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# Mechanics of seabed liquefaction and resolidification

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The problem of pore water pressure changes in the seabed is considered. Two mechanisms of pore pressure changes are distinguished. The first is caused by external excitations, such as earthquakes, when pore pressure is gradually generated, leading to liquefaction. The second mechanism is caused by water waves, and it leads to cyclic changes in pore water pressure and the mean effective stress. Under certain conditions, when the effective stress path tends to exceed the failure condition, the regrouping of effective stresses takes place, as the soil should accommodate to new conditions. Then, the mechanism of resolidification of the seabed is described. It is concluded that after resolidification, the seabed is in a virgin state, as liquefaction erases the previous history of the seabed structure. A critical discussion of selected existing approaches to the problem of pore-pressure changes and the mechanism of liquefaction is presented in detail, in the form of extensive appendices. Some of these appendices deal with the crucial aspects of the mechanics of liquefaction such as, for example, the drained/undrained conditions.

**Key words:** liquefaction, resolidification, seabed, granular soils, dissipation, pore-pressure changes.



## 1. Introduction

This paper deals with some aspects of the behaviour of a granular seabed, such as pore water pressure changes, liquefaction, pore pressure dissipation and resolidification. These phenomena influence the behaviour of marine structures founded on the seabed, as they may lead to the instability of such structures, their sinking in the liquefied seabed, the floating of pipelines buried in the seabed, massive slides, etc. Therefore, the understanding of the mechanics of pore water pressure changes in the seabed is of basic importance in both marine and geotechnical engineering. In some parts of this text, the designation of "pore water pressure" is replaced just by "pore pressure", for the sake of abbreviation, that is commonly used in geomechanics.

Massive liquefaction of the seabed is possible only during strong external excitations, such as earthquakes. It leads to serious damage to marine infrastructure, as in the case of the Kocaeli earthquake, see Groot *et al* [1], or other catastrophic events, such as the earthquake in Kobe. In such cases, the sequence of events is the following: *external cyclic shearing of saturated soil under undrained conditions → pore water pressure generation and the corresponding degradation of the soil's macroscopic properties, such as the shear modulus and its strength → the liquefaction of the seabed → the consequences of liquefaction, such as the loss of stability of marine structures, the sinking of structures, massive flows of the liquefied seabed*, etc. Changes in excess pore pressure in the situation described above will be designated for simplicity as *passive pore water pressure changes*. Pore water is trapped in pores, and an increase in its pressure is a result of the reaction of the soil skeleton to external excitations. In such a situation, pore water is passive, as it only reacts to external loadings, which increase its pressure. The above mechanism is mainly studied in soil mechanics.

The duration of an earthquake is relatively short, measured in tens of seconds. When seismic excitations cease, some amount of energy is stored in the seabed in the form of excess pore pressure. This pressure dissipates after the earthquake, tending to the static equilibrium of the seabed. This process is called the *resolidification* of the seabed. In the case of on-land resolidification, one can observe geysers of pore water and volcano craters on the land surface, caused by excess pore water pressure. Probably similar phenomena take place during the resolidification of the seabed, but there is no information supporting this supposition. The mechanics of resolidification is still not well understood, and attempts to describe this phenomenon will be presented in this paper. Some researchers ask why seabeds, subjected in their history to numerous earthquakes and liquefactions, liquefy again during subsequent earthquakes. In their opinion, such seabeds should be compacted enough to prevent further liquefaction, Sumer [2]. We shall try to answer this question on the basis of current soil mechanics arguments, both experimental and theoretical.

Pore pressures in the seabed may also vary as a result of such excitations as water wave actions. Water waves cause cyclic pore pressure changes in the seabed, but they do not generate excess pore pressure, and therefore cannot cause liquefaction. Pore pressure oscillates around a certain mean level, and may sometimes exceed the Coulomb–Mohr yield condition (defined in terms of effective stresses). In such a situation, the regrouping of effective stresses takes place, which is explained by Sawicki and Staroszczyk [3]. The sequence of events is the following: *water waves change pressures at the mudline → these changes influence pore water pressures in the seabed → the soil skeleton reacts to such pore pressure changes*. Note that there is a basic difference between these two mechanisms. In geotechnical engineering, the sequence of events is the following: *active action of the soil skeleton → passive reaction of pore water*. In the marine approach, the sequence of events is different: *active action of water waves and subsequent changes in pore pressure → passive reaction of the soil* *skeleton*. Therefore, it seems that the designation *active pore pressure changes* is adequate for this situation, at least for practical reasons.

This paper describes these two mechanisms of pore pressure changes in the seabed, and discusses the problem of the seabed liquefaction. Then, the problem of the resolidification of the seabed is analyzed. Some of the existing approaches to the problem of pore-pressure changes in the seabed and its liquefaction are extensively discussed, in the appendices. The original features of the present paper can be summarized as follows:

(a) Explanation of two mechanisms of pore-pressure changes in the seabed.

(b) Critical discussion of some erroneous approaches to this problem.

(c) Explanation of the process of resolidification of the seabed, including the hypothesis that liquefaction erases the previous history of seabed formation.

## 2. Understanding liquefaction

Liquefaction is a process that transforms an initially solid material into a liquid. This phenomenon is typical of saturated granular soils, as it does not take place in classical materials. The saturated granular soil consists of the soil skeleton, the pores of which are filled with water. Such a material can serve as a good foundation soil for marine structures. Under certain conditions, however, such as earthquake excitations or water waves actions, this initially solid foundation soil begins to behave like a liquid. Structures sink in such a foundation soil, the seabed flows, etc.

Theoretical description of this phenomenon has for decades been a great challenge in geomechanics, geotechnical and marine engineering, and even in geophysics. Thousands of papers, on various aspects of liquefaction, have been published, but no consensus has been achieved, as in the classical mechanics of materials, just to mention elasticity, plasticity or rheology. Several attempts to describe this fascinating phenomenon are presented in state-of-the-art publications, for example Been and Jefferies [4], Ishihara [5], Lade and Yamamuro [6] or Sawicki and Mierczyński [7]. In other recent books on soil mechanics, the phenomenon of liquefaction is practically overlooked, see Gudehus [8].

A common assumption in soil mechanics is that liquefaction takes place under undrained conditions. Pore-pressure build-up precedes this phenomenon, reducing effective stresses, and subsequently the shearing resistance of the soil skeleton. These phenomena have been confirmed experimentally by dozens of independent laboratories. On the basis of their findings, a simple definition of liquefaction has been formulated, according to which liquefaction takes place when the mean effective stress equals zero. This definition is physically obvious, as granular soils cannot support negative (extension) effective stresses. However, some researchers, mainly marine engineers, contest these geotechnical achievements, and suggest different approach to the problem of liquefaction, see Sumer and Fredsoe [9], Sumer *et al.* [10], Kirca *et al.* [11]. Their method is discussed in detail in the Appendices to this paper.

The first shortcoming concerns the derivation of the consolidation equation with the "source term", which is supposed to describe pore-pressure generation, according to the authors' opinion. An elementary derivation of the consolidation equation is presented in Appendix 1. The same technique is applied to derive the consolidation equation with the "source term". It has been shown that such an equation follows from a false assumption about a vague Darcy's law.

Another problem is a false interpretation of cyclic effective stress components, see Appendix 3. Recall, that the behavior of materials depends on stress invariants, certainly not on particular stress components, expressed in chosen co-ordinate systems. Note that the mechanical properties of materials do not depend on the choice of a co-ordinate system. They are objective, so constitutive equations should be formulated in terms of stress and strain invariants.

One of the aims of this paper is to build bridges between civil and marine engineering, by finding a common language. The above critical remarks about the "marine" approach to the problem of seabed liquefaction should initiate professional discussion, which may lead to a better understanding of this important phenomenon, and subsequently to the development of practical countermeasures.

## 3. Pore pressure generation and liquefaction in the case of passive pore-pressure changes

As already explained, passive pore pressure changes take place when the soil skeleton transfers some part of external excitations to pore water. The reaction of trapped pore water can be observed mainly as the development of excess pore pressure and subsequent liquefaction. From the theoretical point of view, the process of pore pressure generation needs original constitutive equations, Sawicki and Mierczyński [7], in contrast to the marine approach, in which wave-induced pore pressure changes can be determined from classical models such as filtration or consolidation theories, see Jeng [12]. In order to explain the passive mechanism of pore pressure changes, let us consider an experiment performed in the triaxial apparatus on a cylindrical sample of saturated sand. The total stresses acting on the sample are denoted as $\sigma_z$ (vertical) and $\sigma_x$ (horizontal–radial). The corresponding effective stresses are denoted as follows:

$$\sigma_z' = \sigma_z - u, \qquad \sigma_x' = \sigma_x - u, \tag{3.1}$$

where $u$ denotes pore pressure.

The following stress invariants are useful in the analysis of triaxial investigations:

$$p' = \frac{1}{3}(\sigma'_z + 2\sigma'_x), \qquad q = \sigma'_z - \sigma'_x, \tag{3.2}$$

where $p'$ denotes the mean effective stress and $q$ is the stress deviator.

The clearest interpretation of experimental data can be performed in the effective stress space, as shown in Fig. 1. In this space, there are three important objects, namely the Coulomb-Mohr failure line (C-M), the instability line and the phase transformation line (PTL). Effective stress path cannot exceed the C-M line, which corresponds to the plastic flow of soil. The effective stress states above this line are physically inadmissible.

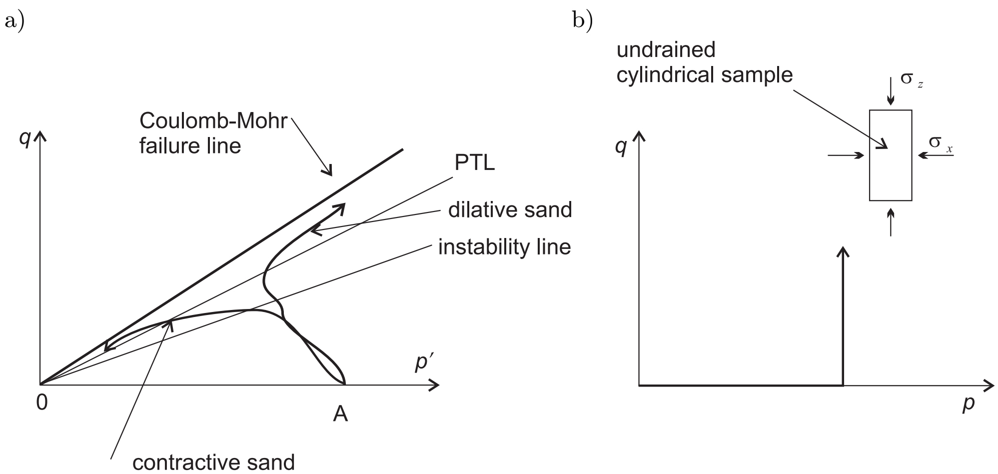


Fig. 1. a) Effective stress paths followed during the undrained shearing of saturated sand. b) The total stress path corresponding to the behavior shown in Fig. 1a.

During the experiment, total stresses change in such a way that the mean total stress $p = (\sigma_z + 2\sigma_x)/3$ is kept constant, and only the stress deviator $q$ changes. Pore pressure is recorded during experiments. Figure 1 illustrates two characteristic effective stress paths followed during such experiments, one for initially contractive samples and another for initially dilative ones. The notion of initially contractive and dilative states is also quite new in soil mechanics, and it introduces a kind of parallel classification of the initial states of granular soils, aside from the classical distinction between initially loose, medium dense, and dense sands, which is rather arbitrary. The definition of initially contractive and dilative states is more precise, as it is based on the location of these states on the $e, p'$ plane (where $e$ denotes the voids ratio) with respect to the steady-state line (SSL), see Fig. 2. The SSL corresponds to the plastic flow of soil, at constant stresses and volume. The methods of preparation of the dilative/contractive samples are described in Sawicki and Świdziński [13].

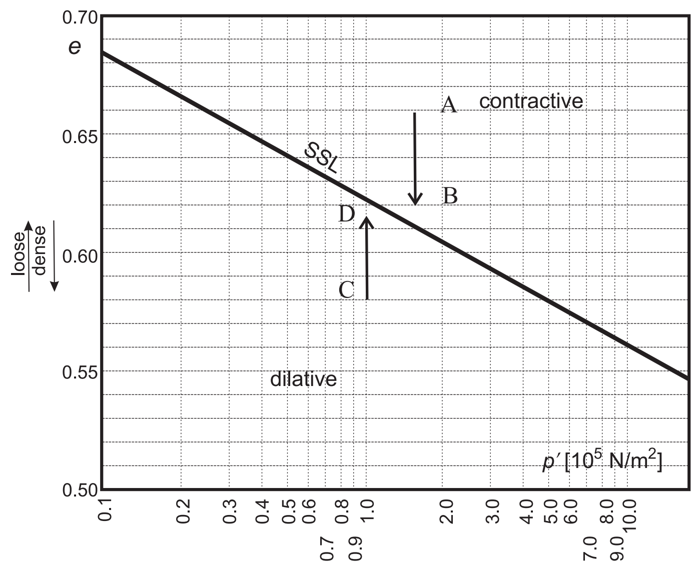


Fig. 2. The steady state line and initially contractive and dilative states of granular soil.

The states above the SSL are initially contractive, and those below this line are initially dilative. Contractive behaviour means that the soil densifies during shearing. An initially dilative soil first compacts during shearing and then dilates. The determination of the SSL requires several experiments, and therefore it is much more difficult than standard geotechnical tests for determination whether a sample is loose or dense.

After this short explanation, let us consider the kind of behaviour shown in Fig. 1. In the case of initially contractive sand, the effective stress path corresponds to two stages. During the first stage, the deviatoric stress increases up to the IL, where it attains its maximum. Then, it rapidly decreases reaching the C-M failure line. This phenomenon is designated as static liquefaction. The behaviour of initially dilative sand is different, as it initially displays the contractive behavior, but after reaching the phase transformation line this behavior changes to dilative.

## 4. The case of active pore pressure changes

In the case of active pore pressure changes, they are the forcing element, and the behaviour of the soil skeleton is the reaction to such excitations, unlike in the previously described mechanism. In the seabed, these active pore pressure changes are caused by the action of water waves, as schematically shown in Fig. 3. The problem of wave-induced pore pressure changes has been studied in geophysics and marine engineering since the 1950s, by means various classical models. Each of these models leads to different solution, as shown in Table 1.

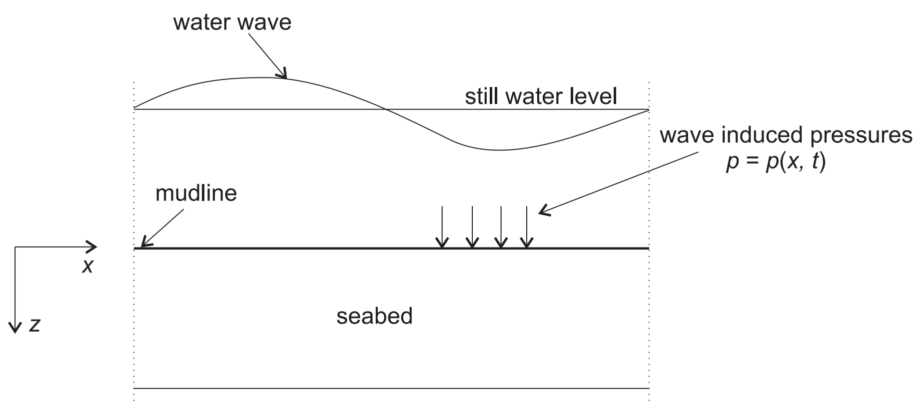


Fig. 3. Water waves induce cyclic changes of pressure at the mudline.

**Table 1. Classical models of the seabed and their consequences.**

| Model | Consequences | References |
|---|---|---|
| Rigid soil skeleton, incompressible pore fluid (filtration theory). | The Laplace equation for $u$, which is independent of the porosity and permeability of the seabed. | Liu [15], Martin [16], Massel [17], Putnam [18], Reid and Kajiura [19], Sleath [20] |
| Rigid soil skeleton, compressible pore fluid. | Diffusion equation for $u$. Fluid compressibility important near the mudline. | Moshagen and Torum [21] |
| Elastic soil skeleton, undrained conditions. | Pore pressure changes important at each depth of the seabed. | Prevost *et al.* [22] |
| Elastic soil skeleton, compressible pore fluid (Biot type approach). | Statically inadmissible effective stresses. | Yamamoto *et al.* [23] |
| Boundary layer | Valid near the mudline. | Mei and Foda [24] |

At present, the Biot-type approach seems to be standard in marine engineering, Jeng [12]. However, this approach has a serious shortcoming, which was discovered and eliminated by Sawicki and Staroszczyk [3]. The problem is that the Biot-type approaches may lead to statically inadmissible effective pressures. In order to rectify this shortcoming, a simple plasticity model was added to the framework of Biot-type equations, to control the effective stress field. The idea of this improvement will be explained on the basis of a triaxial experiment, for the configuration described in the previous section, see Fig. 4. Assume that water waves change pore pressure in such a way that the effective stress path should follow the path ABC in Fig. 4. There is $du = -dp'$. Point B corresponds to the C-M failure line, which cannot be exceeded for physical reasons. Therefore, the sector BC can be considered virtual, as it is unrealistic. On the other hand, the mean effective stress should reach the value of $p'_1$ because it is forced by an external excitation. It is possible only when the shear stress $q$ drops to the point D, which belongs to the C-M failure line. This means that the regrouping of effective stresses has taken place, which is equivalent to an increase in the horizontal effective stress, or an increase in the $K_0$ coefficient , which means the same, see Craig [25]. Recall the well-known relationship: $\sigma'_x = K_0\sigma'_z$. Also note that $q = \sigma'_z - \sigma'_x = (1 - K_0)\sigma'_z$. It follows from this formula that $K_0$ should increase as $q$ decreases.

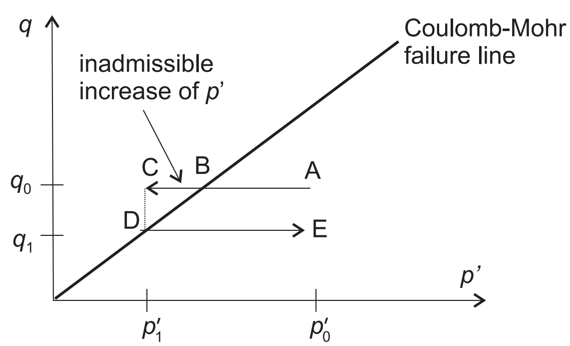


Fig. 4. Regrouping of effective stresses under wave-induced pore pressures.

If the point C does not tend to exceed the C-M line, practically nothing happens except for the cyclic changes in the mean effective stress inside the statically admissible region, i.e. below the C-M line. In the case of a large external excitation, i.e. large pore pressure changes, the same procedure as that shown in Fig. 4 should be repeated. This mechanism may be designated as plastification rather than "momentary liquefaction." In the extreme case when the point C reaches the origin of the co-ordinate system, real liquefaction takes place. Recall that in soil mechanics nomenclature, liquefaction takes place when the mean effective stress is equal to zero, i.e. $p' = 0$.

## 5. Pore pressure dissipation in the seabed

Saturated soils, including the seabed, liquefy because of the generation of excess pore pressure caused by external excitations, such as earthquakes. The release of energy during earthquakes, causes extensive damage to nature and infrastructures. Some of this energy is stored in the form of excess pore pressure, and released when the earthquake is over. This process is known as the dissipation of excess pore pressure and resolidification of the subsoil (seabed). Recall that liquefied soil behaves macroscopically as a viscous liquid. The theoretical condition $p' = 0$ means that there are no contacts between grains. During resolidification these intergranular contacts are gradually rebuilt, and a new soil skeleton is formed. This is possible because of the dissipation of excess pore pressure.

The simplest approach to this problem is based on Biot-type equations, in which excess pore pressure is the initial condition for the pore pressure changes during resolidification. The equation that governs this process is of the following form:

$$\frac{k}{\gamma_w \kappa_s} \nabla^2 u = \frac{\partial u}{\partial t}, \tag{5.1}$$

where $k$ = coefficient of permeability; $\gamma_w$ = unit weight of pore water; $\kappa_s$ = compressibility of the soil skeleton; $\nabla^2$ = Laplace operator. In a simple case of a soil layer, such as the seabed, there is $\nabla^2 = \partial^2/\partial z^2$, where $z$ denotes the vertical co-ordinate.

Eq. (5.1) can be solved for a given geometry of the subsoil and a given distribution of the excess pore pressure $u_{ex}$, which is the initial condition for the problem considered. This means that the problem of pore pressure generation and liquefaction should be solved first. Equation (5.1) can also be applied to model pore pressure generation under partly drained conditions, when it is coupled with the equation for pore pressure generation. Such a coupling decreases the rate of pore pressure generation and delays the liquefaction phenomenon. The sequence of events during pore pressure generation, possible liquefaction and the subsequent dissipation of excess pore pressure is shown in Fig. 5.

The initial state, before an earthquake, is shown in Fig. 5A. Pore pressure has a hydrostatic distribution, not sketched in this figure, in which only the distribution of the initial mean effective stress is shown. Assume that during an earthquake some excess pore pressure has been generated, which is depicted by a solid line in Fig. 5B. Some of the subsoil has liquefied ($u = p'_0$). The distribution of excess pore pressure in Fig. 5B is the initial condition for the problem of pore pressure dissipation, described by Eq. (5.1). After a period of time, excess pore pressure is reduced, as symbolically shown in Fig. 5C. Figure 5D shows the end

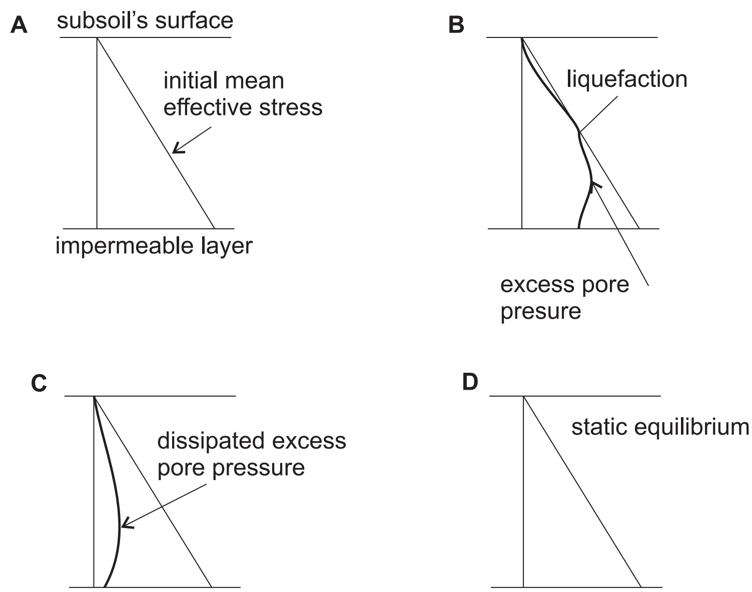


FIG. 5. Sequence of events from liquefaction to the dissipation of excess pore pressure.

of the dissipation process, when a static equilibrium, as that shown in Fig. 5A, is achieved.

Figs. 5A and 5D show the same hydrostatic states of pore pressure, before an earthquake and some time after it has ceased, when there has been enough time for the dissipation of excess pore pressure generated during the earthquake. In terms of the distribution of pore pressures, Figs. 5A and 5D are the same. The basic question, however, is whether the states of the subsoil (the seabed) before and after the earthquake are the same. Equation (5.1) describes only the process of pore pressure dissipation. It does not provide any information about the distribution of effective pressures before and after the earthquake, such as the changes in $K_0$, since the vertical effective stress remains nearly the same because it follows from the own weight of the saturated soil.

EXAMPLE. The distribution of excess pore pressure shown in Fig. 5B should be computed for given geometry and properties of the seabed, see SAWICKI and ŚWIDZIŃSKI [26]. Then, the problem of pore pressure dissipation should be solved using Eq. (5.1), which takes the following form in a 1D case:

$$\frac{\partial u}{\partial t} - a^2 \frac{\partial^2 u}{\partial z^2} = 0, \tag{5.2}$$

$$a^2 = \frac{k}{\gamma_w \kappa_s}, \tag{5.3}$$

where $k$ = soil permeability; $\gamma_w$ = unit weight of water; $\kappa_s$ = compressibility of

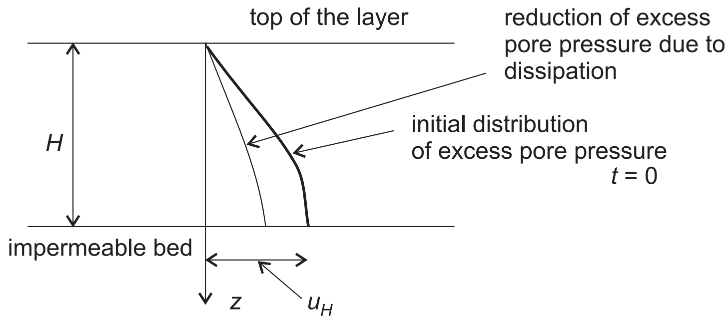


Fig. 6. Example of pore pressure dissipation in a saturated soil layer after an earthquake.

the soil skeleton. Note that $\kappa_s$ is the drained compressibility of the soil skeleton. Its typical value is of the order of $10^{-8}$ m$^2$/N. Assume, for the sake of simplicity, the following shape of excess pore pressure generated at the end of an earthquake, just before dissipation, see Fig. 6:

$$u(t=0) = u_H \sin \lambda z; \qquad \lambda = \frac{\pi}{2H}, \tag{5.4}$$

where $H$ denotes the depth of the soil layer, and $u_H$ is the value of excess pore water pressure at the bottom of the layer considered. Equation (5.4) is the initial condition for the initial-boundary value problem described by Eq. (5.2). The boundary condition is:

$$u(z=0) = 0. \tag{5.5}$$

The solution of Eq. (5.2) with the corresponding initial and boundary conditions (5.4) and (5.5) is the following:

$$u = u_H \exp(-a^2 \lambda^2 t) \sin \lambda z. \tag{5.6}$$

Equation (5.6) means that the shape of the distribution of excess pore pressure remains unchanged. Note that this is only an example, for the sake of illustration, as it has an analytical form. In a general case, the problem should be solved numerically, for different initial distributions of $u$. The above example can be used for the testing of numerical codes.

## 6. Resolidification

The basic question is how the effective stress state in the saturated subsoil (seabed) changes after the earthquake. Some engineers, such as Sumer [2], asked this question after the Kocaeli earthquake, when large parts of the seabed had liquefied in spite of a long history of earthquakes and liquefactions in this region. It is commonly believed that succesive earthquakes should densify the subsoil, making it more resistant to liquefaction. However, is not the case, as these seabeds continue to liquefy, and this phenomenon requires theoretical explanation. Note that there is no sufficient empirical information about the actual behaviour of such seabeds, as excess pore pressures have not been measured during such unexpected events as earthquakes.

It is also difficult to study this phenomenon in the laboratory, as during liquefaction, cf. Fig. 1, the soil sample behaves like a liquid, so its deformations are too great to be measured by available gauges. Also for technical reasons, the process of soil resolidification could not be measured. Therefore, a theoretical analysis of the resolidification of liquefied soil is the only possible way to understand this phenomenon, at least at present. Consider a hypothetical situation, corresponding to the triaxial experiment, shown in Fig. 7.

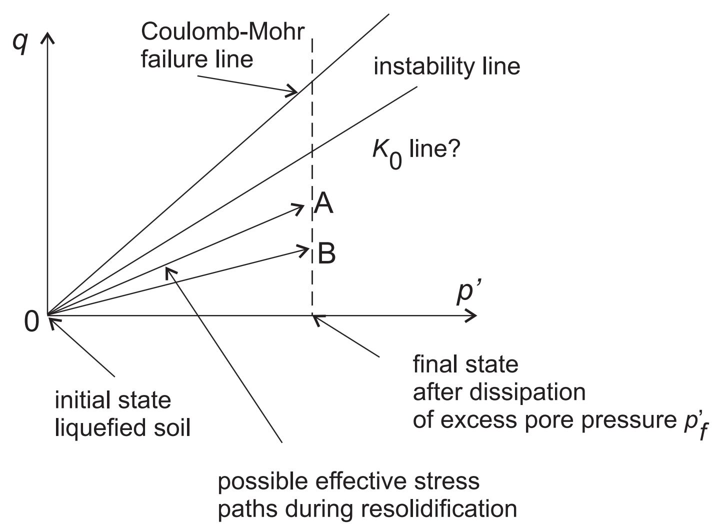


FIG. 7. Possible effective stress paths during the resolidification of liquefied soil.

The problem is considered in the plane of effective stresses, without the analysis of deformations. Assume that the initial condition corresponds to purely liquefied soil ($p' = 0$) which is represented by the origin of the co-ordinate system. Pore pressure is dissipated, and the mean effective stress increases, according to the formula $dp' = -du$. The final value of the mean effective stress is denoted as $p'_f$. These states are marked by the broken line in Fig. 7. The problem is that we do not know how the deviatoric stress $q$ rebuilds during resolidification. The effective stress paths 0A and 0B are two possible candidates among an infinite number of possibilities. How to choose the most probable effective stress path?

It seems that the simplest way is to realize that the liquefaction phenomenon erases the history of the soil sample. Such a history, in practice, is usually based on geological information about the pre-consolidation of sand deposits in a distant past, etc. This is only general information, providing mere hints regarding the soil structure. No concrete data can be found in the available literature. Liquefaction erases such histories, and during the resolidification a virgin structure of the soil is rebuilt. This means that during resolidification, the virgin $K_0$ line is followed. This is similar to the formation of virgin soil samples in triaxial experiments, the history of which begins with the beginning of their preparation.

## 7. Discussion and conclusions

The main results presented in this paper can be summarized as follows:

a) There are two mechanisms of pores pressure changes in the seabed. The first, designated as passive, is caused by external excitations, such as earthquakes. Such excitations lead to pore pressure generation in the seabed and subsequent liquefaction. The second mechanism, designated as active, is caused by water waves. In this case, pore pressures change cyclically, and no liquefaction takes place until extreme conditions are reached. By extreme conditions we understand a situation in which the effective stress path tends to exceed the Coulomb–Mohr failure line. The seabed adapts to such a conditions by changing the horizontal normal stress (an increase in the $K_0$ coefficient). This process can be designated as plastification rather than liquefaction. A situation similar to liquefaction ($p' = 0$) may take place only under very large changes in wave-induced pore pressure, but it is quickly followed by unloading, i.e. an increase in $p'$. That is probably why this short phenomenon is designated, by some authors, as *momentary liquefaction*. Note that subsequent resolidification is also *momentary*. These two phenomena appear cyclically, according to the frequency of water waves. Recall that the first mechanism (passive pore pressure changes) is not cyclic. Both mechanisms need to be described by separate analytical methods.

b) The mechanism of excess pore pressure generated by an earthquake is described. The governing equation was solved analytically for specific conditions, and then numerically for more realistic conditions. It is shown that such a simple approach can also be used to model geysers of pore water, which are sometimes observed after earthquakes.

c) An analysis of seabed resolidification was performed. The basic question was why seabeds, subjected to many liquefactions in the past, continue to liquefy during subsequent earthquakes. We found a simple explanation of this problem. Our explanation is based on the analysis of the process of resolidification. It was observed that liquefaction erases the history of seabed formation. During resolidification the structure of the soil skeleton is built from the beginning. This means that the state of the seabed after resolidification is characterized by the virgin $K_0$ coefficient. Subsequent earthquakes may increase this coefficient, as a result the mechanism described in point a), but liquefaction and subsequent resolidification change this situation. Recall that $K_0 = \sigma'_z/\sigma'_x$ .

## Appendix 1. Elementary derivation of the 1D consolidation equation

Consider the saturated soil elementary volume shown in Fig. 8. The vertical deformation and pore-fluid flow are considered. The horizontal deformations, in the x and y directions, are equal to zero. There is no flow of pore-water in these directions. $V_1$ denotes the volume of water flowing into the element. $V_2$ is the volume of pore-water flowing out of the elementary volume. $dV$ is a change in the volume of the soil skeleton (the saturated soil element). The mass balance of pore-water is the following:

$$V_1 + dV - V_2 = 0, \tag{A1-1}$$

where: $V_1 = v_z\, dxdydt$, $V_2 = (v_z + dv_z)\, dxdydt$, $v_z =$ vertical velocity of the pore fluid, $dV = n\varepsilon\, dxdydz$, $n =$ porosity, $\varepsilon = \varepsilon_z =$ the volumetric strain equal to the vertical strain.

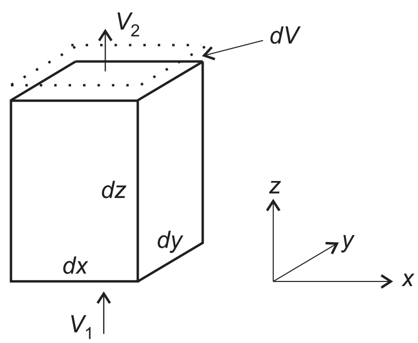


FIG. 8. 1D elementary volume of saturated soil.

After simple algebraic manipulations one obtains the following form of the mass balance:

$$\frac{\partial v_z}{\partial z} = n\frac{\partial \varepsilon}{\partial t}. \tag{A1-2}$$

Note that $\varepsilon = \varepsilon_z = m_v\sigma' = -m_v u$, where $m_v$ is the soil compressibility (in this case, under oedometric conditions), $\sigma'$ = the vertical effective stress, $u$ = excess pore-water pressure. Therefore:

$$\frac{\partial \varepsilon}{\partial t} = -m_v \frac{\partial u}{\partial t} \tag{A1-3}$$

and

$$\frac{\partial v_z}{\partial z} = -n m_v \frac{\partial u}{\partial t}. \tag{A1-4}$$

Let us introduce the Darcy law, in the following form:

$$v_z = -K\frac{\partial u}{\partial z}; K = k/\gamma_w, \tag{A1-5}$$

where $k$ denotes the coefficient of permeability.

Substitution of (A1-5) into (A1-4) leads to the following classical equation:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}, \tag{A1-6}$$

where $c_v = k/n\gamma_w m_v$.

We have obtained, in an elementary way, the classical equation describing the process of 1D consolidation, cf. CRAIG [25]. Note, that no source term appears in Eq. (A1-6).

## Appendix 2: False consolidation equation with the source term

In many publications, a false version of Eq. (A1-6) with the "source term" $s$, is presented, see MCDOUGAL *et al.* [26], SUMER and FREDSOE [7]. It has the following form:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} + s. \tag{A2-1}$$

The above authors treat the function $s$ as a pore water pressure source term, which may be time dependent, and may even depend on the vertical co-ordinate $z$, as well, cf. MCDOUGAL *et al.* [26]. Subsequently, they use Eq. (A2-1) to solve various problems of marine engineering, including the problems of pore-pressure changes and seabed liquefaction. It will be shown here, that Eq. (A2-1) follows from a false assumption and is physically unrealistic. It will also be shown that some interpretations of the "source term" do not conform to reality.

Assume the following form of the false "Darcy law", see (A1-5):

$$v_z = -K\frac{\partial u}{\partial z} + F, \tag{A2-2}$$

where $F$ is a certain function. In the case of $\partial u/\partial z = 0$, i.e. when the phenomenon of filtration does not take place, some vertical flow of the pore-fluid occurs, which cannot be explained physically. A formal differentiation of Eq. (A2-2) with respect to $z$, and then substitution to Eq. (A1-4) lead to Eq. (A2-1), where:

$$s = -\frac{1}{nm_v}\frac{\partial F}{\partial z}. \tag{A2-3}$$

This "source term" has no physical meaning, but McDougal *et al.* [26] and Sumer and Fredsoe [9] build certain concepts about seabed behavior on this false assumption.

## Appendix 3: False interpretation of the mechanism of wave-induced build-up of pore-pressures

Figure 10 illustrates the mechanism of seabed deformation due to water waves, according to Kirca *et al.* [11], see also Sumer and Fredsoe [9]. Water waves impose some bed pressure at the mudline, which in turn causes some deformations of the seabed. These deformations are vaguely identified with a simple shear of the soil element, see Chapter 10.3 of Sumer and Fredsoe [9]. They assume that there is only a single element of the stress tensor, designated as $\tau$, that is responsible for the process of pore-pressure build-up, ignoring the cyclic changes of the normal effective stresses $\sigma'_z$ and $\sigma'_x$. Recall that the effective stress tensor $\sigma'$ has the following form in the case of the plane strain state:

$$\boldsymbol{\sigma}' = \begin{bmatrix} \sigma'_x & \tau \\ \tau & \sigma'_z \end{bmatrix}. \tag{A3-1}$$

The total stress tensor $\boldsymbol{\sigma}$, denoted without "primes", has the same form Also recall that the effective stress tensor is defined as:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - u\mathbf{1}, \tag{A3-2}$$

where $\mathbf{1}$ is a unit tensor, and the soil mechanics sign convention is used (compression is positive). In many papers, the continuum mechanics sign convention is applied (extension is positive), so caution is recommended.

In simple shear tests, applied in soil mechanics, such as those quoted by Sumer and Fredsoe [9], the stress $\tau$ does changes cyclically, whereas the normal stresses $\sigma_x$ and $\sigma_z$ are kept constant. But this is not so in the case shown in Fig. 9, where all components of the stress tensor change cyclically. Respective formulae for wave-induced effective stresses were derived by Yamamoto *et al.* [23] on the basis of Biot's theory, see also Chapter 10.2 in Sumer and Fredsoe [9]. This means that the stress deviator, and particularly its second invariant, is responsible for possible pore-pressure generation. It has the following form in the plane strain state considered:

$$J_2 = \frac{1}{4}(\sigma'_z - \sigma'_x)^2 + \tau^2. \tag{A3-3}$$

Note that the first term in Eq. (A3-3) is ignored by Sumer and Fredsoe [9] after McDougal *et al.* [26].

In the case of simple shear tests applied in soil mechanics, there is $J_2 = \tau^2$, which means that pore-pressure generation induced by cyclic loading depends solely on a single component of the stress tensor, namely $\tau$. Sumer and Fredsoe [9] misuse this obvious fact, which leads to subsequent errors.

For example, they propose, also after McDougal *et al.* [26], the following form of the source term $s$ (denoted as $f$ in their Eq. 10.85):

$$s = \frac{\sigma'_0}{N_l T}, \tag{A3-4}$$

where $T$ = wave period and $N_l$ denotes the number of cycles causing liquefaction, given by the following empirical formula [9, Eq. 10.87]:

$$N_l = \left(\frac{\tau}{\alpha\sigma'_0}\right)^{1/\beta}, \tag{A3-5}$$

where $\sigma'_0$ is the initial effective stress, while $\alpha$ and $\beta$ are empirical constants.

Equation (A3-5) can be accepted as an empirical approximation of geotechnical simple shear tests, performed under undrained conditions. But certainly not for a situation shown in Fig. 9. Another shortcoming of the source term

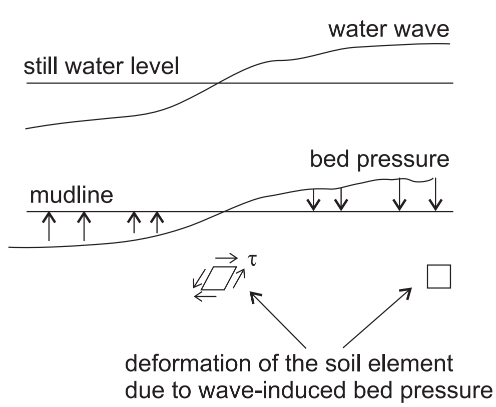


Fig. 9. Bed pressures caused by water waves and the resultant cyclic deformations of the seabed.

(A3-4) is that it is frequency dependent, as $T = 2\pi/\omega$. In classical geotechnical engineering, the behavior of soils subjected to cyclic loadings is frequency independent. Therefore, the approach presented in [9] cannot be applied to analyze the behavior of the seabed subjected to water waves.

Another serious error is a misleading explanation of the liquefaction process. On [9, p. 471], we read "…the pore pressure is generated through the source term …and it spreads in the soil according to a diffusion process…". This statement is unjustified as it follows from a wrong interpretation of the consolidation equation (A1-6) with the source term, represented by Eq. (A2-1).

## Appendix 4. Definition of liquefaction

The saturated soil consists of the soil skeleton, the pores of which are filled with water. It displays features of a solid body due to intergranular forces (effective stresses). Under certain conditions such as cyclic loadings or shocks, the pore pressure may increase causing a decrease in effective stresses. In the extreme case, the effective stresses disappear and the saturated soil behaves like a fluid. This phenomenon is designated as soil liquefaction. A commonly accepted definition of liquefaction is therefore:

$$p' = \frac{1}{3}(\sigma'_x + \sigma'_y + \sigma'_z) = 0, \tag{A4-1}$$

where $p'$ denotes the mean effective stress, and $\sigma'_{x,y,z}$ are normal effective stresses, which cannot be negative in granular soils (recall that the plus sign means compression according the soil mechanics convention).

Sumer *et al.* [10], however, criticize this definition, see point 3 of their conclusions, without proposing any alternative. It seems that their position follows from a vague interpretation of experiments, performed in a wave-flume, cf. [11]. A simplified scheme of their experiments is shown in Fig. 10. A box with satu-

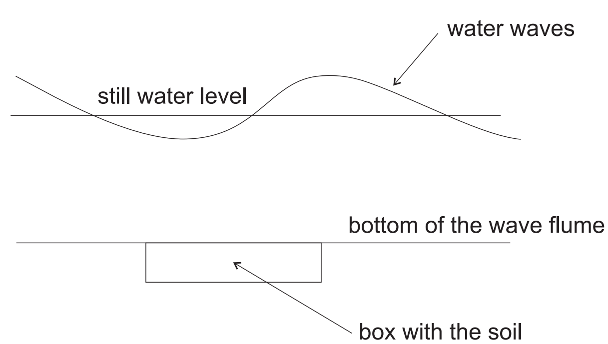


Fig. 10. A scheme of experiments, dealing with pore-pressure changes caused by water-waves, after [11]. Pore-pressure gauges were installed in the box.

rated soil was placed below the bottom of the wave flume. Pore-pressure gauges were installed in this box, and the values of wave-induced pore-pressure changes were recorded. A typical record is shown in Fig. 11.

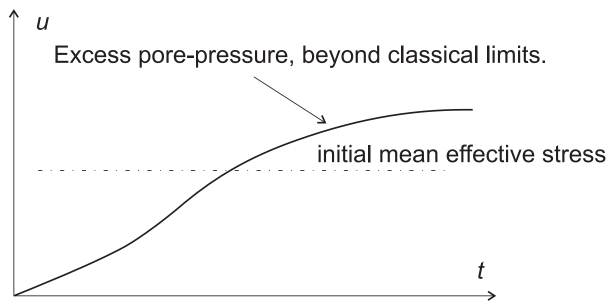


Fig. 11. The excess pore pressure recorded during wave-flume experiments by Kirca *et al.* [11].

We believe that such records, as those presented in Fig. 11, are realistic, i.e. the measured pore-pressure changes are exactly the same as those in the wave channel. A similar situation can be modeled in the triaxial apparatus, as shown in Fig. 12.

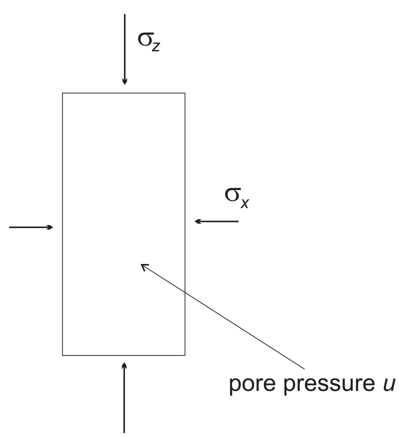


Fig. 12. A sample investigated in the triaxial apparatus.

Imagine a saturated soil sample subjected to the constant global stresses $\sigma_x$, $\sigma_z$. There is the initial pore pressure $u_0$ and the corresponding initial effective stresses are $\sigma'_{x,z} = \sigma_{x,z} - u_0$. This initial state is represented by the point A in Fig. 13. Assume that the pore pressure increases, while the global stresses are kept constant. The idealized effective stress path is shown in Fig. 13. In the first stage, the effective stresses decrease, reaching the point B, lying on the Coulomb–Mohr yield surface. Note that this surface cannot be exceeded for physical reasons, i.e. the stress states outside this surface are statically inadmissible, as the material cannot support such loads. If the pore-pressure still increases, the effective stress path should be tangent to the Coulomb–Mohr yield surface, down to the origin of co-ordinates 0, which means liquefaction. That is why the condition (A4-1) denotes liquefaction. The effective stresses are zero and the saturated soil behaves like a liquid.

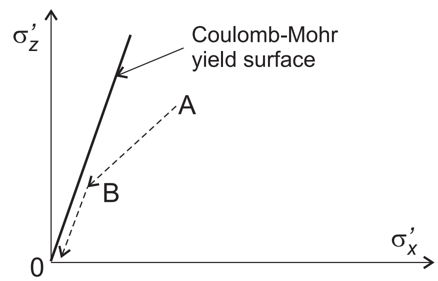


Fig. 13. The effective stress path followed during an increase in pore-pressure, under constant total stresses.

Recall that this is an idealized picture of soil behavior. In fact, along the path AB0, a sample failure may take place under triaxial conditions. In spite of that, the experiment, i.e. a further increase in pore pressure may continue, but will not produce any relevant results.. We shall have a mixture of soil grains and water, already liquefied with increasing values of the pore-water pressure. Under wave flume conditions, we can induce pore pressure higher than that causing liquefaction for geometrical reasons (oedometric conditions which prevent failure), but it is artificial. Therefore, some conclusions by Sumer *et al.* [10] are not justified.

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