

## Design of a turbulent wing for small aircraft using multidisciplinary optimisation\*

W. STALEWSKI, J. ŻÓŁTAK

*Instytut Lotnictwa (Institute of Aviation)  
Aleja Krakowska 110/114  
02-256 Warszawa, Poland  
e-mails: stal@ilot.edu.pl, geor@ilot.edu.pl*

DESIGN PROCESS OF A TURBULENT WING for small aircraft, using multidisciplinary and multi-objective optimisation, based on a genetic algorithm was presented. A generic parametric model of small aircraft wing geometry was developed. In the model, a wide class of wing geometries, with and without high lift devices, was described by a relatively small number of parameters. The optimisation method used the objectives and constraints typical for multidisciplinary wing design, and was applied to the design and optimisation of turbulent wing dedicated for small, two-propeller aircraft. The research was conducted within European Project CESAR. The results of the research have been discussed.

**Key words:** multi-criteria design, multi-disciplinary design, genetic algorithm, aerodynamic of wing.

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### 1. Introduction

THE COST OF SMALL- AND MEDIUM-SIZED DISTANCE AIR TRANSPORT is significantly higher than the cost of alternative means of transport such as the car and/or train, making it one of the major barriers to the improvement of competitiveness of this sector. Reduction of costs generated in different phases of aircraft life cycle seems to be the necessary solution. The CESAR project (Cost-Effective Small Aircraft), supported by the EU in the 6th Framework Programme, focused on the development of technologies that both shorten the market entry time for small aircraft and reduce the cost associated with the process. Basic aircraft configurations [1] were selected as demonstrators: AC1 – low-speed aircraft (turboprop) and AC2 – transonic speed very light business jet. The present study focuses on the results of research performed at *Instytut Lotnictwa (Institute of Aviation – Warsaw, Poland)* on the development of numerical methods in wing design for small aircraft. The proposed methodology is based on parametric mod-

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elling of complex aerodynamic objects [2], numerical multi-criteria and multidisciplinary optimisation, taking into account various types of design constraints [3]. The method was used to design a turbulent wing for a type of small aircraft. The result, related to AC2 configuration, was discussed in the previous work [4]. The present paper discusses research results concerning AC1 configuration.

Wing design is an inherently multidisciplinary activity including analyses in aerodynamics, structure, flight control, manufacturing, etc. The results of integrated aerodynamic and structural wing design process for a conventional civil transport aircraft configuration are presented in [5–8] for high subsonic speed, transonic range and supersonic speed respectively. Similar procedure for an unconventional configuration – blended wing body aircraft – was presented in [9]. In [10] there is presented the wing design process including structural and aerodynamic analyses, power units selection and aircraft missions analysis.

Optimisation approach most frequently uses evolutionary methods. The study [11], however, offers a solution to the same problem via an adjoint method.

While the previously mentioned papers concerned civil transport aircraft, the present study focuses on the multidisciplinary design of a small aircraft wing.

## 2. Design and optimisation methodology

Most of the design and optimisation problems may generally be defined using three groups of parameters (Fig. 1):

- the design variables defining geometrical parameters which may be changed in the design process,
- the environmental variables describing physical properties of the environment in which the designed object works, and
- the objectives and constraints defining the expected properties of the designed object.

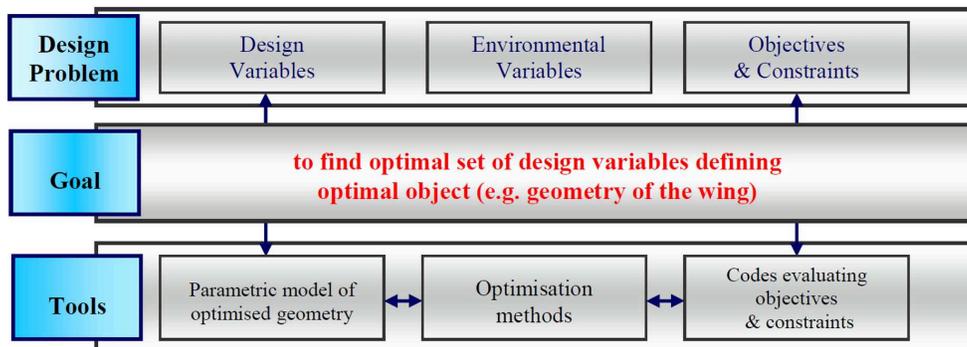


FIG. 1. Scheme of a typical parametric design and optimisation problem solving.

The goal of the design is to find the optimal set of design variables defining the optimal object, i.e., the object for which the objectives reach possibly optimal values and which fulfils all the constrains. To achieve the goal of the design, optimisation tools are typically used, all of which can be categorised in three types of applications: (see Fig. 1):

- utilities dedicated to modification and parameterisation of an object's geometry, including commercial CAD software and specialised codes realising a parametric model of the designed object,
- numerical optimisation methods,
- utilities dedicated to the evaluation of the objectives and constrains.

In the present problem, the method for multi-objective optimisation based on a genetic algorithm [12] was adapted to the multi-disciplinary design and optimisation of small aircraft. A general scheme of applied methodology is shown in Fig. 2.

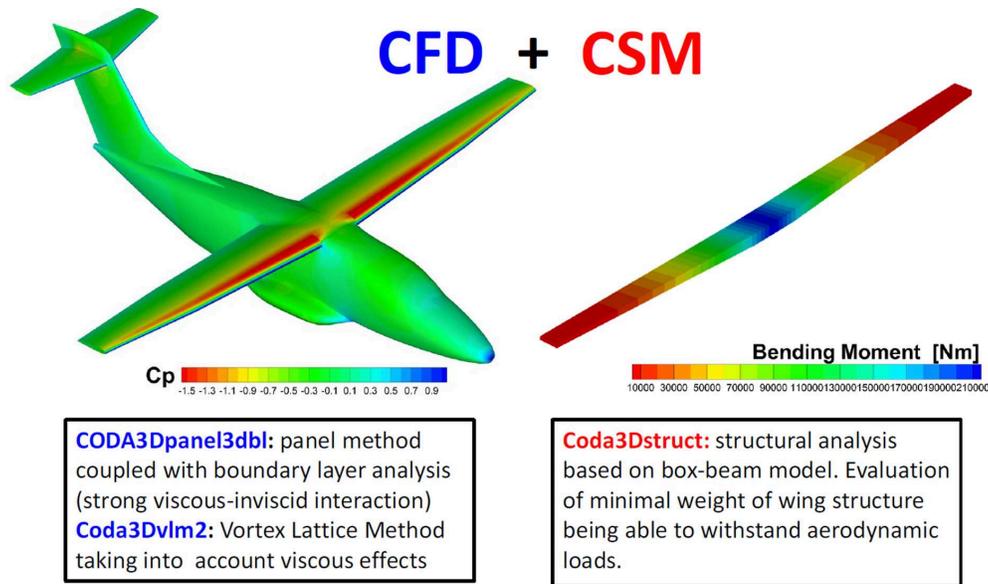


FIG. 2. Scheme of simplified CFD and CSM codes used in turbulent wing optimisation process.

A generic parametric model, based on in-house methodology [2], applied to small aircraft wing geometry was introduced. In the model, a wide class of wing geometries with and without high-lift devices is described by a relatively small number of parameters. The NURBS (Non-Uniform Rational B-Splines) technique is used for geometry modelling and design space description. The optimisation method used the objectives and constraints typical for aerodynamic design. The basic aerodynamic performance was determined using the 3D panel

method coupled with the 2D boundary layer analysis (integral method) [13]. Although the optimisation referred solely to the wing, the aerodynamic computations were performed with reference to the whole aircraft. The box-beam model of the wing structure [3] was used to estimate the weight of the wing.

### 3. Design and optimisation of turbulent wing

The multi-disciplinary, multi-objective optimisation method presented above was used to design and optimise the AC1 turbulent wing for small aircraft defined in project CESAR. A designed wing was supposed to fulfil the requirements and constraints defined by the other participants of project CESAR [14].

#### 3.1. Wing parameterisation

The general assumptions concerning the planform of optimised wing are presented in Fig. 3.

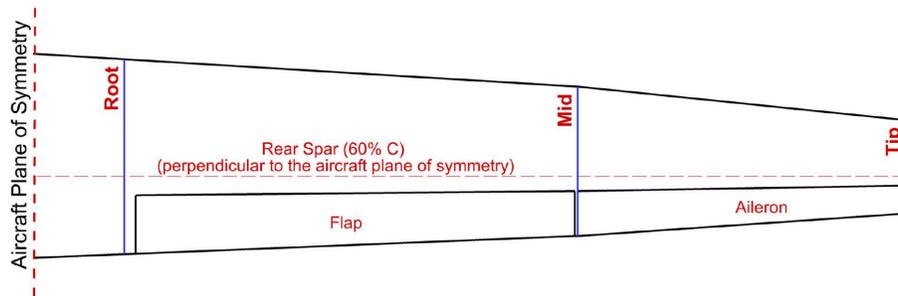


FIG. 3. The planform of optimised wing.

It was assumed that several parameters of the wing planform would be fixed and others would be changeable. The following wing planform parameters were considered to be fixed: the root chord, the area of the aileron zone and positions of front and rear spar. Other wing planform parameters were assumed to be changeable during the design process.

It was considered that wing panels should have a classical, cost saving-oriented manufacturability. It was assumed that the wing would consist of two segments (each modelled as a ruled surface): the flap zone and the aileron zone. Considering possible degrees of freedom, three design parameters were established as the design variables defining the wing planform. They were: the tip chord, the mid chord and the wing area. Three end-sections of the wing segments completely defined the wing surface: the root section, the mid section which is the border section between the flap and aileron zones (the inner and outer wing), and the tip section.

The ILL 5XX airfoil family [15] was chosen as a basic airfoil for the design process. The ILL5XX airfoils have the same mean line and scalable distribution of thickness. Finally,

- the relative thickness (the maximum thickness referenced to the chord),
- the camber of the mean line referenced to the chord, and
- the airfoil twist angle

were used to describe geometry in basic cross-sections of the wing.

The twist angle of the root section was set to 0 degrees. Apart from the wing external shape, the wing box was also parameterised and its geometry was the input data for the wing weight evaluation.

According to the design of high lift system for airfoil ILL518 [15], it was assumed that within the flap zone the designed wing would be equipped with the Fowler flap. The chord of the flap was set to 30% of the wing chord. Taking into account cost saving-oriented manufacturability, it was assumed that the flap panel would be a ruled surface defined by its limiting cross-sections. Typically, the shapes of the flap cross-sections were assumed to be in accordance with flap of the airfoil ILL518. Therefore, the shape of the flap was assumed as fixed. As a result, the problem of high-lift system design and optimisation was reduced to a search for optimal values of the design parameters defining the positions (gap and overlap) and deflections of the Fowler flap for both the takeoff and landing configurations. The design of the high lift system was performed independently after finishing the design process of the clean wing. Finally, 11 and 5 design parameters were defined for the design of the clean wing and the design of the high-lift system respectively.

### 3.2. Objectives and constraints

At the initial stage of the design process, the clean wing was designed. This was achieved by solving appropriate multi-disciplinary and multi-objective optimisation problems. The main goal of the optimisation was to design a possibly low-drag and light weight wing, having possibly high values of maximum lift coefficient  $C_{Lmax}$ . For the optimisation process, the following design flight conditions were established [14]:

- flight conditions 1 (**FC1**):  $M = 0.34$ , ISA, altitude = 10 000ft, Lift = MTOW,
- flight conditions 2 (**FC2**):  $M = 0.34$ , ISA, Altitude = 10 000 ft, Lift =  $2.5 \cdot$  MTOW,
- flight conditions 3 (**FC3**):  $M = 0.13$ , ISA, sea level,

where ISA and MTOW are International Standard Atmosphere and the aircraft maximum take-off weight respectively. Based on formerly performed analyses,

the following three objective functions  $F_i$  were defined:

$$(3.1) \quad F_1 = \frac{L_1}{D_{W1}}, \quad F_2 = \frac{L_2}{W_{W2}}, \quad F_3 = C_{Lmax},$$

where:

- $L_1$  lift of the aircraft in flight conditions **FC1**,
- $D_{W1}$  drag of the wing in flight conditions **FC1**,
- $L_2$  lift of the aircraft in flight conditions **FC2**,
- $W_{W2}$  minimum weight of the wing structure being able to withstand an aerodynamic load in flight conditions **FC2**,
- $C_{Lmax}$  maximum of  $C_L$  in flight conditions **FC3**.

The following wing planform parameters were considered as fixed:

- the root chord,
- the area of aileron zone,
- the line of 20% the of wing chord was assumed as the front spar position, while the line of 60% of the wing chord as the rear spar position. The latter line was assumed to be perpendicular to the aircraft plane of symmetry.

The geometrical constraints were taken into consideration within the parametric model of the wing; therefore, for every created geometry of the wing they were fulfilled automatically. An additional, assumed aerodynamic constraint expressed a limitation of the clean-wing pitching moment coefficient (at the zero lift coefficient):

$$(3.2) \quad C_{m0W} > -0.070$$

The main goal of the optimisation was to maximise the three objectives (3.1). Maximisation of objectives  $F_1$  and  $F_2$  means searching for the wing geometry characterised by:

- possibly low drag in typical cruise flight conditions,
- possibly low weight of the structure which would be able to withstand overloads 2.5 g in typical cruise flight conditions.

Both parameters: the drag and the wing weight influence the direct operating costs of the aircraft and, especially for low-cost aircraft, should be minimised [11]. Maximisation of objective  $F_3$  aims at increasing flight safety and shortening of take-off and landing distances, which is important from the point of view of environmental and utility factors.

Aerodynamic and structural properties of the designed wings/aircraft were evaluated using cost-effective, simplified codes:

- the panel method coupled with the boundary layer analysis [13],
- the structural analysis based on the box-beam model [3], [16],
- the vortex lattice method taking into account viscous effects [17] (used for fast evaluation of  $C_{Lmax}$ ).

An appropriate high-lift system was designed after optimising the clean wing. At the beginning, the shape of the original high-lift system of ILL518 airfoil [15] was adopted to the designed clean wing geometry, which was conducted using specialised in-house software.

Next, using the interactive design and optimisation method, the optimal take-off and landing configurations were designed. In this case the design constraints expressed a condition of feasibility of movement of the Fowler flap from closed position, through take-off position up to landing position. The design objectives expressed requirement of achievement of satisfactory, possibly the highest values of:

- $C_{Lmax}$  for landing configuration,
- $C_{Lmax}$  and  $C_L/C_D$  for takeoff configuration,

where  $C_{Lmax}$  is a maximum lift coefficient and  $C_L/C_D$  is an aerodynamic efficiency. Evaluations of these coefficients were performed using the panel method [13] for the following take-off and landing flight conditions:

- Flight conditions 4 (**FC4**):  $M = 0.11$ , ISA + 20°C, altitude = 6560 ft.

### 3.3. Results

**3.3.1. Clean-wing design.** The multi-objective genetic algorithm was used to solve the optimisation problem. Starting from a random wing population, the genetic algorithm generated subsequent generations of wings which increasingly better fitted the design criteria. Each generation consisted of 48 wings. The optimisation process was stopped after 300 generations, which required 14 400 objective-constraint evaluations performed for a single wing. Application of the parallel-computing technique allowed for objective-constraint evaluation for 24 wings simultaneously. According to the applied genetic algorithm, the wings not satisfying all the constraints were not eliminated from the optimisation process automatically. Instead, their probability of becoming parents of next generations was significantly decreased. The solution to the multi-objective optimisation issue was the Pareto set which included 1093 Pareto-optimal wings.

From the Pareto set, the wing called AC1T-IOA-01 was downselected as the final version of the designed turbulent wing for the two-propeller, low-speed aircraft. The values of the objectives calculated for the selected wing AC1T-IOA-01 are presented in Figs. 4, 5 and 6 and compared with the objectives calculated for the baseline wing AC1T-BASELINE and for all obtained Pareto-optimal wings.

Geometry of wing AC1T-IOA-01 satisfies all geometrical constraints and preferences, particularly concerning cost saving-oriented classical manufacturability. Wing AC1T-IOA-01 consists of two segments, corresponding to the flap and aileron zones. The surface of each segment is assumed as the ruled surface, defined by its end sections. Three basic wing sections (root, mid and tip sections), were obtained by modifications of the basic airfoil ILL518. The geometry of the

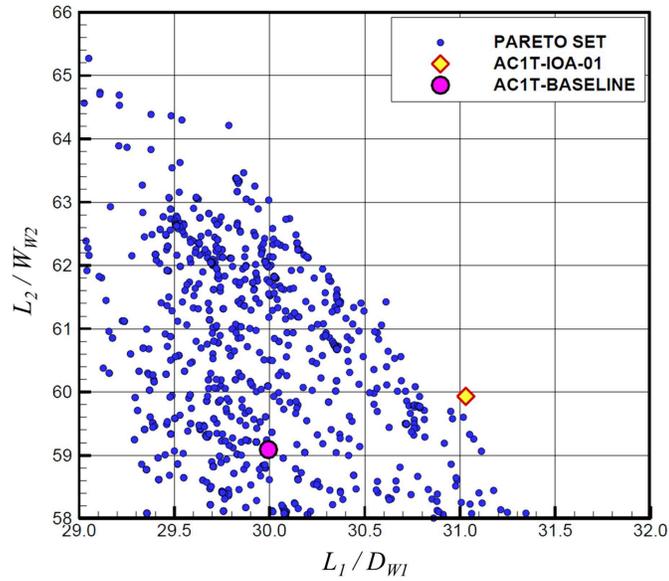


FIG. 4. Projection of Pareto set on F1–F2 space. AC1T-IOA-01 – selected optimal wing.

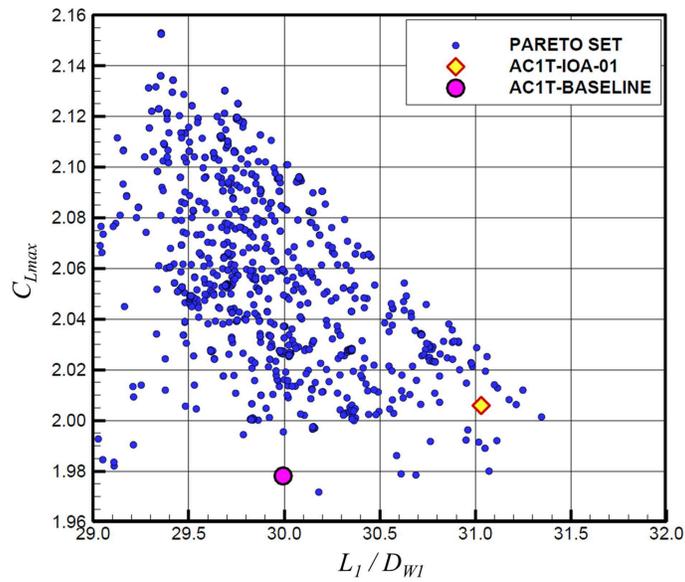


FIG. 5. Projection of Pareto set on F1–F3 space. AC1T-IOA-01 – selected optimal wing.

selected wing is described by spanwise distributions of the following basic parameters referring to the shape and position of the airfoils – wing cross-sections:

- the relative thickness – the maximum thickness of the airfoil referenced to its chord,

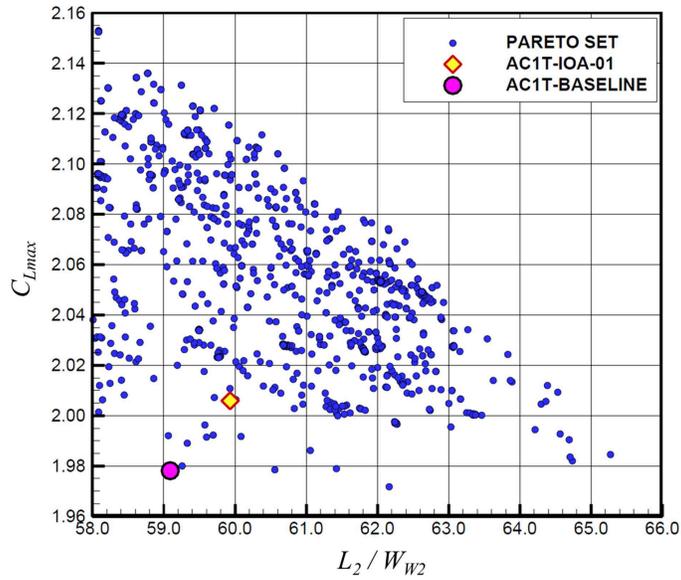


FIG. 6. Projection of Pareto set on F2–F3 space. AC1T-IOA-01 – selected optimal wing.

- the maximal camber of the mean line of the airfoil referenced to its chord,
- the twist angle – the angle between the airfoil chord and the assumed horizontal plane of the wing.

Spanwise distributions of the above parameters describing the geometry of the wing AC1T-IOA-01 are shown in Fig. 7.

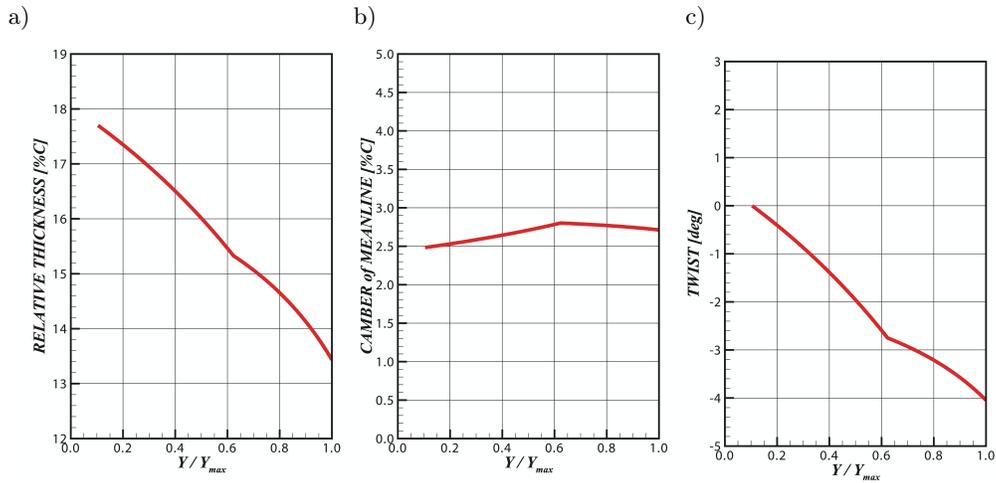


FIG. 7. Geometry of wing AC1T-IOA-01. Spanwise distributions of: a) the wing section relative thickness, b) the camber of the wing section mean line, c) the wing section twist angle.

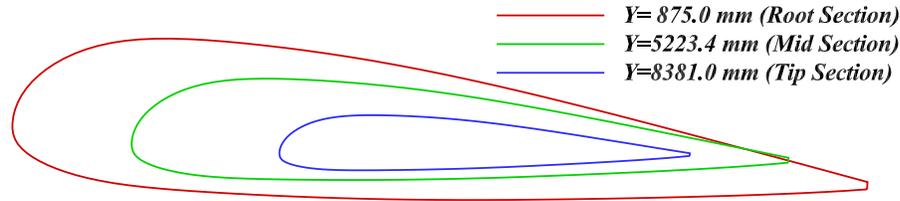


FIG. 8. The root, mid and tip sections of the wing AC1T-IOA-01 – a projection on aircraft plane of symmetry.

The basic cross-sections of the wing are shown in Fig. 8.

**3.3.2. Clean-wing aerodynamic and structural properties.** The main goal of optimisation was to design a possibly low-drag and low-weight wing, having possibly high values of  $C_{Lmax}$ . To check whether the goal had been achieved, a wide spectrum of aerodynamic characteristics for wing AC1T-IOA-01 was analysed. The results of the analysis were compared with aerodynamic properties of the initially defined geometry – AC1T-BASELINE. All calculations were performed for the complete aircraft configuration, although certain analysed characteristics referred solely to the wing. Figure 9 shows the wing drag polars calculated

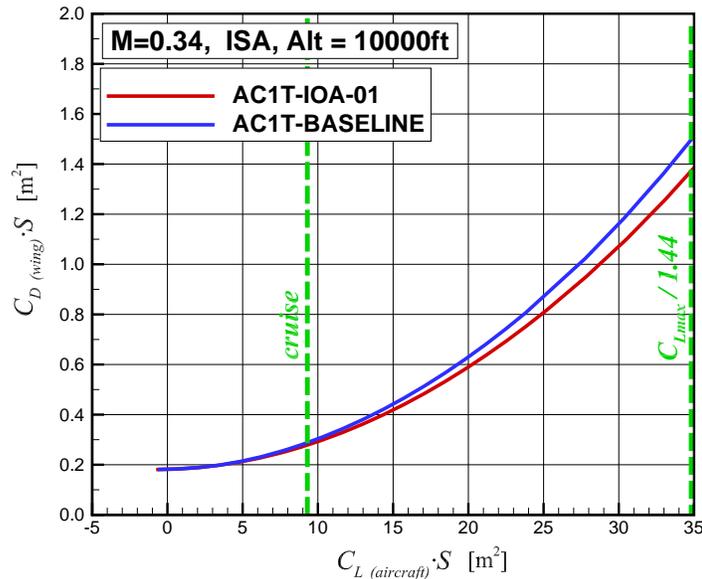


FIG. 9. Comparison of wing drag polars calculated at cruise flight conditions **FC1** for wings AC1T-IOA-01 and AC1T-BASELINE.

for wings AC1T-IOA-01 and AC1T-BASELINE. In the graph, a drag coefficient refers solely to the drag of the wing, but the lift coefficient concerns the complete aircraft. The calculations were performed for the full aircraft configuration, at cruise flight conditions **FC1**.

According to [14], the wing-optimisation process should aim at minimum drag in a range of the lift coefficient  $C_L$  from cruise design point to  $C_{Lmax}/1.44$ . Figure 9 shows that, in this range of  $C_L$  obtained for the wing AC1T-IOA-01, the reduction of drag in comparison to the AC1T-IOA-BASELINE changes from 3.3% to 8.3%.

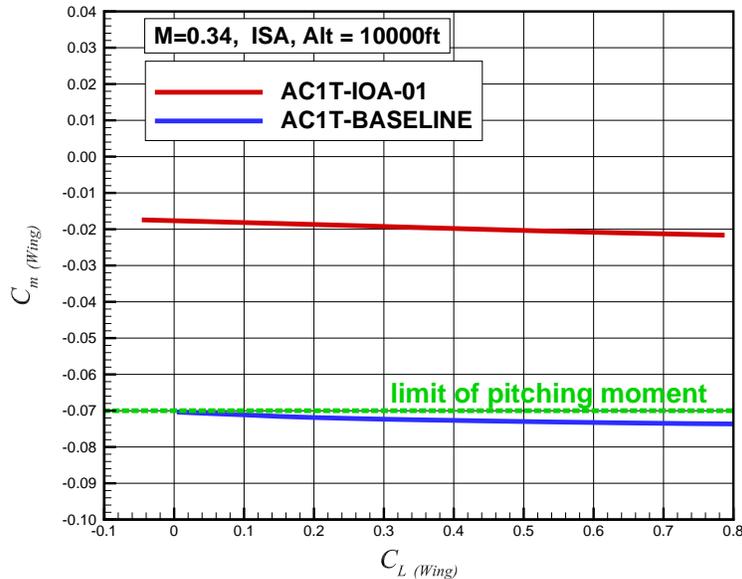


FIG. 10. Comparison of pitching moment coefficient  $C_m$  versus lift coefficient  $C_L$  curves calculated at cruise flight conditions **FC1** for wings AC1T-IOA-01 and AC1T-BASELINE.

Based on [14], the optimised wing should fulfil the constraint concerning the minimum pitching moment related to the wing aerodynamics centre in cruise configuration for **FC1** condition. Figure 10 shows comparison of pitching moment coefficient  $C_m$  versus lift coefficient  $C_L$  curves calculated at cruise flight conditions for wings AC1T-IOA-01 and AC1T-BASELINE. Wing AC1T-IOA-01 is characterised by a considerably low negative pitching moment completely fulfilling the above condition. In comparison with the baseline wing the reduction of negative pitching moment is 72%. Moreover, the low negative pitching moment of wing the AC1T-IOA-01 favours a reduction of the aircraft total drag, enabling reduction of the drag of the horizontal tail.

Figure 11 shows the comparison of curves of lift coefficient ( $C_L$ ) versus angle of attack ( $\alpha$ ), calculated for wings AC1T-IOA-01 and AC1T-BASELINE. The calculations were performed at flight conditions **FC3**. The presented lift coefficient refers solely to the isolated wing, but the calculations were performed for the complete aircraft. For the optimised wing the  $C_{Lmax}$  coefficient is similar to the baseline but is by 1.4% greater and it fulfils the constraint defined in [14].

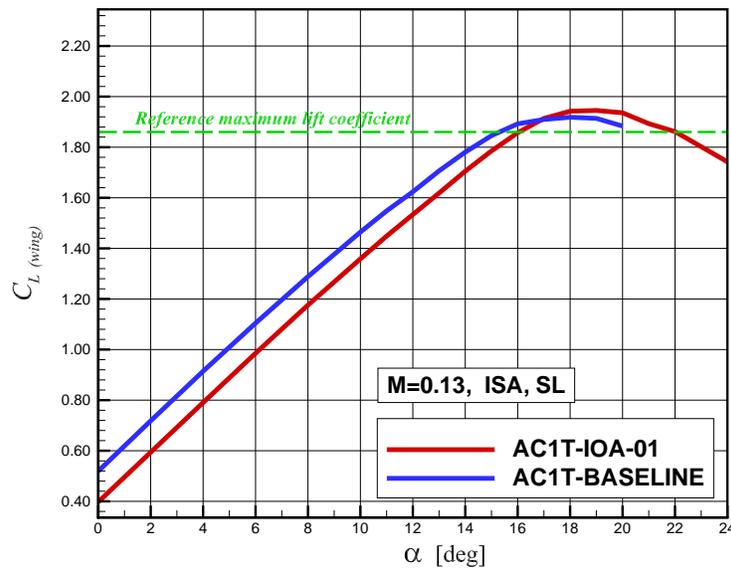


FIG. 11. Comparison of lift coefficient  $C_L$  versus angle of attack  $\alpha$  curves calculated for flight condition **FC3**.

Analysing presented in Figs. 5 and 6 values of the objective  $F_2 = L_2/W_{W2}$  (3.1) calculated for optimised and baseline wing, one may conclude that both compared wings have similar weight, although the wing AC1T-IOA-01 is by 1.4% lighter than wing AC1T-BASELINE.

**3.3.3. Geometry of wing with high lift system.** The flap zone of the wing AC1T-IOA-01 is limited by the flap-inner section and the flap-outer section. The chord of the designed Fowler Flap equals 30% of the wing chord. Taking into account cost saving-oriented manufacturability, the surfaces of the flap and the main wing are designed as ruled surfaces, defined by their limiting cross-sections, which are shown in Fig. 12.

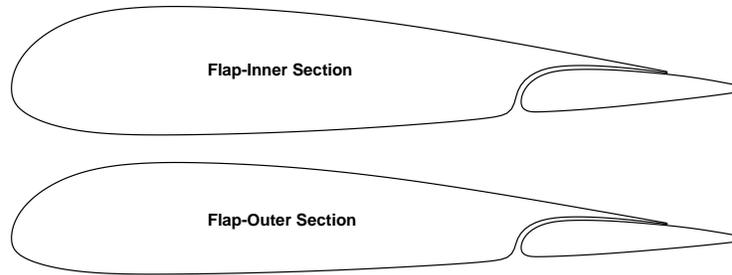


FIG. 12. Flap-inner and flap-outer cross-sections of wing AC1T-IOA-01 with high lift system.

The system of flap positioning was established to define the 3D position of the deflected Fowler flap. The system uses positions of the flap nose in two limiting cross-sections of the flap. A way of positioning in given cross-section is presented in Fig. 13. First, the non-deflected flap is moved to the given position.

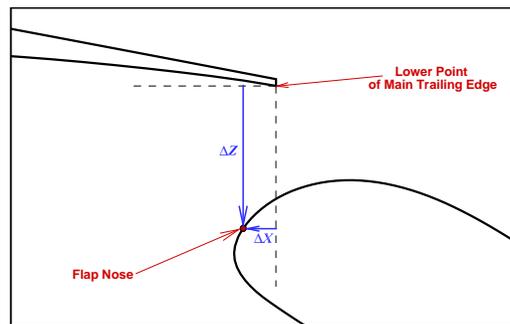


FIG. 13. The definition of flap position in given wing cross-section.

For both the limiting sections of the flap the shift is described by two parameters ( $\Delta X$ ,  $\Delta Z$ ) where:  $\Delta X$  – is a distance between the main trailing-edge lower point and the flap nose in direction parallel to the wing local chord in the given cross section,  $\Delta Z$  – is a distance between main trailing-edge lower point and flap nose in direction normal to the wing local chord in the given cross section. Next, the flap is rotated by the given deflection angle ( $\delta_{FL}$ ), where the rotation axis is simply the nose-line of the flap. The optimal position of the flap for the landing and take-off conditions was obtained using the interactive design and optimisation approach. Two cross-sections of the wing limiting the flap zone, corresponding to the optimal configuration are shown in Fig. 14.

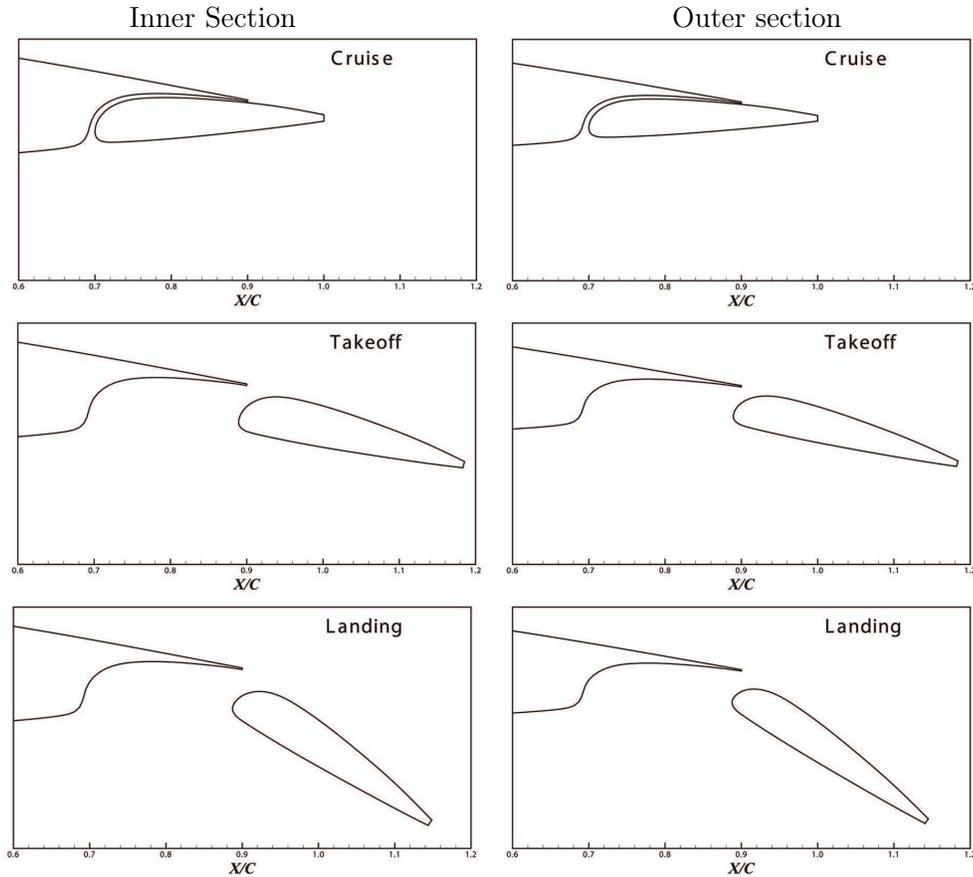


FIG. 14. Optimal positions of Fowler flap in flap-inner (left) and flap-outer (right) cross-sections.

**3.3.4. Aerodynamic properties of high lift configurations.** CFD calculations for high-lift configurations of aircraft AC1 with wing AC1T-IOA-01 were performed using the panel method coupled with the boundary layer analysis [13] for flight condition **FC4**. Figure 15 presents relation of lift coefficient  $C_L$  to the angle of attack  $\alpha$  of the complete aircraft for both the take-off and landing configurations. The optimal high lift configuration seems to be highly satisfactory in comparison with the design criteria formulated in [14]. The obtained increase in maximum lift coefficient ( $C_{Lmax}$ ) was approximately 0.3 for the landing and take-off conditions.

#### 4. Conclusions

There was developed and implemented the cost efficient methodology of aircraft/wing design based on the multi-objective and multi-disciplinary optimisation. To improve the efficiency of the design process, a parametric model

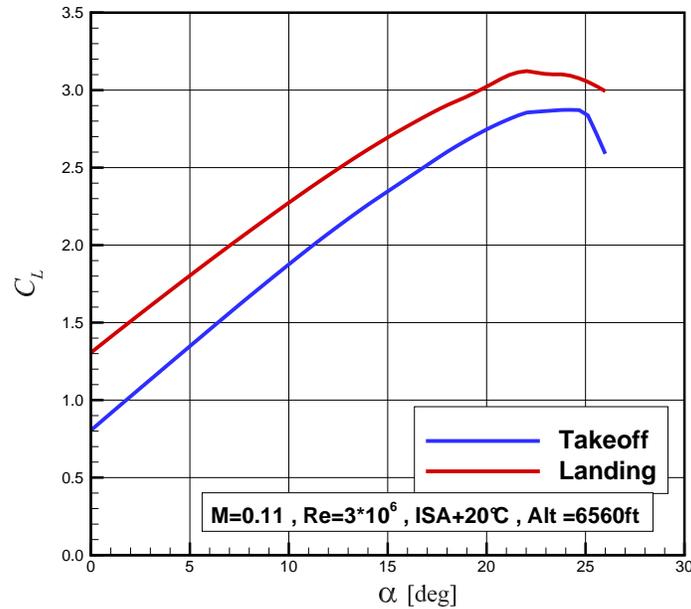


FIG. 15. Calculated dependency lift coefficient  $C_L$  versus angle of attack  $\alpha$  for take-off and landing configuration of aircraft AC1 with wing AC1T-IOA-01 (FC4 flight conditions).

of AC1 aircraft/wing was developed for both the clean wing and the wing with the high-lift system.

The presented optimisation technique was applied to a turbulent wing designed for AC1 concept of small aircraft considered in CESAR project. The final result of the design process are both clean wing AC1T-IOA-01 and the wing-dedicated high-lift system (the Fowler flap).

Using the parametric model of the wing, the positions and deflections of the flap were optimised for the take-off and landing configurations.

For designed wing AC1T-IOA-01 with the high lift system, values of defined objectives are satisfactory and the defined constraints are predominantly fulfilled. Within the range of lift coefficient  $C_L$  from the cruise design point to  $C_{Lmax}/1.44$ , in comparison to the baseline, the reduction of drag for the optimised clean wing varies from 3.3% to 8.3%. Additionally, wing AC1T-IOA-01 is characterised by:

- considerably lower negative pitching moment, reduced by 72% in comparison with the baseline. Moreover, the low negative pitching moment reduces the aircraft total drag by drag decrease of the horizontal tail,
- maximum lift coefficient, similar to the baseline, but greater by 1.4%,
- weight, similar to the baseline, but lighter by 1.4%,
- highly-effective high-lift system.

Based on the presented results, the optimised wing AC1T-IOA-01 seems to be a very interesting solution for a cost-effective, low-speed, small aircraft.

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