrm{i}\nu_e}{k v_{Te}}\right)} + \frac{\omega_{pi}^2}{k^2 v_{Ti}^2} \frac{1 - I_+\left(\frac{\omega + \mathrm{i}\nu_i}{k v_{Ti}}\right)}{1 - \frac{\mathrm{i}\nu_i}{\omega + \mathrm{i}\nu_i} I_+\left(\frac{\omega + \mathrm{i}\nu_i}{k v_{Ti}}\right)} = 0, \tag{2.12}$$

The following asymptotic formulae for the plasma dispersion function can be easily obtained

$$I_+(x) \approx -\mathrm{i}\sqrt{\frac{\pi}{2}}x + x^2 + \mathrm{i}\frac{1}{4}\sqrt{\frac{\pi}{2}}x^3 + \ldots, \quad \text{for } |x| \ll 1, \tag{2.13a}$$

$$I_+(x) \approx 1 + \frac{1}{x^2} + \frac{3}{x^4} + \cdots - \mathrm{i}\sqrt{\frac{\pi}{2}}x\mathrm{e}^{-x^2/2} \quad \text{for } |x| \gg 1 \text{ and } -\pi/4 < \arg x < 5\pi/4, \tag{2.13b}$$

$$I_+(x) \approx -\mathrm{i}\sqrt{2\pi}x\mathrm{e}^{-x^2/2} \quad \text{for } |x| \gg 1 \text{ and } |\Im\mathfrak{m}x| \gg |\Re\mathfrak{e}x|. \tag{2.13c}$$

Furthermore, since the observed filamentation of plasma within the TLE discharges is predominantly transverse we assume perpendicularity of the propagation velocity and the perturbation wave-vector, $\mathbf{k}\cdot\mathbf{u} = 0$. There are three possible types of asymptotic limits allowing for analytical progress

$$1^0)\ \left|\frac{\mathfrak{Im}\,\omega+\nu_e}{kv_{Te}}\right| \gg \left|\frac{\mathfrak{Re}\,\omega}{kv_{Te}}\right|, \qquad \left|\frac{\mathfrak{Im}\,\omega+\nu_i}{kv_{Ti}}\right| \gg \left|\frac{\mathfrak{Re}\,\omega}{kv_{Ti}}\right|, \tag{2.14}$$

$$2^0)\ \left|\frac{\mathfrak{Im}\,\omega+\nu_e}{kv_{Te}}\right| \gg \left|\frac{\mathfrak{Re}\,\omega}{kv_{Te}}\right|, \qquad \left|\frac{\mathfrak{Im}\,\omega+\nu_i}{kv_{Ti}}\right| \ll \left|\frac{\mathfrak{Re}\,\omega}{kv_{Ti}}\right|, \tag{2.15}$$

$$3^0)\ \left|\frac{\mathfrak{Im}\,\omega+\nu_e}{kv_{Te}}\right| \ll \left|\frac{\mathfrak{Re}\,\omega}{kv_{Te}}\right|, \qquad \left|\frac{\mathfrak{Im}\,\omega+\nu_i}{kv_{Ti}}\right| \ll \left|\frac{\mathfrak{Re}\,\omega}{kv_{Ti}}\right|. \tag{2.16}$$

However, using formulae (2.13a)–(2.13c) we can calculate the root with the greatest growth rate, $\mathfrak{Im}\,\omega$, which in case 1^0) gives $\mathfrak{Im}\,\omega \approx -\nu_i(1+v_{Ti}^2\omega_{pe}^2/v_{Te}^2\omega_{pi}^2)$, in case 2^0) gives $\mathfrak{Im}\,\omega \approx -\nu_i/2$ and in case 3^0) $\mathfrak{Im}\,\omega \approx -\nu_i(1+\omega_{pe}^2/\nu_i\nu_e)/2$, hence all the three cases are stable. This means, that longitudinal oscillations do not lead to an instability.

2.4. Transverse oscillations

The general dispersion relation for transverse oscillations, analogous to (2.12), in the studied case with $\mathbf{k}\cdot\mathbf{u}=0$ takes the following form (cf. Eqs. (2.10) and (2.11a-c))

$$\frac{k^2c^2}{\omega^2} - 1 + \frac{\omega_{pe}^2}{\omega(\omega+\mathrm{i}\nu_e)} I_+\left(\frac{\omega+\mathrm{i}\nu_e}{kv_{Te}}\right) + \frac{\omega_{pi}^2}{\omega(\omega+\mathrm{i}\nu_i)} I_+\left(\frac{\omega+\mathrm{i}\nu_i}{kv_{Ti}}\right) - \frac{k^2u^2}{2\omega^2}\frac{\omega_{pe}^2}{k^2v_{Te}^2}\frac{1-I_+\left(\frac{\omega+\mathrm{i}\nu_e}{kv_{Te}}\right)}{1-\frac{\mathrm{i}\nu_e}{\omega+\mathrm{i}\nu_e}I_+\left(\frac{\omega+\mathrm{i}\nu_e}{kv_{Te}}\right)} = 0. \tag{2.17}$$

Similar asymptotic limits as in the case of longitudinal oscillations, given in (2.14)–(2.16) can be considered here. The straightforward calculation of case 2^0) gives the maximal growth rate $\mathfrak{Im}\,\omega \approx -\nu_e$, which means that in this case the system is stable. Case 3^0) is slightly more complicated, however, within the considered asymptotic limits it can be shown, that the maximal growth rate in this case satisfies $\mathfrak{Im}\,\omega \ll \nu_i$. The only interesting limit that remains is given by the case 1^0) relations in (2.14), which, as it will turn out leads to $\mathfrak{Im}\,\omega \approx 0.4\nu_i$. Let us now consider case 1^0) in detail. Under the assumptions (2.14) the dispersion relation (2.17) can be greatly simplified to

$$0 = -\mathrm{i}\sqrt{\frac{\pi}{2}}\left[\frac{\omega_{pe}^2}{\omega kv_{Te}}\mathrm{e}^{-(\omega+\mathrm{i}\nu_e)^2/2k^2v_{Te}^2} + \frac{\omega_{pi}^2}{\omega kv_{Ti}}\mathrm{e}^{-(\omega+\mathrm{i}\nu_i)^2/2k^2v_{Ti}^2}\right] + \frac{k^2c^2}{\omega^2} + \mathrm{i}\frac{k^2u^2}{2\omega^2}\frac{\omega_{pe}^2}{k^2v_{Te}^2}\frac{\omega+\mathrm{i}\nu_e}{\nu_e} + \frac{\omega_{pe}^2}{\omega(\omega+\mathrm{i}\nu_e)} + \frac{\omega_{pi}^2}{\omega(\omega+\mathrm{i}\nu_i)} - 1. \tag{2.18}$$

Taking into account (2.5) we see, that the exponential terms clearly dominate and hence we may approximate the above dispersion relation by

$$\frac{\omega_{pe}^2}{v_{Te}}\mathrm{e}^{-(\omega+\mathrm{i}\nu_e)^2/2k^2v_{Te}^2}+\frac{\omega_{pi}^2}{v_{Ti}}\mathrm{e}^{-(\omega+\mathrm{i}\nu_i)^2/2k^2v_{Ti}^2}=0, \tag{2.19}$$

which leads to the following equation for ω, at leading order,

$$\omega^2\left(\frac{1}{k^2v_{Ti}^2}-\frac{1}{k^2v_{Te}^2}\right)+2\mathrm{i}\omega\left(\frac{\nu_i}{k^2v_{Ti}^2}-\frac{\nu_e}{k^2v_{Te}^2}\right)+\frac{\nu_e^2}{k^2v_{Te}^2}-\frac{\nu_i^2}{k^2v_{Ti}^2}\approx 0, \tag{2.20}$$

the solutions of which can be expressed in the form

$$\omega=\pm\mathrm{i}\nu_i\frac{\frac{v_{Ti}}{v_{Te}}\frac{\nu_e}{\nu_i}\mp 1}{1\mp\frac{v_{Ti}}{v_{Te}}}. \tag{2.21}$$

The only complex growth rate with a positive imaginary part at the leading order can be approximated by

$$\omega\approx\mathrm{i}\nu_i\left(\frac{v_{Ti}\nu_e}{v_{Te}\nu_i}-1\right)\equiv\mathrm{i}\nu_i(\chi-1),\quad\text{where}\quad\chi=\frac{v_{Ti}\nu_e}{v_{Te}\nu_i}, \tag{2.22}$$

which is, indeed, positive as long as $\chi > 1$. We emphasize, that the growth rate crucially depends on the dissipative collisional process and can only be positive, when χ, which very roughly can be estimated by the ratio of collisional cross-sections σ_e/σ_i is greater than one. This is possible only because the conditions in streamers of the TLE's, under the action of the strong electrical field E, which accelerates the electrons, correspond to the $^2\Pi_g$ resonance, greatly increasing the electrons-neutrals collisional cross-section σ_e. Note, that in the absence of the electric field the electron energies are necessarily lower and the $^2\Pi_g$ resonance is absent as well, implying $\mathfrak{Im}\,\omega < 0$, thus stability. Therefore the diffusive process is essential for triggering the instability and its development (a situation, when instability develops only in the presence of diffusive effects has also been reported in fluid mechanics and diffusive instabilities, e.g. in thin current sheets, are known).

The above result (2.22) is independent of the wave vector $\mathbf{k}$. Further analysis of the dispersion relation (2.18) by introducing $\omega=\mathrm{i}\nu_i(\chi-1)/(1-v_{Ti}/v_{Te})+\delta\omega(k)$, with $\delta\omega(k)\ll\omega$ leads to

$$\begin{aligned}\mathfrak{Im}\,[\delta\omega](k)\approx{}&\frac{k^2v_{Te}^2}{\nu_e^2}\chi\nu_i\Bigg\{\ln\left(\frac{\omega_{pe}^2v_{Ti}}{\omega_{pi}^2v_{Te}}\right)\\&-\sqrt{\frac{1}{2\pi}}\frac{1}{(\chi-1)}\frac{kv_{Ti}}{\nu_i}\left(\frac{u}{v_{Te}}\right)^2\left(\frac{\omega_{pe}}{\omega_{pi}}\right)^2\left[\frac{2k^2c^2v_{Te}^2}{u^2\omega_{pe}^2}-1\right]\mathrm{e}^{-\chi^2\nu_i^2/2k^2v_{Ti}^2}\Bigg\},\end{aligned} \tag{2.23}$$

which can be expressed in the form

$$\mathfrak{Im}\,[\delta\omega](k) \approx \nu_i \frac{\chi^2}{\chi - 1} K^2 [\kappa - \xi K (K^2 - K_0^2) \mathrm{e}^{1/2K^2}], \tag{2.24}$$

where

$$K = \frac{k v_{Te}}{\nu_e}, \qquad K_0 = \frac{1}{\sqrt{2}} \frac{u \omega_{pe}}{c \nu_e}, \tag{2.25a}$$

$$\kappa = \frac{\chi - 1}{\chi} \ln\left(\frac{\omega_{pe}^2 v_{Ti}}{\omega_{pi}^2 v_{Te}}\right), \qquad \xi = \sqrt{\frac{2}{\pi}} \frac{c^2 \nu_e^2}{v_{Te}^2 \omega_{pi}^2}. \tag{2.25b}$$

Note, that according to (2.5) $|K| \ll 1$, thus indeed $\delta\omega(k) \ll \omega$. At the altitude of $h = 60$ km,

$$K_0 \approx 2.03 \times 10^{-5}, \quad \kappa \approx 1.27, \quad \xi \approx 7.4 \times 10^{11}. \tag{2.26}$$

The consistency of the perturbational approach (at the leading and higher orders) requires, that the growth rate must not depend on the sign of the wave vector K of the perturbations, which is suggested by the second term in square brackets in the formula (2.24) and hence this entire term must vanish. This implies

$$K \approx \pm K_0, \tag{2.27a}$$

$$\mathfrak{Im}\,[\delta\omega](k) \approx \nu_i \frac{\kappa \chi^2}{\chi - 1} K_0^2. \tag{2.27b}$$

It follows from (2.27b), that the growth rate $\mathfrak{Im}\,[\omega]$ increases with κ and K_0 at constant ξ. The K_0 dependence is particularly interesting, since it is the only parameter dependent on speed of the electric discharge, u, hence the instability is more vigorous, if the discharge is faster.

An estimate of the wavelength of the transverse oscillations, i.e. of the thickness of filaments is provided by formula (2.27a) (similar as in the case of ion-acoustic instability; cf. [38]),

$$\lambda \approx \frac{2\pi}{K_0} \frac{v_{Te}}{\nu_e} = 2^{3/2} \pi \frac{c v_{Te}}{u \omega_{pe}} \approx 5 - 850 \text{ m}, \tag{2.28}$$

(and about 70 m at altitude $h = 60$ km) which is similar or less than the thickness of the streamer. In that way the thickness of filaments is proportional to the thermal velocity of electrons, i.e. increases with the electron temperature and inversely proportional to the speed of the discharge.

2.5. Discussion of the results

A precise estimate of the growth rate is difficult, since the electron and ion collision frequencies are poorly known. The equation $\nu_\alpha = N_n \int_0^\infty \sigma_\alpha(v) v f_\alpha \mathrm{d}^3p$, allows for a crude estimate $\nu_\alpha \approx N_n \sigma_\alpha \tilde{v}_\alpha$, where $\tilde{v}_\alpha$ is the averaged velocity magnitude of α species. Even a more crude approach leads to a substitution of $v_{T\alpha}$ in place of $\tilde{v}_\alpha$, which allows to calculate $\omega \approx \mathrm{i}\nu_i(\sigma_e/\sigma_i - 1)$. Using also very rough estimates of the cross sections provided at the beginning of Section 2, i.e. $\sigma_i \approx 24.45 \times 10^{-20}$ m^2 and $\sigma_e \approx 34 \times 10^{-20}$ m^2 the maximal growth rate can be estimated by $\omega \approx 0.4\mathrm{i}\nu_i$. Let us stress at this point the fact, that the electron energies within the TLE's, $1-10$ eV, correspond very well to the energy range for the electron-nitrogen collisional ${}^2\Pi_g$ resonance, leading to an increase of the electron-nitrogen collisional cross section by roughly a factor of 3 and even greater, as depicted in [33, Fig. 2] (data taken also from [34] and [35]). This means that the $e^- - N_2$ collisional ${}^2\Pi_g$ resonance is fundamental for the studied instability to develop, since in its absence the cross-section σ_e would be smaller than the ions-neutrals collisional cross section, leading to $\mathfrak{Im}\,[\omega] < 0$ and hence to decay of perturbations and stability.

Furthermore, using a somewhat more precise estimate of the collisional frequencies, $\nu_\alpha \approx N_n \sigma_\alpha \tilde{v}_\alpha$, leads to $\omega \approx \mathrm{i}\nu_i[(v_{Ti}/\tilde{v}_i)(\tilde{v}_e/v_{Te})(\sigma_e/\sigma_i) - 1]$, and under the conditions in TLEs $(v_{Ti}/\tilde{v}_i)(\tilde{v}_e/v_{Te}) > 1$, so that the growth rate is even more positive. The reason for the factor $(\tilde{v}_e/v_{Te})$ being greater than unity is obvious since $(\int v f_e \mathrm{d}\mathbf{v})^2 \geq \int (\mathbf{v} - \langle \mathbf{v} \rangle_e)^2 f_e \mathrm{d}\mathbf{v}$, with $v = \sqrt{(v_x^2 + v_y^2 + v_z^2)} > 0$. This also means that $(v_{Ti}/\tilde{v}_i) < 1$, however, under current assumptions relating to negative discharges the mean ion velocity is zero, $\langle v \rangle_i \approx 0$ whereas for electrons $\langle \mathbf{v} \rangle_e \approx u\hat{\mathbf{e}_z}$ with $u \sim 10^2$ km/s of the same order as the thermal electron velocity, thus the ratios have to satisfy $(\tilde{v}_i/v_{Ti}) < (\tilde{v}_e/v_{Te})$.

On the other hand, in the case of electrons it is important to use the full expression $\nu_e = N_n \int_0^\infty \sigma_e(v) v f_e \mathrm{d}^3p$, since both σ_e and f_e are expected to depend significantly on v and moreover the cross section σ_i for ions-neutrals collisions also requires more precise estimates. The latter can be achieved with the aid of the Sutherland potential (Maxwell's attracting potential $-\mathrm{Const}/r^4$ with a hard core of radius r_m; cf. [39, p. 108]), which provides a simple model for interactions of an ion with a neutral molecule polarized by the ion. The polarizabilities, which determine the strength of interactions are also provided by [39], hence the cross section can be simply calculated from basic principles of classical mechanics, namely that $\sigma_i = \pi b^2$, where b is calculated from an implicit relation

$$\frac{\pi}{2} = \int_0^{b/r_m} \frac{\mathrm{d}r}{\sqrt{1 - r^2 - \left(\frac{rR}{b}\right)^4}}, \tag{2.29}$$

with $r_m = 2.79 \times 10^{-10}$ m denoting the sum of covalent radii of N_2 and NO^+ molecules; in the above $R = (\mathrm{Const}/\mathscr{E})^{1/4}$, where $\mathrm{Const} \approx 12.504 \times 10^{-40}$ eVm^4 (corresponds to C_4 in [39]) and $\mathscr{E}$ is the collision energy. For a typical of TLEs value of the collision energy, $\mathscr{E} \sim 1$ eV the cross section for ions-neutrals collisions turns out to be $\sigma_i \approx 28.5 \times 10^{-20}$ m^2, which is slightly larger than the value obtained from a ballistic estimate $\sigma_i \approx 24.45 \times 10^{-20}$ m^2, making the growth rate slightly smaller. It is clear, therefore, that more precise observational data is required that is mainly for the ion and electron temperatures and cross sections for the $\mathrm{NO}^+ - \mathrm{N}_2$ collisions, which would allow for further, thorough computational analysis of the found instability.

Finally, consistency with the assumptions (2.14) follows from the fact, that the real part of ω satisfies $\mathfrak{Re}\, \omega \ll \nu_i$ (in fact it is of the order $\pi k^2 v_{Te} v_{Ti}/\nu_e \ll \nu_i$ by virtue of (2.5)). On the other hand,

$$\frac{1}{\nu_i} \sim 10^{-8} - 10^{-6} \text{ s} \ll \tau \approx 10 - 100 \text{ ms}, \tag{2.30}$$

so that the time of growth of the found instability $\sim 10/4\nu_i$ is much quicker than the life-time of the TLE.

3. Concluding remarks

We have briefly reviewed the state-of-art regarding observational facts on Transient Luminous Events, i.e. electric discharges between clouds and the ionosphere. The subtle filamentary structure of TLE's, with filaments roughly aligned with the electric field, was numerously reported in observations. The physical conditions of plasma within the TLE's are such, that the magnetic effects seem to be negligible at least at the leading order and thus no standard filamentation mechanisms based on interactions of plasma with the magnetic field should be effective in the case of cloud to ionosphere discharges.

The study of the stability of plasma in the parameter regime relevant to plasma conditions inside propagating discharges at the heights of 40–80 km, revealed that there exists a dominant type of instability, dissipative in nature, which is vigorous enough to develop in 10^{-8}–10^{-5} s, i.e. much faster than the TLE lifetime. This instability depends crucially on the cross sections for scattering of electrons on nitrogen molecules N_2, i.e. on the electron energies, which in the case of TLE's typically fall in the range of the well-known collisional $^2\Pi_g$ resonance, which increases the values of the scattering cross-sections roughly by a factor of 3. It is only under the resonance conditions, achieved in the presence of the strong electrical field accelerating the electrons, that the growth rate of disturbances can be positive, leading to development of instability, which is dissipative (and resonant) in nature.

The identified dissipative-resonant instability was shown to lead to a 'wavy' density distribution in the form of stripes aligned with the electric field. This suggests, that evolution of this quickly growing instability might contribute to physical mechanisms behind filamentation of streamers in the TLE's.

Acknowledgements

The financial support of the Polish National Science Centre, grant no. NCN 2014/13/B/ST10/01285, is gratefully acknowledged. This work was also partially supported within statutory activities no. 3841/E-41/S/2015 of the Ministry of Science and Higher Education of Poland.

References

1. A.L. Peratt, *Physics of the Plasma Universe, Appendix C. Dusty and Grain Plasmas*, Springer, New York, 1992.
2. H. Alfvén, *On the Filamentary Structure of the Solar Corona. The Solar Corona*, Proceedings of IAU Symposium no. 16, International Astronomical Union, Academic Press, New York, p. 35, 1963.
3. N.Y. Kotsarenko, O. Verkhoglyadova, K.I. Churyumov, *Three-dimensional model of ray structure formation in cometary plasma tails*, Planetary and Space Science, **42**, 9, 733–736, 1994, doi: 10.1016/0032-0633(94)90114-7.
4. J. Blecki, S. Savin, N. Cornilleau-Wehrlin, K. Kossacki, M. Parott, H. Rothkaehl, K. Stasiewicz, R. Wronowski, O. Santolik, J.-A. Sauvaud, *Fine structure of the polar cusp as deduced from the plasma wave and plasma measurements*, Advances in Space Research, **32**, 3, 315–321, 2003, doi:10.1016/S0273-1177(03) 90269-9.
5. J. Blecki, R. Wronowski, S. Savin, N. Cornilleau-Wehrlin, M. Parott, J. Safrankova, Z. Nemecek, O. Santolik, J.-A. Sauvaud, *Filamentation of the space plasmas*, Proceedings of WDS Conference, pp. 371–380, Prague, 2003.
6. J. Blecki, S. Savin, N. Cornilleau-Wehrlin, H. Rothkaehl, M. Parott, K. Stasiewicz, R. Wronowski, O. Santolik, J.-A. Sauvaud, Z. Nemecek, J. Safrankova, *Filamentary structure of the magnetospheric plasma*, Proceedings of Galperin Conference "Auroral Phenomena and Solar terrestrial Relations", Moscow, Russia, CAWSES Handbook-1, Boulder, 2004, pp. 350–354.
7. C.T.R. Wilson, *The electric field of a thundercloud and some of its effects*, Proceedings of the Physical Society of London, **37**, 1, 32D, 1924, doi: 10.1088/1478-7814/37/1/314.
8. O.H. Vaughan, B. Vonnegut, *Recent observations of lightning discharges from the top of a thundercloud into the clear air above*, Journal of Geophysical Research, **94**, 13179–13182, 1989.
9. R.C. Franz, R.J. Nemzek, J.R. Winckler, *Television image of a large upward electrical discharge above a thunderstorm system*, Science, **249**, 1990, doi: 10.1126/science.249.4964.48.
10. M. Stanley, M. Brook, S. Cummer, C. Barrington-Leigh, E. Gerken, *Broadband detection and characterization of day-time sprites and of negative CGs which initiated*

sprites, Eos Transactions American Geophysical Union, **79**, Fall Meeting Supplement, F177, 1998.

11. M. Stanley, M. Brooks, P. Krehbiel, S.A. Cummer, *Detection of day-time sprites via a unique sprite ELF signature*, Geophysical Research Letters, **27**, 871–874, 2000, doi: 10.1029/1999GL010769.
12. M.G. McHarg, R.K. Haaland, D. Moudry, H.C. Stenbaek-Nielsen, Altitude-time development of sprites, Journal Geophysical Research, **107**, A11, 1364, 2002, doi: 10.1029/2001JA000283.
13. C.L. Kuo, A.B. Chen, J.K. Chou, L.Y. Tsai, R.R. Hsu, H.T. Su, H.U. Frey, S.B. Mende, Y. Takahashi, L.C. Lee, *Radiative emission and energy deposition in transient luminous events*, Journal of Physics D: Applied Physics, **41**, 234014-234028, 2008, doi: 10.1088/0022-3727/41/23/234014.
14. V.V. Surkov, M. Hayakawa, *Underlying mechanisms of transient luminous events: a review*, Annals of Geophysics, **30**, 1185–1212, 2012, doi: 10.5194/angeo-30-1185-2012.
15. M. Sato, Y. Takahashi, S. Watanabe, T.P. Ushio, T. Morimoto, Z.-I. Kawasaki, M. Suzuki, T. Takashima, H. Ohya, H. Nakata, K. Odo, Y. Hiraki, T. Adachi, Y. Hobara, *JEM-GLIMS mission, coupling of thunderstorms and lightning discharges to near-earth space*, TLE Workshop, Corte, France, 2008, doi: 10.1063/1.3137707.
16. E.A. Gerken, U.S. Inan, C.P. Barrington-Leigh, *Telescopic imaging of sprites*, Geophysical Research Letterts, **27**, 17, 2637–2640, 2000, doi: 10.1029/2000GL000035.
17. M.G. McHarg, H.C. Stenbaek-Nielsen, T. Kammae, *Observations of streamer formation in sprites*, Geophysical Research Letterts, **34**, L06804, 2007, doi: 10.1029/2006 GL027854.
18. H.C. Stenbaek-Nielsen, T. Kanmae, M.G. McHarg, R. Haaland, *High-speed observations of sprite streamers*, Surveys in Geophysics, **34**, 6, 769–795, 2013, doi: 10.1007/ s10712-013-9224-4.
19. E.A. Gerken, U.S. Inan, *Observations of decameter-scale morphologies in sprites*, Journal of Atmosospheric and Solar-Terrestrial Physics, **65**, 567–572, 2003, doi: 10.1016/S1364-6826(02)00333-4.
20. R.A. Marshall, U.S. Inan, *High-speed measurements of small-scale features in sprites: sizes and lifetimes*, Radio Science, **41**, RS6S43, 2006, doi: 10.1029/2005RS003353.
21. V.P. Pasko, U.S. Inan, T.F. Bell, *Spatial structure of sprites*, Geophysical Research Letterts, **25**, 12, 2123–2126, 1998, doi: 10.1029/98GL01242.
22. U. Ebert, S. Nijdam, Ch. Li, A. Luque, T. Briels, E. van Veldhuizen, *Review of recent results on streamer discharges and discussion of their relevance for sprites and lightning*, Journal of Geophysical Research, **115**, A00E43, 2010, doi: 10.1029/2009JA014867.
23. V.P. Pasko, *Recent advances in theory of transient luminous events*, Journal of Geophysical Research, **115**, A00E35, 2010, doi: 10.1029/2009JA014860.
24. M.G. McHarg, H.C. Stenbaek-Nielsen, T. Kammae, R.K. Haaland, *Streamer tip splitting in sprites*, Journal of Geophysical Research, **115**, A00E53, 2010, doi: 10.1029/2009JA014850.
25. V.P. Pasko, Y. Yair, Ch.-L. Kuo, *Lightning related transient luminous events at high altitude in the Earth's atmosphere: phenomenology, mechanisms and effects*, Space Science Reviews, **168**, 475–516, 2012, doi: 10.1007/s11214-011-9813-9.
26. R. Morrow, J.J. Lowke, *Streamer propagation in air*, Journal of Physics D: Applied Physics, **30**, 614–627, 1997, http://iopscience.iop.org/0022-3727/30/4/017.

27. T. Neubert, *On sprites and their exotic kin*, Science, **300**, 5620, 747–749, 2003, doi: 10.1126/science.1083006.

28. Y. Hiraki, Y. Kasai, H. Fukunishi, *Chemistry of sprite discharges through ion-neutral reactions*, Atmospheric Chemistry and Physics, **8**, 3919–3928, 2008, doi: 10.5194/acp-8-3919-2008.

29. D.D. Sentman, H.C. Stenbaek-Nielsen, M.G. McHarg, J.S. Morrill, *Plasma chemistry of sprite streamers*, Journal of Geophysical Research, **113**, D11112, 2008, doi:10.1029/2007JD008941.

30. D.K. Sharma, J. Rai, M. Israil, P. Subrahmanyam, P. Chopra, S.C. Garg, *Enhancement in ionospheric temperatures during thunderstorms*, Journal of Atmosospheric and Solar-Terrestrial Physics, **66**, 51–56, 2004, doi:10.1016/j.jastp.2003.07.013.

31. T. Kovács, J.M.C. Plane, W. Feng, T. Nagy, M.P. Chipperfield, P.T. Verronen, M.E. Andersson, D.A. Newnham, M.A. Clilverd, D.R. Marsh, *D-region ion-neutral coupled chemistry (Sodankylä Ion Chemistry, SIC) within the Whole Atmosphere Community Climate Model (WACCM 4) – WACCM-SIC and WACCM-rSIC*, Geoscientific Model Development, **9**, 3123–3136, 2016, doi: 10.5194/gmd-9-3123-2016.

32. N.V. Smirnova, A.D. Danilov, *Rocket data on the D-region positive ion composition*, Journal of Atmospheric and Terrestrial Physics, **56**, 8, 887–892, 1994, doi: 10.1016/0021-9169(94)90150-3.

33. Y. Itikawa, *Cross-sections for electron collisions with nitrogen molecules*, Journal of Physical and Chemical Reference Data, **35**, 1, 31–53, 2006, doi: 10.1063/1.1937426.

34. R.E. Kennerly, *Absolute total electron scattering cross sections for N2 between 0.5 and 50 eV*, Physical Review A, **21**, 1876–1883, 1980, doi: 10.1103/Phys-RevA.21.1876.

35. W. Sun, M.A. Morrison, W.A. Isaacs, W.K. Trail, D.T. Alle, R.J. Gulley, M.J. Brennan, S.J. Buckman, *Detailed theoretical and experimental analysis of low-energy electron-N2 scattering*, Physical Review A, **52**, 1229–1256, 1995, doi: https://doi.org/10.1103/PhysRevA.52.1229.

36. M.J. Brunger, P.J.O. Teubner, A.M. Weigold, S.J. Buckman, *Vibrational excitation of* N_2 *in the* $^2\Pi_g$ *resonance region*, Journal of Physics B: Atomic, Molecular and Optical Physics, **22**, 1443–1453, 1989.

37. A.F. Alexandrov, L.S. Bogdankevich, A.A. Rukhadze, *Principles of Plasma Electrodynamics*, Springer, Berlin, Heidelberg, 1984.

38. B. Shokri, S.M. Khorashadi, M. Dastmalchi, *Ion-acoustic filamentation of a current-driven plasma*, Physics of Plasmas, **9**, 8, 3355–3358, 2002, doi: http://dx.doi.org /10.1063/1.1490133.

39. G. Schmidtke, K. Suchy, K. Rawer, *Encyclopedia of Physics*, Vol. XLIX/7, Geophysics III/7, Springer, Berlin, Heidelberg, 1984.

40. K. Słomińska, J. Błęcki, J. Słomiński and the JEM-EUSO Collaboration, *Science of atmospheric phenomena with JEM-EUSO*, Experimental Astronomy, **40**, 239–251, 2015, doi: 10.1007/s10686-014-9431-0.

Received July 12, 2018; revised version September 20, 2018.