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Analysis of micropolar porous thermoelastic circular plate by eigenvalue approach

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The present paper examined a two-dimensional axi-symmetric problem of thick circular plate in a micropolar porous thermoelastic medium due to thermomechanical sources. An eigenvalue approach has been employed after applying the Laplace and Hankel transforms to investigate the problem. The expressions of displacements, stresses, microrotation, volume fraction field and temperature distribution are obtained in the transformed domain. A numerical inversion technique has been used to obtain the resulting quantities in the physical domain. The numerical simulated resulting quantities are shown graphically to depict the effects of thermal forces and porosity. Particular cases of interest are also studied and presented.

Key words: micropolar, thermoelasticity, volume fraction field, eigenvalue, Laplace and Hankel transforms.

1. Introduction

Classical theory of elasticity deals with a theoretical, ideal and simplified model of a solid in the form of an elastic material that is regarded as a continuum in the mathematical sense, and thus the molecular, atomic structure of the body is neglected. The continuous distribution of a matter in a given region of the body is characterized by a single quantity – the density. This theory describes well the behavior of construction materials such as steel, aluminum and concrete, with stresses remaining within the material elastic limits. In various situations, some basic differences are evident between the classical theory of elasticity and experiments in the problems where stress gradient effects occur. In many dynamical problems, such as elastic vibrations with high frequencies and short wavelengths or the ultrasonic waves characterized by high frequencies and short wavelengths, the discrepancy between the classical theory of elasticity and the experiments is clearly observed. Therefore, the classical elasticity fails to obtain good results in the case of granular body vibrations. Granular materials are construction materials, polymers and polycrystalline structures.

In order to eliminate the shortcomings of classical elasticity, several generalized continuum theories have been introduced, which have additional degrees of freedom. One of these theories is the micropolar continuum theory introduced and developed by Eringen [1]. Micropolar continuum mechanics is a scientific discipline concerned with the mechanics of oriented bodies whose primitive element consists of rigid particles. Here, the constituents of materials are allowed to rotate independently without stretch. Materials consisting of fiber or elongated grains, wood, certain rocks, etc. come under this category.

Eringen [2] and Nowacki [3] developed the linear theory of micropolar thermoelasticity by extending the theory of micropolar continua to include thermal effect. Using the Green–Lindsay (G-L) theory [4], the generalized theory of micropolar thermoelasticity was developed by Dost and Tabarrok [5]. Chandrasekhariah [6] presented a micropolar thermoelasticity theory by including heat-flux among the constitutive variables.

A porous material is a material whose solid portion is continuously connected through the whole volume to form a solid matrix with voids through which the liquid or gas may flow. Many materials such as rocks, sand, soil, limestone, sandstones, etc., which occur on and below the surface of the earth, and are an integral part of the human life, are known as porous materials. Nanaziato and Cowin [7, 8] investigated the linear and nonlinear theories of elastic materials with voids. Iesan [9] derived the basic equations of the linear theory of micropolar elastic medium with voids and studied the shock waves in this medium.

Iesan [10] developed the theory of thermoelastic materials with voids and established the uniqueness, reciprocal and variational theorem for micropolar thermoelasticity. Iesan [11] presented a theory of initially stressed thermoelastic material with voids. Marin [12] established the uniqueness of the solution of an initial value problem in thermoelastic bodies with voids. Iesan and Nappa [13] investigated the axially symmetric problem for a porous elastic solid. Kumar and Chaudhary [14] studied the axi symmetric problem in a micropolar elastic medium with voids due to mechanical sources. Kumar and Partap [15] studied the effect of porosity on the propagation of circular crested waves in micropolar thermoelastic homogeneous isotropic plate subjected to stress-free thermally insulated and isothermal conditions. Kumar and Gupta [16] investigated the axisymmetric problem in a micropolar porous thermoelastic material. Kumar and Kumar [17] studied the reflection and refraction of the elastic waves at the interface of initially stressed thermoelastic half-space with voids and an elastic half-space.

Kumar, Sharma and Garg [18] studied the deformation in micropolar elastic medium with voids under the application of concentrated force, uniformly distributed force, linearly distributed force and moving couple. Sharma and Kumar [19] studied the propagation of plane waves and fundamental solution in thermoviscoelastic medium with voids. Sharma, Sharma and Bhargava [20] studied the plane waves and fundamental solution in an electro-microstretch elastic solid. Sharma and Marin [21, 22] studied the reflection of plane wave in micropolar thermoelastic solid half-space with distinct conductive and thermodynamic temperatures and also studied the relection and refraction of waves from imperfect boundary between two heat conducting micropolar thermoelastic solids. Marin and Florea [23] analyzed the temporal behavior of solutions in the micropolar porous thermoelastic bodies by including the set of independent variables, the time derivative of the voidage to include the inelastic effects. Tripathi, Kedar and Deshmukh [24] presented the problem of a thick circular plate with axisymmetric heat supply in generalized thermoelastic diffusion by using the integral transform technique. Kumar, Kumar and Gourla [25] worked on the axisymmetric problem with the response of thermomechanical sources in a thermoelastic porous medium. Kumar and Abbas [26] investigated the interactions due to various sources in a saturated porous medium having incompressible fluid.

In spite of all these studies, not much work has been done on a micropolar porous thermoelastic circular plate. In this paper, we study a two-dimensional problem of a micropolar thermoelastic porous circular plate by using the eigenvalue approach and the Laplace and Hankel transforms. The components of displacements, microrotation, volume fraction field, temperature distribution and stresses are obtained numerically and depicted graphically for a specific model.

2. Basic equations

Following Kumar and Partap [15], the field equations and the constitutive relations in a micropolar porous thermoelastic medium without body forces, body couples, heat sources and extrinsic equilibrated body force are given by

$$(\lambda+\mu)\nabla(\nabla.\vec{u})+(\mu+K)\nabla^2\vec{u}+K\nabla\times\vec{\varphi}+b\nabla\varphi^*-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \tag{2.1}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\varphi})-\gamma\nabla\times(\nabla\times\vec{\varphi})+K\nabla\times\vec{u}-2K\vec{\varphi}=\rho j\frac{\partial^2\vec{\varphi}}{\partial t^2}, \tag{2.2}$$

$$\alpha_1\nabla^2\varphi^*-b(\nabla.\vec{u})-\xi_1\varphi^*-\omega_0\frac{\partial\varphi^*}{\partial t}+m\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\rho\chi\frac{\partial^2\varphi^*}{\partial t^2}, \tag{2.3}$$

$$K_1^*\nabla^2 T - \nu \mathrm{T}_0\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla.\vec{u}) - \mathrm{mT}_0\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\varphi^* = \rho C^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T, \tag{2.4}$$

$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j}+u_{j,i}) + K(u_{j,i}-\varepsilon_{ijk}\varphi_k) - \nu\left(1+\tau_1\frac{\partial}{\partial t}\right)T\delta_{ij} + b\delta_{ij}\varphi^*, \tag{2.5}$$

$$m_{ij} = \alpha\varphi_{r,r}\delta_{ij} + \beta\varphi_{i,j} + \gamma\varphi_{j,i}, \tag{2.6}$$

where λ, μ, K, α, β, γ are the micropolar constants, α_1, b, xi_1, ω_0, m and χ are the elastic constants due to the presence of voids, $\vec{u}$ is the displacement vector, $\vec{\varphi}$ is the microrotation vector, ρ is the density, j is the micro-inertia, K_1^* is the coefficient of thermal conductivity, $\nu = (3\lambda + 2\mu + K)\alpha_t$, α_t is the coefficient of linear thermal expansion, T is the change in the temperature of the medium at any time, C^* is the specific heat at constant strain, τ_0 and τ_1, are the thermal relaxation times, t_{ij}, m_{ij} are the stress tensor and couple stress tensor respectively, and $\nabla^2 = \frac{\partial^2}{\partial \mathrm{r}^2} + \frac{1}{\mathrm{r}}\frac{\partial}{\partial \mathrm{r}} + \frac{1}{\mathrm{r}}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial \mathrm{z}^2}$ is the Laplacian operator.

For Lord–Shulman (L-S) theory, $\tau_1 = 0$, $\tau_0 > 0$, and $\eta_0 = 1$.

For G-L theory, $\tau_1 \geq \tau_0 > 0$ and $\eta_0 = 0$.

3. Formulation of the problem

We consider an infinite, homogeneous isotropic micropolar thermoelastic porous circular plate of a thickness $2d$ occupying the region defined by $\leq r \leq \infty$, $-d \leq z \leq d$ Let (r, θ, z) be the cylindrical polar coordinates, and the problem two-dimensional with all the quantities depending only on (r, z, t). The plate is axisymmetric with the z-axis as the axis of the symmetry. The origin of the coordinate system (r, θ, z) is taken as the middle surface of the plate and z-axis normal to it along the thickness. We take $r - z$ plane as the plane of incidence. The initial temperature in the thick plate is given by a constant temperature T_0.

For two-dimensional problem, let

$$\vec{u} = (u_r, 0, u_z), \qquad \vec{\varphi} = (0, \varphi_\theta, 0). \tag{3.1}$$

To facilitate the solution, we take

$$\begin{gathered} r' = \frac{\omega^* r}{c_1}, \quad z' = \frac{\omega^* z}{c_1}, \quad u_r' = \frac{\rho c_1\omega^* u_r}{\nu T_0}, \quad u_z' = \frac{\rho c_1 \omega^* u_z}{\nu T_0}, \quad \varphi_\theta' = \frac{\rho c_1^2 \varphi_\theta}{\nu T_0}, \\ \varphi^{*\prime} = \frac{\rho c_1^2 \varphi^*}{\nu T_0}, \quad T' = \frac{T}{T_0}, \quad t' = \omega^* t, \quad \tau_0' = \omega^*\tau_0, \quad \tau_1' = \omega^*\tau_1, \\ h' = \frac{c_1 h}{\omega^*}, \quad t_{ij}' = \frac{t_{ij}}{\nu T_0}, \quad m_{ij}' = \frac{\omega^*}{c_1 \nu T_0} m_{ij}, \end{gathered} \tag{3.2}$$

where

$$c_1^2 = \frac{\lambda + 2\mu + K}{\rho}, \qquad \omega^* = \frac{K}{\rho j}.$$

We define the Laplace and Hankel transforms as

$$\bar{f}\,(r,z,s) = L\{\bar{f}(r,z,t)\} = \int_0^\infty f(r,z,t)e^{-st}dt, \tag{3.3}$$

$$\widetilde{f}\,(\xi,z,s) = H\left\{\bar{f}\,(x,z,s)\right\} = \int_0^\infty r\bar{f}\,(x,z,s)\,J_n\,(\xi r)\,dr. \tag{3.4}$$

Equations (2.1)–(2.4), with the aid of Eqs. (3.1)–(3.4), yield

$$\widetilde{u_r''} = a_{11}\widetilde{u}_r + a_{14}\widetilde{\varphi^*} + a_{15}\widetilde{T} + b_{12}\widetilde{u_z'} + b_{13}\widetilde{\varphi_\theta'}, \tag{3.5}$$

$$\widetilde{u_z''} = a_{22}\widetilde{u}_z + a_{23}\widetilde{\varphi}_\theta + b_{21}\widetilde{u_r'} + b_{24}\widetilde{\varphi^{*\prime}} + b_{25}\widetilde{T}', \tag{3.6}$$

$$\widetilde{\varphi_\theta''} = a_{32}\widetilde{u}_z + a_{33}\widetilde{\varphi}_\theta + b_{31}\widetilde{u_r'}, \tag{3.7}$$

$$\widetilde{\varphi^{*\prime\prime}} = a_{41}\widetilde{u}_r + a_{44}\widetilde{\varphi^*} + a_{45}\widetilde{T} + b_{42}\widetilde{u_z'}, \tag{3.8}$$

$$\widetilde{T''} = a_{51}\widetilde{u}_r + a_{54}\widetilde{\varphi^*} + a_{55}\widetilde{T} + b_{52}\widetilde{u_z'}, \tag{3.9}$$

where

$$a_{11} = (\frac{\xi^2 + s^2}{\delta^2}), \quad a_{14} = \frac{p_0\xi}{\delta^2}, \quad a_{15} = -\frac{\xi}{\delta^2}(1+\tau_1 s), \quad a_{22} = (\xi^2\delta^2 + s^2),$$

$$a_{23} = -p\xi, \quad a_{32} = -\xi{\delta^*}^2, \quad a_{33} = \left(\xi^2 + \frac{s^2}{\delta_1^2} + 2{\delta^*}^2\right), \quad a_{41} = p_0\delta_1^*\xi,$$

$$a_{44} = (\xi^2 + \delta_3^* s^2 + p_1\delta_1^* + \delta_2^* s), \quad a_{45} = -\bar{\nu}\delta_1^*(1+\tau_1 s),$$

$$a_{51} = \xi\epsilon(s + \eta_0\tau_0 s^2), \quad a_{54} = \bar{\nu}\epsilon(s + \eta_0\tau_0 s^2),$$

$$a_{55} = (\xi^2 + Q^*(s + \tau_0 s^2)), \quad b_{12} = \frac{\xi(1-\delta^2)}{\delta^2}, \quad b_{13} = \frac{p}{\delta^2},$$

$$b_{21} = -\xi(1-\delta^2), \quad b_{24} = -p_0, \quad b_{25} = (1+\tau_1 s), \quad b_{31} = -{\delta^*}^2,$$

$$b_{42} = p_0\delta_1^*, \quad b_{52} = \epsilon\left(s + \eta_0\tau_0 s^2\right),$$

$$c_2^2 = \frac{\mu + K}{\rho}, \quad \delta^2 = \frac{c_2^2}{c_1^2}, \quad p = \frac{K}{\rho c_1^2}, \quad p_0 = \frac{b}{\rho c_1^2}, \quad \delta^{*2} = \frac{K c_1^2}{\gamma {\omega^*}^2}, \quad \delta_1^2 = \frac{c_3^2}{c_1^2},$$

$$c_3^2 = \frac{\gamma}{\rho j}, \quad \delta_1^* = \frac{\rho c_1^4}{\alpha_1\omega^{*2}}, \quad \bar{\nu} = \frac{m}{\nu}, \quad p_1 = \frac{\xi_1}{\rho c_1^2},$$

$$\delta_2^* = \frac{\omega_0 c_1^2}{\alpha_1\omega^*}, \quad \delta_3^* = \frac{\rho\chi c_1^2}{\alpha_1}, \quad Q^* = \frac{\rho C^* c_1^2}{K_1^*\omega^*}, \quad \epsilon = \frac{\nu^2 T_0}{\rho K_1^*\omega^*}.$$

The system of equations (3.5)-(3.9) can be written as

$$\frac{d}{dz}W(\xi, z, s) = A(\xi, s)W(\xi, z, s), \tag{3.10}$$

where

$$W = \begin{bmatrix} U \\ DU \end{bmatrix}, \qquad A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix}, \qquad U = \begin{bmatrix} \widetilde{u}_r \\ \widetilde{u}_z \\ \widetilde{\varphi}_\theta \\ \widetilde{\varphi^*} \\ \widetilde{T} \end{bmatrix}, \qquad D = \frac{d}{dz},$$

$$O = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} a_{11} & 0 & 0 & a_{14} & a_{15} \\ 0 & a_{22} & a_{23} & 0 & 0 \\ 0 & a_{32} & a_{33} & 0 & 0 \\ a_{41} & 0 & 0 & a_{44} & a_{45} \\ a_{51} & 0 & 0 & a_{54} & a_{55} \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0 & b_{12} & b_{13} & 0 & 0 \\ b_{21} & 0 & 0 & b_{24} & b_{25} \\ b_{31} & 0 & 0 & 0 & 0 \\ 0 & b_{42} & 0 & 0 & 0 \\ 0 & b_{52} & 0 & 0 & 0 \end{bmatrix}.$$

We take the solution of Eq. (3.10) as

$$W(\xi, z, s) = X(\xi, s)e^{qz}, \tag{3.11}$$

such that

$$A(\xi, s)W(\xi, z, s) = qW(\xi, z, s),$$

which leads to the eigenvalue problem. The characteristic equation corresponding to the matrix A is given by

$$\det(A - qI) = 0,$$

an expansion gives

$$q^{10} - \lambda_1 q^8 + \lambda_2 q^6 - \lambda_3 q^4 + \lambda_4 q^2 - \lambda_5 = 0, \tag{3.12}$$

where λ_1, λ_2, λ_3, λ_4 and λ_5 are given in Appendix.

The roots of Eq. (3.12) are $\pm q_i$, $i = 1, 2, 3, 4, 5$.

The eigenvectors $X_i(\xi, s)$ corresponding to the eigenvalues q_i may be obtained by solving

$$[A - qI]X_i(\xi, s) = 0.$$

The set of eigen-vector $X_i(\xi, s)$ can be written as

$$X_i(\xi,s) = \begin{bmatrix} X_{i1}(\xi,s) \\ X_{i2}(\xi,s) \end{bmatrix},$$

where

$$X_{i1}(\xi,s) = \begin{bmatrix} a_i q_i \\ b_i \\ -\xi \\ d_i \\ e_i \end{bmatrix}, \quad X_{i2}(\xi,s) = \begin{bmatrix} a_i q_i^2 \\ b_i q_i \\ -\xi q_i \\ d_i q_i \\ e_i q_i \end{bmatrix}, \quad q = q_i,\ i = 1,2,3,4,5,$$

$$X_{j1}(\xi,s) = \begin{bmatrix} -a_i q_i \\ b_i \\ -\xi \\ d_i \\ e_i \end{bmatrix}, \quad X_{j2}(\xi,s) = \begin{bmatrix} a_i q_i^2 \\ -b_i q_i \\ \xi q_i \\ -d_i q_i \\ -e_i q_i \end{bmatrix}, \quad \begin{array}{l} j = i+5,\ q = -q_i, \\ i = 1,2,3,4,5, \end{array}$$

where

$$\begin{aligned}
a_i &= \frac{\xi}{\Delta_i}[r_1^2\{r_2(r_3(1-\delta^2) + p\delta^{*^2}) - p_0^2\delta_1^* r_3\} \\
&\quad + \epsilon s r_1 r_5\{r_3(r_2 + \bar{\nu}^2\delta_1^*(1-\delta^2) - 2\bar{\nu}p_0\delta_1^* - \bar{\nu}\delta_1^*) + p\bar{\nu}^2\delta_1^*\delta^{*^2}\}], \\
b_i &= \frac{-1}{\Delta_i}[r_1^2\{r_2(r_3 r_4 + p\delta^{*^2} q_i^2) - p_0^2\delta_1^*\xi^2 r_3\}, \\
&\quad + \epsilon s r_3 r_1 r_5(\xi^2 r_2 + \bar{\nu}^2\delta_1^* r_4 - 2\bar{\nu}p_0\delta_1^*\xi^2) + \epsilon s p\bar{\nu}^2\delta_1^*\delta^{*^2} q_i^2 r_1 r_5], \\
d_i &= \delta_1^*(p_0 r_1 + \epsilon s\bar{\nu} r_5)(\xi a_i + b_i)q_i/(-r_2 r_1 - \epsilon s\bar{\nu}^2\delta_1^* r_5), \\
e_i &= [\epsilon r_5\{(r_2 r_1 + \epsilon s\bar{\nu}^2\delta_1^* r_5) \\
&\quad - \bar{\nu}\delta_1^*(p_0 r_1 + \epsilon\bar{\nu} r_5)\}](\xi a_i + b_i)q_i/\{r_1(-r_2 r_1 - \epsilon s\bar{\nu}^2\delta_1^* r_5)\}], \\
\Delta_i &= \delta^{*^2}[r_1^2\{r_2(\xi^2 + s^2 - q_i^2) + p_0^2\delta_1^*(q_i^2 - \xi^2)\} \\
&\quad + \epsilon s r_2 r_1 r_5(\xi^2 - q_i^2) + \epsilon s\delta_1^* r_1 r_5\{\bar{\nu}^2(\xi^2 + s^2 - q_i^2) - 2p_0\bar{\nu}(\xi^2 - q_i^2)\}], \\
r_1 &= (\xi^2 + Q^*(s + \tau_0 s^2) - q_i^2), \\
r_2 &= (\xi^2 + \delta_3^* s^2 + p_1\delta_1^* + \delta_2^* s - q_i^2), \\
r_3 &= \left(\xi^2 + \frac{s^2}{\delta_1^2} + 2\delta^{*^2} - q_i^2\right), \qquad r_4 = (\xi^2 + s^2 - \delta^2 q_i^2), \\
r_5 &= (1 + \tau_1 s)(s + \eta_0\tau_0 s^2).
\end{aligned}$$

We take the solution of Eq. (3.11) as

$$W(\xi,z,s) = \sum_{i=1}^{5} N_i X_i(\xi,s)\cosh(q_i z), \tag{3.13}$$

where N_1, N_2, N_3, N_4 and N_5 are arbitrary constants.

4. Boundary conditions

The boundary conditions at the surface $z = \pm d$ of the plate is given by

$$\frac{dT}{dz} = \pm g_0 F(r,z), \tag{4.1}$$

$$t_{zz} = \delta(t)H(a-r), \tag{4.2}$$

$$t_{zr} = 0, \tag{4.3}$$

$$m_{z\theta} = 0, \tag{4.4}$$

$$\frac{d\varphi^*}{dz} = 0. \tag{4.5}$$

where $F(r,z) = z^2 e^{-\omega r}$, $\omega > 0$.

$\delta()$ is a Dirac delta function and $H()$ is the Heavi side (the unit step) function and t_{zz}, t_{zr} and $m_{z\theta}$ are given by

$$t_{zz} = (\lambda + 2\mu + K)\frac{\partial u_z}{\partial z} + \lambda\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) - \nu\left(1 + \tau_1\frac{\partial}{\partial r}\right)T + b\varphi^*, \tag{4.6}$$

$$t_{zr} = (\mu + K)\frac{\partial u_r}{\partial z} + \mu\frac{\partial u_z}{\partial r} - K\varphi_\theta, \tag{4.7}$$

$$m_{z\theta} = \gamma\frac{\partial \varphi_\theta}{\partial z}. \tag{4.8}$$

The expressions of displacements, microrotation, volume fraction field, temperature distribution and stresses in the transformed domain are obtained with the aid of Eqs. (3.2)–(3.4) and (3.13)–(4.8) as

$$(\widetilde{u}_r, \widetilde{u}_z, \widetilde{\varphi_\theta}, \widetilde{\varphi^*}, \widetilde{T}) = \frac{1}{\Delta}\sum_{i=1}^{5}(a_i q_i, b_i, -\xi, d_i, e_i)\Delta_i \cosh(q_i z), \tag{4.9}$$

$$(\widetilde{t_{zz}}, \widetilde{t_{zr}}, \widetilde{m_{z\theta}}) = \frac{1}{\Delta}\sum_{i=1}^{5}(L_i, M_i, P_i)\Delta_i \cosh(q_i z), \tag{4.10}$$

where

$$\Delta = \begin{vmatrix} S_1 & S_2 & S_3 & S_4 & S_5 \\ T_1 & T_2 & T_3 & T_4 & T_5 \\ U_1 & U_2 & U_3 & U_4 & U_5 \\ V_1 & V_2 & V_3 & V_4 & V_5 \\ W_1 & W_2 & W_3 & W_4 & W_5 \end{vmatrix}$$

and Δ_i $(i = 1, 2, 3, 4, 5)$ are obtained from Δ by replacing ith column of Δ with

$|Q, R, 0, 0, 0|^{tr}$ also

$$
\begin{aligned}
&S_i = e_i q_i \sinh(q_i d), \quad T_i = L_i \cosh(q_i d), \quad U_i = M_i \cosh(q_i d), \quad i = 1,2,3,4,5,\\
&V_i = P_i \cosh(q_i d), \quad W_i = d_i q_i \sinh(q_i d), \quad i = 1,2,3,4,5,\\
&Q = \pm g_0 \frac{z^2\omega}{(z^2+\omega^2)^{3/2}}, \qquad R = \frac{aJ_1(\xi a)}{\xi},\\
&L_i = \left[\frac{\lambda\xi a_i q_i}{\rho c_1^2} + p_0 d_i - (1+\tau_1 s)e_i + b_i q_i\right], \qquad i = 1,2,3,4,5,\\
&M_i = \left[-\frac{\mu\xi b_i}{\rho c_1^2} + \xi p + \left(\frac{\mu}{\rho c_1^2} - p\right) a_i q_i^2\right], \qquad i = 1,2,3,4,5,\\
&P_i = \frac{-\gamma\xi {\omega^*}^2 q_i}{\rho c_1^4}, \qquad i = 1,2,3,4,5.
\end{aligned}
$$

Particular cases

(i) Take $\tau_1 = 0$ and $\eta_0 = 1$, then Eq. (4.9)–(4.10), yield the corresponding expressions for a micropolar porous thermoelastic with one relaxation time.

(ii) Take $\tau_1 > 0$ and $\eta_0 = 0$, then Eqs. (4.9)–(4.10), yield the corresponding expressions for a micropolar porous thermoelastic with two relaxation times.

(iii) In the absence of thermal effect, the boundary conditions (4.1)–(4.5) for micropolar porous medium reduce to the following forms:

$$
\begin{aligned}
t_{zz} &= \delta(t)H(a-r),\\
t_{zr} &= 0,\\
m_{z\theta} &= 0,\\
\frac{d\varphi^*}{dz} &= 0,
\end{aligned}
$$

and by following the same procedure, we obtain the corresponding expressions for displacements, microrotation, volume fraction field and stresses for micropolar porous medium as

$$
(\widetilde{u}_r, \widetilde{u}_z, \widetilde{\varphi_\theta}, \widetilde{\varphi^*}) = \frac{1}{\Delta^*}\sum_{i=1}^{4}(a_i q_i, b_i, -\xi, d_i)\Delta_i^* \cosh(q_i z),
$$

$$
(\widetilde{t_{zz}}, \widetilde{t_{zr}}, \widetilde{m_{z\theta}}) = \frac{1}{\Delta^*}\sum_{i=1}^{4}(L_i, M_i, P_i)\Delta_i^* \cosh(q_i z),
$$

where

$$
\Delta^* = \begin{vmatrix} T_1^* & T_2^* & T_3^* & T_4^* \\ U_1^* & U_2^* & U_3^* & U_4^* \\ V_1^* & V_2^* & V_3^* & V_4^* \\ W_1^* & W_2^* & W_3^* & W_4^* \end{vmatrix},
$$

and Δ_i^* $(i=1,2,3,4)$ are obtained from Δ^* by replacing ith column of Δ^* with $|R,0,0,0|^{tr}$, also

$$\begin{aligned} T_i^* &= L_i^* \cosh(q_i d), \quad U_i^* = M_i^* \cosh(q_i d), \qquad i=1,2,3,4,\\ V_i^* &= P_i^* \cosh(q_i d), \quad W_i^* = d_i q_i \sinh(q_i d), \quad i=1,2,3,4,\\ L_i^* &= \left[\frac{\lambda\xi a_i q_i}{\rho c_1^2} + p_0 d_i + b_i q_i\right], \quad i=1,2,3,4,\\ M_i^* &= \left[-\frac{\mu\xi b_i}{\rho c_1^2} + \xi p + \left(\frac{\mu}{\rho c_1^2} - p\right) a_i q_i^2\right], \quad i=1,2,3,4,\\ P_i^* &= \frac{-\gamma\xi {\omega^*}^2 q_i}{\rho c_1^4}, \quad i=1,2,3,4. \end{aligned}$$

(iv) By neglecting the porous effect, i.e., α_1, b, ξ_1, ω_0, χ and φ^* tend to zero, the boundary conditions for micropolar thermoelastic medium are given by

$$\begin{aligned} \frac{dT}{dz} &= \pm g_0 F(r,z),\\ t_{zz} &= \delta(t) H(a-r),\\ t_{zr} &= 0,\\ m_{z\theta} &= 0, \end{aligned}$$

and the corresponding expressions are given by

$$\begin{aligned} (\widetilde{u}_r, \widetilde{u}_z, \widetilde{\varphi_\theta}, \widetilde{T}) &= \frac{1}{\Delta^{**}} \sum_{i=1}^{4} (a_i q_i, b_i, -\xi, e_i) \Delta_i^{**} \cosh(q_i z),\\ (\widetilde{t_{zz}}, \widetilde{t_{zr}}, \widetilde{m_{z\theta}}) &= \frac{1}{\Delta^{**}} \sum_{i=1}^{4} (L_i, M_i,\ P_i) \Delta_i^{**} \cosh(q_i z), \end{aligned}$$

where

$$\Delta^{**} = \begin{vmatrix} S_1^{**} & S_2^{**} & S_3^{**} & S_4^{**}\\ T_1^{**} & T_2^{**} & T_3^{**} & T_4^{**}\\ U_1^{**} & U_2^{**} & U_3^{**} & U_4^{**}\\ V_1^{**} & V_2^{**} & V_3^{**} & V_4^{**} \end{vmatrix},$$

and Δ_i^{**} $(i=1,2,3,4)$ are obtained from Δ^{**} by replacing ith column of Δ^{**} with $|Q,R,0,0|^{tr}$ also

$$\begin{aligned} S_i^{**} &= e_i q_i \sinh(q_i d), \quad T_i^{**} = L_i^{**} \cosh(q_i d), \quad i=1,2,3,4,\\ U_i^{**} &= M_i^{**} \cosh(q_i d), \quad V_i^{**} = P_i^{**} \cosh(q_i d), \quad i=1,2,3,4,\\ L_i^{**} &= \left[\frac{\lambda\xi a_i q_i}{\rho c_1^2} + p_0 d_i - (1+\tau_1 s) e_i + b_i q_i\right], \quad i=1,2,3,4, \end{aligned}$$

$$M_i^{**} = \left[-\frac{\mu \xi b_i}{\rho c_1^2} + \xi p + \left(\frac{\mu}{\rho c_1^2} - p \right) a_i q_i^2 \right], \quad i = 1, 2, 3, 4,$$

$$P_i^{**} = \frac{-\gamma \xi {\omega^*}^2 q_i}{\rho c_1^4}, \quad i = 1, 2, 3, 4.$$

5. Inversion of transforms

The transformed displacements, microrotation, volume fraction field, temperature distribution and stresses are of the form $\widetilde{f}(\xi, z, s)$ and to obtain the function $f(r, z, t)$, the inversion of the Hankel transform is of the form

$$\widetilde{f}(\xi, z, s) = \int_0^\infty \xi \bar{f}(\xi, z, s) J_n(\xi r) d\xi. \tag{5.1}$$

The inversion formula for the Laplace transform is taken as

$$f(r, z, t) = \frac{1}{2\iota\pi} \int_{c-\iota\infty}^{c+\iota\infty} \bar{f}(r, z, s) e^{-st} ds, \tag{5.2}$$

where c is an arbitrary constant greater than all real parts of the singularities of $\bar{f}(r, z, t)$.

6. Numerical results and discussion

Following ERINGEN [27], the values of micropolar parameters for numerical computation are given as

$$\lambda = 9.4 \times 10^{10}\ \mathrm{N \cdot m^{-2}}, \qquad \mu = 4.0 \times 10^{10}\ \mathrm{N \cdot m^{-2}},$$
$$K = 1.0 \times 10^{10}\ \mathrm{N \cdot m^{-2}}, \qquad \rho = 1.74 \times 10^{3}\ \mathrm{Kg \cdot m^{-3}},$$
$$j = 0.2 \times 10^{-19}\ \mathrm{m^2}, \qquad \gamma = 0.779 \times 10^{-9}\ \mathrm{N}.$$

Following DHALIWAL and SINGH [28], we take the values of thermal parameters as

$$C^* = 1.04 \times 10^{3}\ \mathrm{J \cdot Kg^{-1} \cdot K^{-1}}, \qquad K_1^* = 1.7 \times 10^{6}\ \mathrm{J \cdot m^{-1} \cdot s^{-1} \cdot K^{-1}},$$
$$\alpha_t = 2.33 \times 10^{-5}\ \mathrm{K^{-1}}, \qquad \tau_0 = 6.131 \times 10^{-13}\ \mathrm{s},$$
$$\tau_1 = 8.765 \times 10^{-13}\ \mathrm{s}, \qquad m = 1.13849 \times 10^{10}\ \mathrm{N/m^2 \cdot K},$$
$$T_0 = 0.298 \times 10^{3}\ \mathrm{K}.$$

The values of void parameters are given as

$$\alpha_1 = 3.688 \times 10^{-9}\ \text{N}, \qquad b = 1.138494 \times 10^{10}\ \text{N/m}^2,$$
$$\xi_1 = 1.1475 \times 10^{10}\ \text{N/m}^2, \qquad \chi = 1.1753 \times 10^{-19}\ \text{m}^2,$$
$$\omega_0 = 0.0787 \times 10^{-1}\ \text{N} \times \text{s/m}^2.$$

The variations of normal stress, tangential couple stress, volume fraction field and temperature distribution with distance r in the cases of micropolar thermoelastic porous medium (MTPM), micropolar thermoelastic medium (MTM) and micropolar porous medium (MPM) are shown in Figs. 1–4, respectively. In all these figures, MTPM, MPM and MTM are corresponding to solid line (——), small dash line (- - - - -) and dash line with centered symbol ($- * - * -$), respectively.

Figure 1 shows the variations of normal stress t_{zz} for MTPM, MTM and MPM. Initially, the value of t_{zz} increases for MTPM and MTM, and decreases for MPM and then oscillates with distance r about the origin. The value of t_{zz} for MTPM is greater in comparison to MTM and MPM for the ranges $1 \leq r \leq 2$, $2.8 \leq r \leq 3.6$, $4.4 \leq r \leq 5.2$, $6 \leq r \leq 6.7$, $7.5 \leq r \leq 8$, whereas for the ranges $2 \leq r \leq 2.8$, $3.6 \leq r \leq 4.4$, $5.2 \leq r \leq 6$, $6.7 \leq r \leq 7.5$, its value for MPM is greater in comparison to MTPM and MTM.

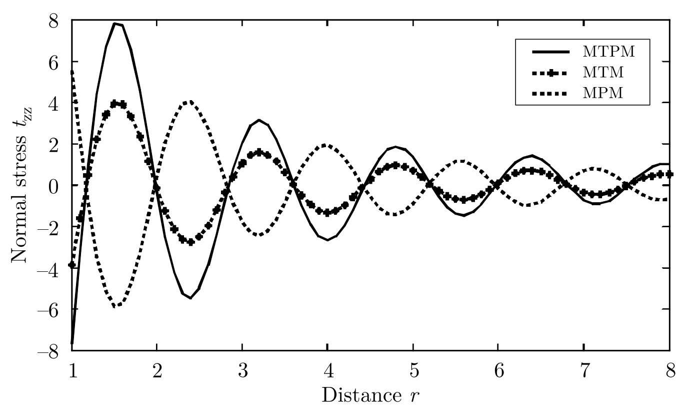

Fig. 1. Variations of normal stress t_{zz}.

Figure 2 depicts the variations of $m_{z\theta}$ for MTPM, MTM and MPM. The value for $m_{z\theta}$ decreases sharply for MTPM, MTM and MPM for the range $1 \leq r \leq 2$ and then oscillates with distance r about the origin. The value of $m_{z\theta}$ for MPM is greater in comparison to MTPM and MTM for $1 \leq r \leq 1.4$, $2.3 \leq r \leq 3$, $3.9 \leq r \leq 4.3$, $7.2 \leq r \leq 7.8$, whereas its value for MPM is smaller in comparison to MTPM and MTM for the ranges $1.7 \leq r \leq 2.2$, $3.2 \leq r \leq 3.8$, $4.7 \leq r \leq 5.3$,

$6.3 \le r \le 6.8$. Although the behavior and variation of $m_{z\theta}$ are similar for MTPM, MTM and MPM for the whole range $1 \le r \le 8$, the magnitude values are distinct.

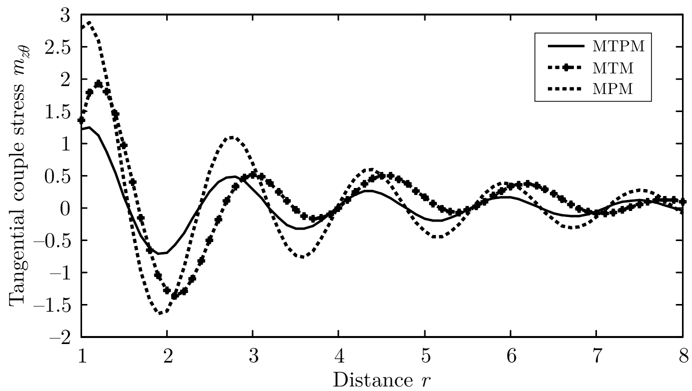

Fig. 2. Variations of tangential couple stress $m_{z\theta}$.

Figure 3 depicts the variations of the volume fraction field φ^*. Initially, the value of φ^* increases and then oscillates with distance r for MTPM. For MPM, this value starts with sharp decrease and then oscillates with distance r about the origin. For the ranges $1 \le r \le 3.8, 4.3 \le r \le 5.6, 7.2 \le r \le 8$, the behavior of φ^* is opposite for MTPM and MPM, while for the ranges $5.6 \le r \le 6.4$ and $6.4 \le r \le 7.2$, the behavior of φ^* is similar for MTPM and MPM.

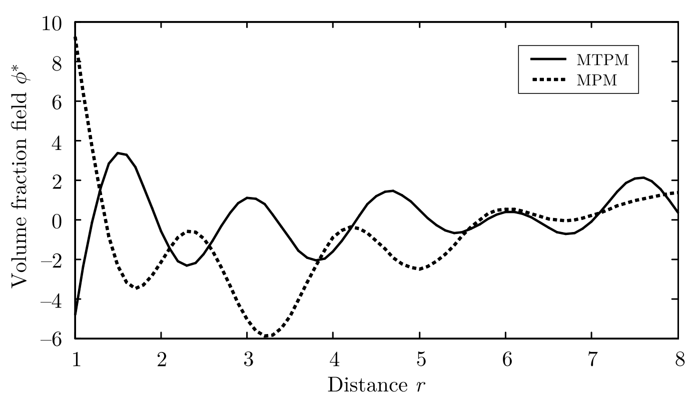

Fig. 3. Variations of volume fraction field φ^*.

Figure 4 shows the variations of the temperature distribution T. Initially, the value of T increases for MTPM and decreases for MTM and then oscillates with distance r about the origin. The variation pattern of T is opposite for the ranges

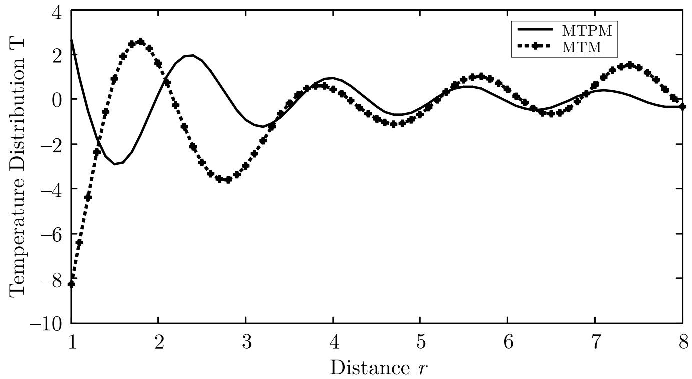

Fig. 4. Variations of temperature distribution T.

$1 \leq r \leq 3.2$ for MTPM and MTM and similar for $3.2 \leq r \leq 8$. The behavior of temperature distribution is oscillatory for the entire range.

7. Conclusions

due to the complicated nature of the governing equations for the thermoelastic micropolar porous theory, the work done in this field by means of the eigenvalue approach, is unfortunately limited in number. In this paper, the eigenvalue approach is used to analyze a two-dimensional problem for a micropolar thermoelastic porous circular plate. This method provides quite a successful approach in dealing with such problems, and gives exact solutions in the transformed domain without any assumed restrictions on the actual physical quantities that appear in the governing equations. Thus, from the above presented numerical results and discussion, a significant effect of thermal forces and porosity on normal stress and tangential couple stress is observed. It is also observed that the variations of volume fraction field and temperature distribution are oscillatory in nature. The problem studied in this paper is a significant problem of continuum mechanics and it provides a breakthrough for the researchers working in the field of micropolar thermoelastic media.

Appendix

$$\lambda_1 = -(a_{11}+a_{22}+a_{33}+a_{44}+a_{55}+b_{12}b_{21}+b_{13}b_{31}+b_{25}b_{52}+b_{24}b_{42}),$$

$$\begin{aligned}\lambda_2 = {} & -a_{14}a_{41}+a_{33}a_{55}+a_{44}a_{55}+a_{11}a_{55}+a_{22}a_{55}+a_{33}a_{44}+a_{11}a_{33}+a_{22}a_{33} \\ & +a_{11}a_{44}+a_{22}a_{44}+a_{11}a_{22}-a_{15}a_{51}-a_{45}a_{54}-a_{23}a_{32} \\ & +(a_{33}+a_{44}+a_{55})b_{12}b_{21}-(a_{14}b_{42}+a_{15}b_{52}+a_{32}b_{13})b_{21}\end{aligned}$$

$$+(a_{11}+a_{33}+a_{55})b_{42}b_{24}+(a_{11}+a_{33}+a_{44})b_{25}b_{52}$$
$$-(a_{41}b_{24}+a_{23}b_{31}+a_{51}b_{25})b_{12}+(a_{22}+a_{44}+a_{55}+b_{42}b_{24}+b_{25}b_{52})b_{31}b_{13}$$
$$-a_{45}b_{52}b_{24}-a_{54}b_{42}b_{25},$$

$$\begin{aligned}\lambda_3 = {} & (a_{11}a_{22}+a_{22}a_{55})(a_{33}+a_{44})-a_{23}a_{32}(a_{11}+a_{44}+a_{55})\\ & +a_{11}a_{55}(a_{22}+a_{33}+a_{44})+a_{33}a_{44}(a_{11}+a_{22}+a_{55})-a_{45}a_{54}(a_{11}+a_{22}+a_{33})\\ & -a_{14}a_{41}(a_{22}+a_{33}+a_{55})-a_{15}a_{51}(a_{22}+a_{33}+a_{44})+b_{42}b_{25}\\ & (a_{14}a_{51}-a_{11}a_{54}-a_{33}a_{54})+b_{52}b_{25}(-a_{14}a_{41}+a_{11}a_{33}+a_{14}a_{44}+a_{33}a_{44})\\ & +b_{52}b_{24}(a_{15}a_{41}-a_{11}a_{45}-a_{33}a_{45})-b_{12}b_{25}(a_{33}a_{51}+a_{44}a_{51}-a_{41}a_{54})\\ & -b_{12}b_{24}(a_{33}a_{41}-a_{45}a_{51}+a_{41}a_{55})\\ & +b_{42}b_{24}(-a_{15}a_{51}+a_{11}a_{33}+a_{11}a_{55}+a_{33}a_{55})\\ & -a_{32}b_{21}b_{13}(a_{44}+a_{55})+b_{31}b_{13}(a_{22}a_{44}+a_{22}a_{55}+a_{44}a_{55}-a_{45}a_{54})\\ & +b_{21}b_{42}(a_{15}a_{54}-a_{14}a_{33}-a_{14}a_{55})+b_{21}b_{52}(a_{14}a_{45}-a_{15}a_{33}-a_{15}a_{44})\\ & +b_{12}b_{21}(-a_{45}a_{54}+a_{33}a_{44}+a_{44}a_{55}+a_{33}a_{55})-b_{12}b_{31}(a_{23}a_{44}+a_{23}a_{55})\\ & -b_{31}b_{13}(a_{54}b_{42}b_{25}-a_{44}b_{52}b_{25}+a_{45}b_{52}b_{24}-a_{55}b_{24}b_{42})+a_{32}b_{13}\\ & (a_{51}b_{25}+a_{41}b_{24})+a_{15}a_{41}a_{54}+a_{14}a_{45}a_{51}+(a_{14}b_{42}+a_{15}b_{52})a_{23}b_{31},\end{aligned}$$

$$\begin{aligned}\lambda_4 = {} & (a_{44}a_{55}-a_{45}a_{54})(a_{33}b_{12}b_{21}-a_{23}b_{12}b_{31})+(a_{15}a_{54}-a_{14}a_{55})\\ & (a_{33}b_{21}b_{42}-a_{23}b_{31}b_{42})+(a_{14}a_{45}-a_{15}a_{44})(a_{33}b_{21}b_{52}-a_{23}b_{31}b_{52})\\ & +(a_{45}a_{54}-a_{44}a_{55})(a_{32}b_{21}b_{13}-a_{22}b_{31}b_{13})+a_{33}b_{12}b_{24}(a_{45}a_{51}-a_{41}a_{55})\\ & +a_{33}b_{42}b_{24}(-a_{15}a_{51}+a_{11}a_{55})+a_{32}b_{13}b_{24}(a_{41}a_{55}-a_{45}a_{51})\\ & +(a_{41}a_{54}-a_{44}a_{51})(a_{33}b_{12}b_{25}-a_{32}b_{13}b_{25})+a_{33}b_{42}b_{25}(a_{14}a_{51}-a_{11}a_{54})\\ & +a_{33}b_{52}b_{25}(a_{11}a_{44}-a_{14}a_{41})+a_{15}a_{51}(a_{23}a_{32}-a_{22}a_{33}-a_{22}a_{44}-a_{33}a_{44})\\ & +a_{14}a_{45}a_{51}(a_{22}+a_{33})+a_{15}a_{41}a_{54}(a_{22}+a_{33})\\ & +a_{45}a_{54}(a_{23}a_{32}-a_{11}a_{22}-a_{11}a_{33}\\ & -a_{22}a_{33})+a_{14}a_{41}(a_{23}a_{32}-a_{22}a_{33}-a_{22}a_{55}-a_{33}a_{55})-a_{11}a_{23}a_{32}(a_{44}+a_{55})\\ & +a_{55}(a_{11}a_{22}a_{33}-a_{23}a_{32}a_{44})+a_{11}a_{22}a_{44}(a_{33}+a_{55})+a_{33}a_{44}a_{55}(a_{11}+a_{22})\\ & +(a_{15}a_{41}-a_{11}a_{45})a_{33}b_{24}b_{52},\end{aligned}$$

$$\begin{aligned}\lambda_5 = {} & (a_{22}a_{33}-a_{23}a_{32})(a_{11}a_{44}a_{55}-a_{11}a_{45}a_{54}+a_{14}a_{45}a_{51})\\ & +(a_{15}a_{54}-a_{14}a_{55})(a_{22}a_{33}a_{41}-a_{23}a_{32}a_{41})+(a_{23}a_{32}-a_{22}a_{33})a_{15}a_{44}a_{51}\end{aligned}$$

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