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Fåhræus effect revisited

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A CONSISTENT HYDRODYNAMIC ANALYSIS OF BLOOD flow through capillaries is proposed. The approach, while suggested by empirical observations, is based solely on the properties of Newtonian fluids and suspensions. Blood flow is divided into three phases: the first is a thin erythrocyte-free layer near the wall, the second a core flow of constant hematocrit and the third an intermediate layer wherein the hematocrit varies. Based on the observation that viscosity depends exponentially on the local hematocrit, blood flow velocity profiles are obtained and the direct connection between the Fåhræus and the Fåhræus-Lindqvist effects is established.

Key words: blood viscosity, hemodynamics, Fåhræus effect, Fåhræus–Lindqvist effect.

1. Introduction

Sometime before 1929, Robin Fåhræus (1888–1968), a pathologist at the University of Uppsala in Sweden, conducted a set of experiments on blood flow in capillaries. Although crude by today's standards, these investigations opened a new area of research in hemodynamics. By counting the number of red cells suspended in blood flowing through narrow glass tubes with radii $R < 250$ μm, Fåhræus discovered that their volumetric concentration (i.e., the proportion of blood volume occupied by them) in the tubes, the *tube hematocrit*, $H_T = H_T(R)$, is lower than their *reservoir hematocrit* H_F, or concentration in the feeding reservoir. In the second of his two papers on the subject [1], he wrote that different distribution of blood cells "has been overlooked (...) [because of] the fact that a drop of blood from the finger or the ears shows-at least regarding the red cells-the same composition as blood from an artery or a vein. But the composition of a drop from bleeding narrow vessels is not the same as the composition of the

*Ryszard Herczyński (1926–2009), a prominent researcher at IPPT PAN for over 30 years, made pioneering contributions to bio-fluid mechanics, suspension flows, and rarified gas dynamics. A close collaborator of Władysław Fiszdon, he played an important role in bridging scientific communities in the West and the East by co-organizing biannual Symposia on Advanced Problems of Fluid Mechanics (1952–1989). This paper, left by RH on his desk in an unfinished form, was edited and prepared for publication by Andrzej Herczyński (andrzej@bc.edu). *Note by the Editorial Board.*

blood in the vessels themselves". Fåhræus explained the observed discrepancy by noting that the average velocity of erythrocytes in the capillaries is greater than that of plasma (extracellular liquid).

Fåhræus' painstaking experiments were repeated more than 40 years later by BARBEE and COCKELET [2] using a more reliable apparatus and with more careful preparation of blood (e.g., removing all particles except for erythrocytes, preventing the formation of rouleaux, etc.). The results of both the original experiments, which proved remarkably accurate despite the simplicity of Fåhræus' setup, and those of Barbee and Cockelet's experiments, conducted at the same value of reservoir hematocrit $H_F = 0.405$, were summarized in a plot showing the dependence of the relative hematocrit, defined as $H_r = H_T/H_F$, on the tube radius R. This graph, referred to as the *Barbee graph* in the present work, is reproduced in Fig. 1.

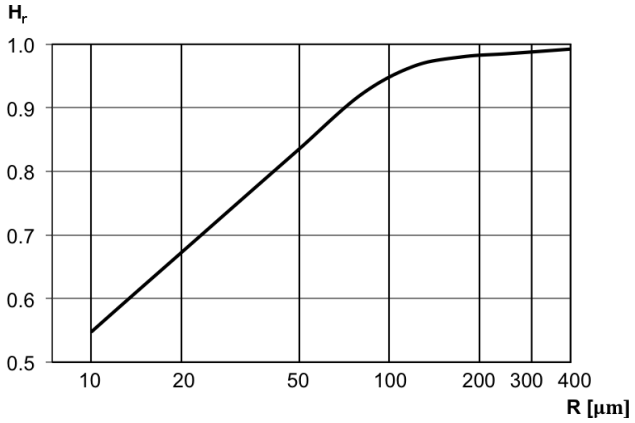

FIG. 1. The Barbee graph.

In his 1929 paper [1], Fåhræus also pointed out that the observed phenomenon must correspond to the reduction of the relative viscosity of blood in capillaries. This insight led to a second series of experiments, conducted by Fåhræus and his coworker, pathologist TORSTEN LINDQVIST [3]. They measured the relative viscosity, η_{rel}, defined as the ratio of the output of blood plasma Q_p – and in later experiments, the ratio of the output of water Q_w at the same value of the pressure drop – to the total output of blood Q. As Q depends on H_F and R, and Q_w depends on R alone, η_{rel} is a function of H_F and R:

$$\eta_{rel}(H_F, R) = \frac{Q_w(R)}{Q(H_F, R)}. \tag{1.1}$$

The results of a large number of such experiments, and for various values of H_F, in addition to the "standard" $H_F \approx 0.4$, were collected and discussed by PRIES *et al.* [4]. A monotonic decrease of the relative viscosity η_{rel} with the diminishing

tube radius R was indeed observed, but there was a considerable scatter to the data overall, caused by the differences in experimental apparatus. PRIES *et al.* [4] proposed an empirical formula for the relative viscosity η_{rel}, which incorporates both the tube radius R and the reservoir hematocrit H_F.

Fåhræus and Lindqvist did not offer a theoretical framework linking the two phenomena they investigated separately – now usually referred to as the Fåhræus effect and the Fåhræus–Lindqvist effect, respectively. Numerous later analyses (see the historical review by GOLDSMITH *et al.* [5] and references therein) endeavored to elucidate the relation between these two effects using either heuristic reasoning or rheological considerations for suspensions.

An attempt to connect the two phenomena was undertaken by SHARAN and POPEL [6], who proposed a two-phase hydrodynamic model (see also WANG and BASSINGTHWAIGHTE [7]). In their approach, the tube is divided into two separate cylindrical regions: the central "core" region containing the axis where the flow viscosity is η_C, and the outer, cell-free boundary layer where the viscosity is that of the plasma η_{Pl}. However, accommodation between these two flows of different viscosities is given in terms of a complicated and purely empirical formula suggested by PRIES *et al.* [4].

The aim of the present paper is to bridge the gap between the Fåhræus and the Fåhræus–Lindqvist effects based entirely on the classical hydrodynamics. In contrast to the model employed by SHARAN and POPEL [6], the flow of blood is treated here as a *flow of suspension* through a tube and no *ad hoc* assumptions are employed except for the well-established functional form of the viscosity of dense suspensions. The proposed approach is based on considering the flow separately in *three* distinct regions of the cylindrical vessel, whose defining radii are determined from experiments. This hydrodynamic model, unlike the two-phase models mentioned above, leads to velocity profiles and other relevant quantities directly from the equation of motion.

The paper is organized as follows. The second section describes how, using the Barbee relationship, the blood vessel (tube) is divided into three regions and suggests expressions for the local hematocrit in each region. In the third section, the Navier–Stokes equation with the appropriate boundary conditions is formulated and solved numerically in each of the regions, leading to expressions for blood viscosity and flow velocity profiles. The relative viscosity is then calculated as a function of the tube's radius and compared with experiments. The last section provides a summary of the proposed approach and concluding remarks.

2. Local hematocrit and the Barbee graph

To capture the salient characteristics of blood flow in a capillary, the tube is divided into three separate regions: a particle-free region close to the wall,

a cylindrical core region around the axis, and an intermediate, accommodation region between the two.

The particle-free region is defined as the layer in which the center of volume of none of the suspended particles is located. The width of this region is determined by the size and form of the suspended particles. For a suspension of rigid spherical particles of uniform radius, this width δ would be equal to their radius. For particles of more complicated shape, like erythrocytes, the effective δ must be measured or estimated. Since erythrocytes are flexible, bi-concave disks with a diameter of about 8 μm, and the maximum cross-section diameter of about 2–3 μm (see, e.g., [8]), it is assumed here that $\delta = 2.5$ μm. Consequently, the inner radius of the particle-free region is taken to be $R_1 = R - \delta$. The hematocrit in the core region of radius R_2 is assumed to be the same as that of the feeding reservoir H_F. The intermediate region is the annular space between radii R_2 and R_1, $R_2 < R_1$. However, the boundary of the core region R_2 is somewhat imprecisely defined, as observed and photographed already by Fåhræus (Fig. 2 in [1]), due to the formation of temporary clusters of erythrocytes.

In each of the three regions delineated above, the *local hematocrit* $h(r)$ is different: $h(r) = H_F$ in the core $r < R_2$, $h(r) = 0$ in the particle-free boundary region $R_1 \le r \le R$, and $h(r)$ is a monotonically decreasing function in the intermediate region $R_2 \le r \le R_1$. Experiments suggest that $h(r)$ should be a smooth function everywhere.

It is sufficient to posit the simplest possible form of the local hematocrit,

$$h(r) = H_F f(r), \tag{2.1}$$

where $f(r)$ is a smooth, continuous function satisfying the following three conditions:

(i) $f(r) = 1$ for $0 \le r \le R_2$;
(ii) $f(r)$ is monotonic for $R_2 \le r \le R_1$, $f'(R_1) = 0$ and $f'(R_2) = 0$;
(iii) $f(r) = 0$ for $R_1 \le r \le R$;

and where $R_2 = R_2(H_r, R)$ is a continuous function of its two arguments. It should be noted that condition (i), which implies that the concentration of erythrocytes in the core remains the same as in the reservoir after they are squeezed into a capillary, is not *a priori* evident and is a basic assumption of the present model.

Since the total volume V_e occupied by erythrocytes within a segment of the tube (capillary) of length l is given by

$$V_e = \int_0^R h(r) 2\pi r l \, dr,$$

and $H_T = V_e/\pi R^2 l$, the relative hematocrit can be expressed as

$$H_r = \frac{2}{R^2}\int_0^R f(r) r\, dr. \tag{2.2}$$

The integral on the right side of (2.2) depends on R_2 and can be evaluated by casting it as a sum of two terms, which will be evaluated separately:

$$\int_0^R f(r) r\, dr = \int_0^{R_2} f(r) r\, dr + \int_{R_2}^{R_1} f(r) r\, dr.$$

Denote now

$$F(H_r, R) = \frac{2}{R^2}\int_0^R f(r) r\, dr$$

and define

$$\Phi(H_r, R) = H_r - F(H_r, R). \tag{2.3}$$

According to the implicit function theorem, it follows from $\Phi(H_r, R) = 0$ that there exists a unique continuous function $H_r = H_r(R)$ satisfying (2.2), provided Φ and $\partial\Phi/\partial H_r$ are continuous functions and $\partial\Phi/\partial H_r \neq 0$. These conditions are satisfied in the case considered here.

The implicit function theorem does not provide the function $H_r(R)$, it only guarantees that the function exists and is unique. However, the dependence of H_r on R is already given by the Barbee relationship, the function implied by the Barbee graph. It follows that the Barbee relationship represented in Fig. 1, although established experimentally only for one particular value of the reservoir hematocrit $H_F = 0.405$ remains valid for all other values of H_F provided only that equation (2.1) and the conditions following it are satisfied. It will be so under the assumption that erythrocytes are not too densely packed and blood flow is adequately described by the suspension theory.

The exact mathematical form of the function $f(r)$ in the intermediate region, describing the transition between the core flow and the particle-free boundary layer, does not substantially change the physically significant results such as flow velocity profiles and the relative blood viscosity, provided f obeys all three conditions listed under (2.1). The function $f(r)$ is assumed here to have the simple form

$$f(r) = \frac{1}{2}\left[1 + \sin\left(\frac{\pi}{2}\frac{R_1 + R_2 - 2r}{R_1 - R_2}\right)\right] \qquad \text{for } R_2 \le r \le R_1. \tag{2.4}$$

Evaluation of the integrals in (2.2) for the above function yields

$$H_r = \frac{R_1^2 + R_2^2}{2R^2} - \frac{2(R_1 - R_2)^2}{\pi^2 R^2}. \tag{2.5}$$

For any R, the value of $R_1 = R - \delta$ is known, and thus comparing $H_r(R)$ obtained in (2.5) with the Barbee relationship one can deduce the value of $R_2 = R_2(R)$. In Table 1, the values of R_2 for $R = 10, 20, 50, 80$, and 250 μm are given showing, as expected, that the local hematocrit becomes increasingly uniform with the rise of tube's diameter.

Table 1. Values of R_2/R, R_1/R, and $(R_1 - R_2)/R$ for various radii R.

R, μm	R_2/R	R_1/R	$(R_1 - R_2)/R$
10	0.73	0.75	1.2×10^{-2}
20	0.76	0.88	6.0×10^{-3}
50	0.88	0.95	1.4×10^{-3}
80	0.95	0.97	2.5×10^{-4}
250	0.989	0.990	4.0×10^{-6}

Furthermore, for any fixed value of R, and with R_2 determined using the Barbee graph, it is now possible to obtain from (2.1) and (2.5) the local hematocrit $h(r)$. Local hematocrit profiles at various values of the reservoir hematocrit (including the traditional value $H_F = 0.405$) are shown in Fig. 2.

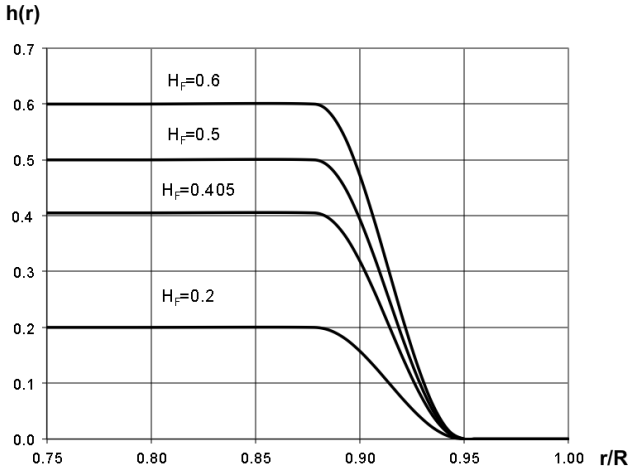

FIG. 2. Profiles of the local hematocrit $h(r)$ for $R = 50$ μm in tubes with the reservoir hematocrits $H_F = 0.2$, $H_F = 0.4$, $H_F = 0.5$ and $H_F = 0.6$.

3. Viscosity and velocity profiles

The link between the first set of experiments conducted by Fåhræus, measuring the relative hematocrit, and the second set of experiments conducted by Fåhræus and Lindqvist, concerning blood viscosity, can be established using the

Navier–Stokes equation. The linearized, steady-state form of this equation, employed by Stokes to derive the Poiseuille formula, in cylindrical coordinates reads

$$\frac{1}{r}\frac{d}{dr}\left(r\eta(r)\frac{dv}{dr}\right) = -\frac{\Delta p}{l}, \tag{3.1}$$

where $\eta(r)$ is the viscosity of the fluid as a function of the distance from the tube axis, $v(r)$ is the fluid velocity (parallel to the axis), p is the pressure, and l is the tube length. As usual, it is assumed that the velocity profile has zero derivative at the center and that the no-slip boundary condition is obeyed at the tube wall,

$$\left.\frac{dv}{dr}\right|_{r=0} = 0 \qquad \text{and} \qquad v(R) = 0. \tag{3.2}$$

In the case of uniform viscosity, $\eta(r) = \eta_0 = \text{const.}$, the Poiseuille formula for the velocity inside the tube is easily recovered by solving (3.1) subject to the boundary conditions (3.2):

$$v(r) = V_0\frac{R^2 - r^2}{R^2}, \tag{3.3}$$

where V_0, the velocity along the tube axis, depends on the pressure gradient $\Delta p/l$, R and viscosity η_o, namely $V_0 = \Delta p R^2/4\eta_0 l$.

In the suspension flow considered here, however, viscosity is not constant but depends on the local concentration of erythrocytes, i.e., on the local hematocrit. Since viscosity will increase if the erythrocytes are packed more densely, η will vary monotonically with the local hematocrit. Furthermore, experiments suggest that viscosity varies exponentially within the intermediate region of the tube. Notably, BARBEE [9] himself reported the exponential dependence of η on the reservoir hematocrit H_F with the shear rate at the wall, $\gamma = dv/dr|_{r=R}$, as a parameter.

Consistent with these observations, it is postulated that viscosity varies exponentially with the local hematocrit:

$$\eta(r) = \eta_0 \exp(\alpha h(r)), \tag{3.4}$$

where η_0 and α are the fitting parameters to be determined from the second series of experiments performed by Fåhræus and Lindqvist. In particular, the value of α can be deduced once the velocity profile is known by comparing the relative viscosity given in (1.1) with the experimental data. Since the scatter in the measured values of the relative viscosity $\eta_{rel}(R)$ for $H_F \approx 0.4$ is relatively small for large values of tube radius R, it is sufficient to choose a *single point* in this region to determine the value of α. This procedure yields the approximate value $\alpha = 3.0$, which corresponds to the shear rate $\gamma \approx 60\ \text{s}^{-1}$ in Barbee's paper [9]. It should be emphasized that the form of (3.4) is an assumption requiring a consistency check.

To find the velocity profile $v = v(r)$, numerical integration of (3.1) with (3.4) subject to the boundary conditions of (3.2) was performed separately in each of the three sub-regions of the tube: in the core ($0 \le r \le R_2$), in the intermediate region ($R_2 \le r \le R_1$), and in the particle-free region near the wall ($R_1 \le r \le R$). In each of these regions, the Newton–Coates sixth-order integration method was employed using 10 evenly spaced grid points. The results are shown in Fig. 3 with $R = 50$ μm and the reservoir hematocrit H_F as the parameter, and in Fig. 4

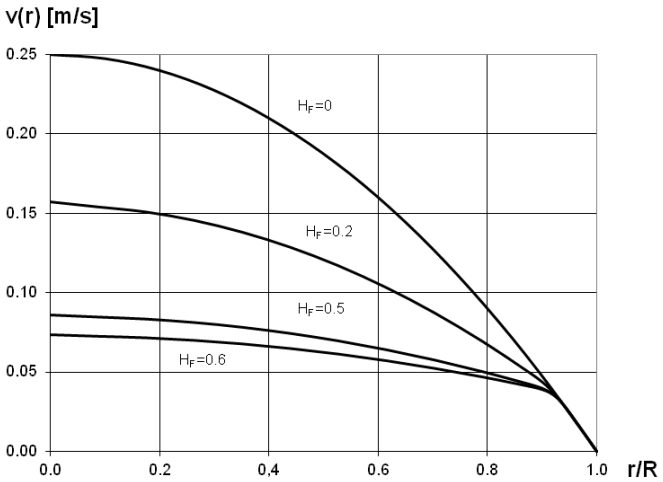

FIG. 3. Profiles of the local velocity of blood $v(r)$ for $R = 50$ μm for reservoir hematocrits $H_F = 0.2$, $H_F = 0.5$ and $H_F = 0.6$ as a function of r/R. The parabolic velocity profile at $H_F = 0$ is added for comparison.

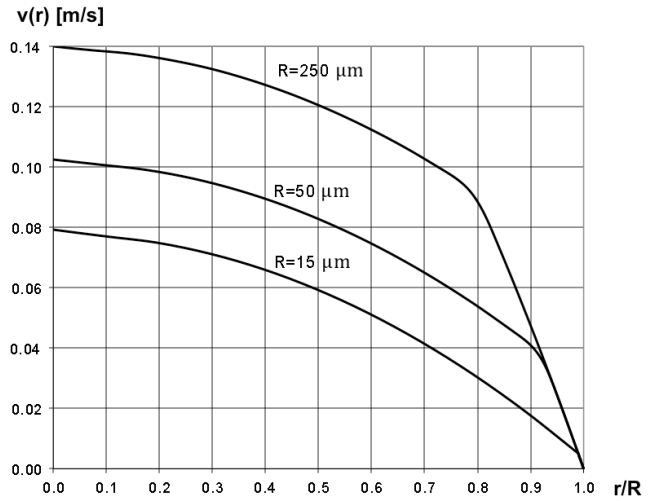

FIG. 4. Profiles of the local velocity of blood $v(r)$ for reservoir hematocrit $H_F = 0.405$ in tubes of different radii: $R = 15$ μm, $R = 50$ μm and $R = 250$ μm.

with $H_F = 0.4$ and the tube's outer radius R as the parameter. These velocity profiles can be described as quasi-parabolic, flattened slightly at the center of the tube. By comparison, COX and MASON [10] reported velocity profiles completely flat at the central axis for suspensions of rigid spherical particles. Note in Fig. 3 that due to the presence of the particle-free region near the wall, the values of $v(r)$ coincide for all H_F in this region, that is, for $R_1/R \leq r/R \leq 1$. It can be seen in Table 1 that $R_1/R \approx 0.95$ in this case. Furthermore, the limiting value of $R\,dv/dr$ as $r/R \to 1$ is the same for all values of R as can be seen in Fig. 4.

Once $v = v(r)$ has been obtained, several other quantities of interest can be calculated in the same loop of the numerical code, which integrates (3.1). The total output of blood can be determined from

$$Q(H_F, R) = 2\pi \int_0^R v(r) r\, dr, \tag{3.5}$$

whereas the output of erythrocytes is given by

$$Q_T(H_F, R) = 2\pi \int_0^R v(r) h(r) r\, dr. \tag{3.6}$$

Note that $v(r)$ and thus Q and Q_T depend on $h(r)$ and therefore on H_F. A comparison of the calculated output of water Q_w with its known theoretical value $Q_{w,th}$ is an indication of the accuracy of the proposed method: the relative difference is

$$|Q_w - Q_{w,th}|/Q_{w,th} \approx 0.03.$$

Furthermore, the output of the erythrocytes $Q_T = Q_T(H_F, R)$ given in (3.6) can be used to compare the hematocrit at the tube's outlet,

$$H_{out} = Q_T(H_F, R)/Q(H_F, R),$$

with that in the feeding reservoir H_F. As noted already by FÅHRÆUS [1], H_{out} should be equal to H_F. The results compiled in Table 2 show that the two values converge with the increase in tube's radius R, and are within 10% for $R \geq 50$ μm and with H_F in the range of 0.1–0.6.

Finally, using (3.5) it is now also possible to calculate the relative viscosity $\eta_{rel}(H_F, R)$ defined by (1.1) and to compare the results with the experiment. Calculations were carried out for three cases: $H_F = 0.2$, $H_F = 0.405$ and $H_F = 0.6$, but experimental data is available only for the standard reservoir hematocrit $H_F = 0.405$. A comparison of η as a function of R and H_F with the experimental data is given in Fig. 5. For standard hematocrit, the calculated relative

Table 2. Values of the output hematocrit H_{out} for a given reservoir hematocrit H_F in tubes with different radii.

H_F	$R = 10$	$R = 20$	$R = 50$	$R = 150$	$R = 350$	$R = 600$
0.1	0.078	0.089	0.087	0.092	0.10	0.10
0.2	0.15	0.192	0.173	0.20	0.20	0.20
0.3	0.23	0.26	0.286	0.30	0.30	0.30
0.405	0.30	0.38	0.403	0.405	0.405	0.405
0.5	0.37	0.42	0.47	0.5	0.5	0.5
0.6	0.44	0.49	0.56	0.6	0.6	0.6

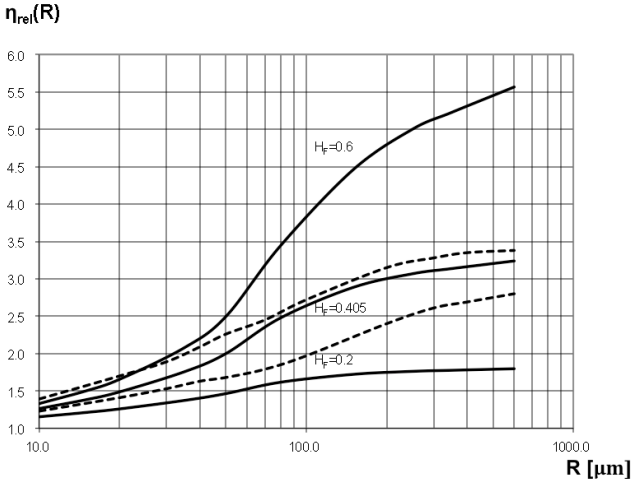

FIG. 5. The relative viscosity of blood, $\eta_{rel}(R)$, as a function of tube radius R for various hematocrit values in the feeding reservoir: $H_F = 0.2$, $H_F = 0.405$ and $H_F = 0.6$. The range of experimental data for $H_F = 0.405$, taken from PRIES *et al.* [4], is delineated by dotted curves.

viscosity curve lies well within the experimentally determined region in Fig. 5, delineated with dash lines.

4. Discussion and concluding remarks

The analysis and results offered here do not provide the full theory of blood flow through capillaries. No satisfactory theory for the flow of dense suspension through the tube has been proposed thus far, not even for the simplest case of the suspension of rigid spheres. The purpose of the present paper, however, goes beyond devising a purely empirical formula that would neatly agree with available experimental data for the suspension of erythrocytes. Rather, the present approach – although suggested by empirical observations and referring to ex-

periments – aims to provide a simple hydrodynamic description based on the properties of Newtonian fluids and suspension flows.

The principal assumptions of the proposed analysis are:

(i) The local hematocrit $h(r)$ can be described by (2.1) and (2.4);
(ii) The flow can be considered separately in three regions defined by $R_1 = R - \delta$ with $\delta \approx 2.5\ \mu m$ and R_2 which is a continuous function of H_r and R;
(iii) Local viscosity depends exponentially on $h(r)$, as given in (3.4) with $\alpha = 3$.

These three assumptions establish a direct link between the Fåhræus and the Fåhræus–Lindqvist effects for blood flow through capillaries based on the Navier–Stokes equation, and lead to the following conclusions and results:

(i) The Barbee graph (Fig. 1) is universal, valid for all H_F and R for which blood could be treated as a suspension.
(ii) The radius of the core region R_2 can be deduced from (2.5) and the Barbee relationship.
(iii) Hematocrit and velocity profiles in the tube can be obtained by integrating (3.1) subject to the boundary conditions ((3.2) and the matching conditions).

For very narrow capillaries, whose diameter is of the same order as the dimension of erythrocytes, the above description is no longer adequate and the flow should instead be treated as that of individual particles (erythrocytes) interacting with one another and with the tube's wall. Experiments show that at the threshold value of the capillary's diameter, the relative viscosity rises sharply as compared to that in slightly larger vessels. A theoretical investigation of such flows, in tubes with a radius R in the range of 3–10 μm, was given by SECOMB *et al.* [11] and SECOMB and PRIES [12].

A few concluding remarks are in order. It seems that more than 80 years after the original Fåhræus experiments, the basic physics underlying his findings has not yet been fully appreciated either in the literature on blood or on fluid dynamics of dense suspensions.

In physiology, the Fåhræus–Lindqvist effect, the phenomenon assuring that the viscosity of blood in capillaries is substantially lower than in large vessels, is critical for the adequate supply of blood to all body organs. The well-established fact that the maximal drop of resistance in the circulatory system is at the level of arterioles was ascribed, from the XIX century onwards, to the large number of capillaries, however, the enhanced flow in them due to decreased

viscosity has been overlooked. Even today, the Fåhræus effect is rarely included in textbooks on physiology, and is sometimes omitted in monographs on the circulatory system.

In hydrodynamics, Fåhræus observations indicate that the very presence of boundaries in a suspension flow forces an ordering of suspended particles close to the vessel's walls. Theories of suspension based on random distribution of particles in the whole domain do not satisfactorily describe such flows, particularly near boundaries. The well-known discrepancies in measurements of the viscosity of suspensions depending on the type of viscometer used are manifestly due to different geometry of these measuring devices, i.e. due to different shapes of their internal boundaries.

This paper concerns a very specific type of suspension, that of erythrocytes in plasma or saline, a suspension of flexible particles of complicated shape. A question arises, therefore, as to whether the proposed approach could be employed more broadly. For example, could one find the equivalent of the Barbee graph for any particular suspension? Although the method presented here is closely tied to the experiments on blood flow, there are reasons to believe that it will prove applicable to other types of suspensions as well.

Acknowledgement

The author would like to thank Professor T. J. Pedley for his helpful comments and for suggestions of additional literature.

Closing Note

This article, written by Ryszard Herczyński mostly in 2008, is his last scientific contribution, which he was not able to complete. The author would have almost certainly objected to some of the changes that were introduced in the manuscript, but it is hoped that they do not detract too much from his ideas and results. (*Andrzej Herczyński*)

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