

## A new adaptable multiple-crack detection algorithm in beam-like structures

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IN THIS ARTICLE, A SIMPLE METHOD FOR DETECTING, localizing and quantifying multiple cracks in beams using natural frequencies is presented. We model cracks as rotational springs and demonstrate a relationship among natural frequencies, crack locations and depths. The main advantage of our method is that it can detect adaptably the unknown number of cracks intervened. Concise, simple calculations and good accuracy are other advantages of this method. We present a number of numerical examples for several beams to validate our method.

**Key words:** vibration-based detection, multiple-crack, damage detection, crack detection algorithm.

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### Notations

- $a$  crack depth,
- $b$  beam width,
- $h$  beam height,
- $k_i$  local stiffness of the  $i$ th massless rotational spring in multiple-crack model,
- $m$  number of cracks,
- $x$  Cartesian coordinate along beam length,
- $E_n$  modal corrected Young's modulus of beam in the  $n$ th mode,
- $I$  second static moment of inertia of the beam cross-sectional area,
- $L$  beam length,
- $M_{i,n}$  resisting modal bending moment developed in the crack location  $i$  and the mode  $n$ ,
- $U_n$  total modal strain energy stored in the uncracked beam in the mode  $n$ ,
- $\rho$  mass density of beam (per unit volume),
- $\beta_i = x_i/L$  normalized  $i$ th crack location,
- $\gamma_i = a_i/h$  normalized  $i$ th crack depth,
- $\omega_n$  the  $n$ th mode undamped natural frequency of uncracked beam,
- $\Delta\omega_n$  difference between the  $n$ th natural frequency of healthy and cracked beam ,
- $\phi_n''(\beta)$  curvature of the  $n$ th mode shape of the uncracked beam.

## 1. Introduction

INCREASING NUMBER OF AGEING STRUCTURES is one of the biggest challenges that mechanical, aerospace and civil engineering are facing. The large-scale engineering structures are very expensive and it is not wisely to tag them “out-of-service” after the passage of their preliminary design life-time, based on their specifications defined initially when they are placed in service. At the same time, it is of a high risk to let them remain to stay in-service without any inspection and monitoring. Non-destructive Testing/Evaluation (NDTE) methods, such as ultrasonic testing, X-ray, acoustic emission (AE), acousto-ultrasonics, Lamb wave, etc. [1] generally use local techniques for localized inspection and monitoring of damage in structures. However, these techniques are neither economical nor feasible for many large-scale and complex structures with limited accessible parts [2]. Automated and on-line structural health monitoring (SHM) methods have received an exponential attention in the last three decades. This is due to an extra-ordinary technological progress in information technology, sensor/actuator technology, MEMS/NEMS and other related digital age technologies and microsystems.

SHM techniques may provide us with important information about the probable presence of damage scenarios like cracks in the following hierarchy: (1) to detect damage (if any), (2) to localize them, (3) to quantify their severity, and (4) to predict the remaining useful life of a structure. Vibration-based damage identification based on modal testing data is one of the methods that are global in the sense that the measured and processed data show the global dynamic characteristics of a structure, such as natural frequencies and vibration mode shapes. A change in modal data, obtained from a structure, before and after a damage occurrence, such as a single-crack or multiple-crack damage, may be used as a measure for detecting, localizing and assessing the crack damage intensities.

A sudden failure of structures may sustain a loss of life and property due to damage propagation. Therefore, considerable research and many studies have been devoted to damage detection in structures. We have classified some damage identification algorithms by NDE methods in Fig. 1. In this figure, we have included the relevant references [1–4]. The propagation of cracks is one of the main causes of structural failure. Therefore, a number of studies have been devoted to crack detection and prevention from the initiation of a crack growth. There are a number of works considering the effects of a crack on the vibrational parameters of structures [5–10]. We have also classified crack detection procedures in the literature on beam-like structures in Fig. 2. In this figure, we have included the relevant references [1–40].

The mechanical behavior of an open crack is different from a closing crack [14]. Most of researchers are interested in considering a fatigue crack as an

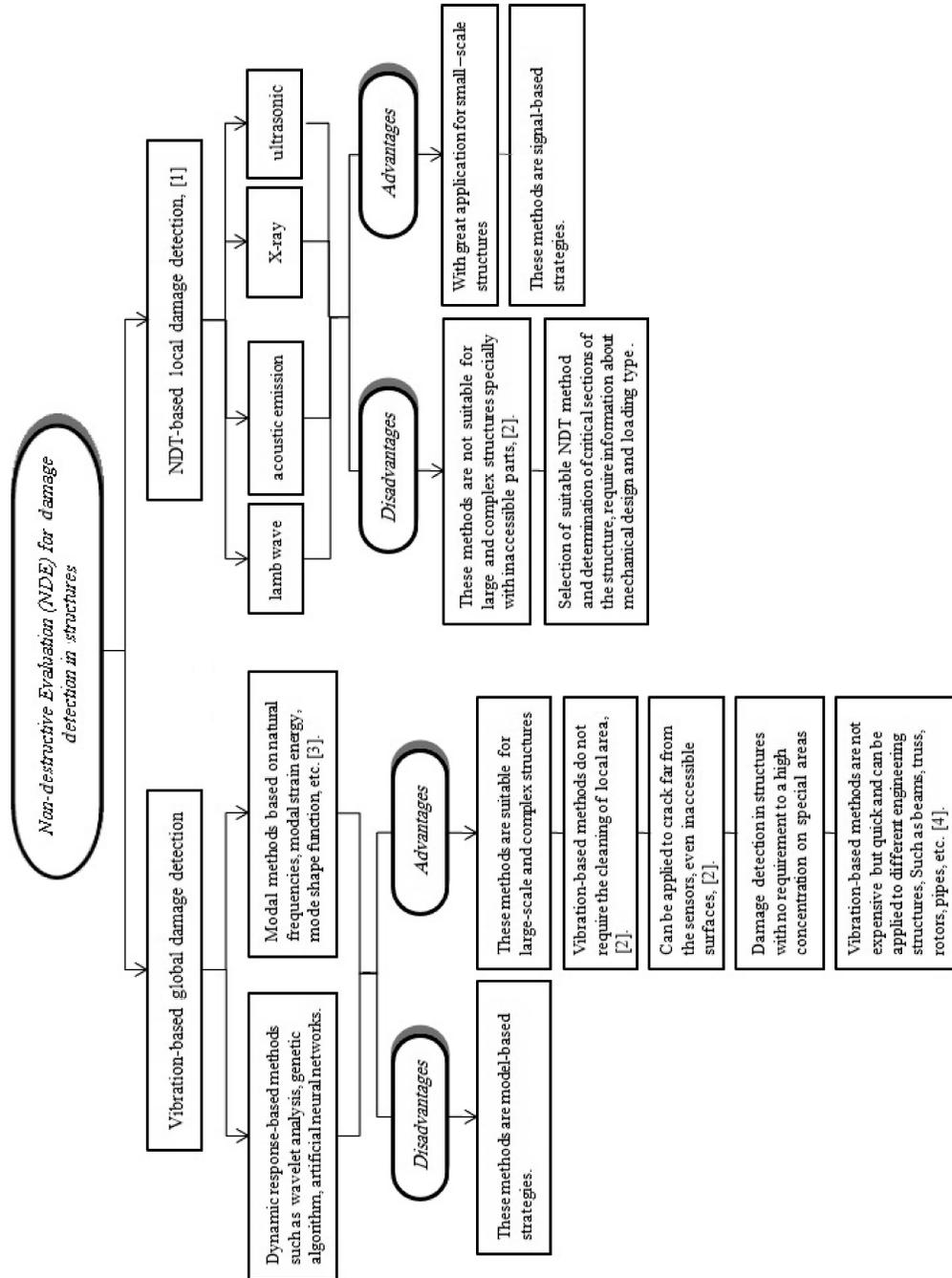
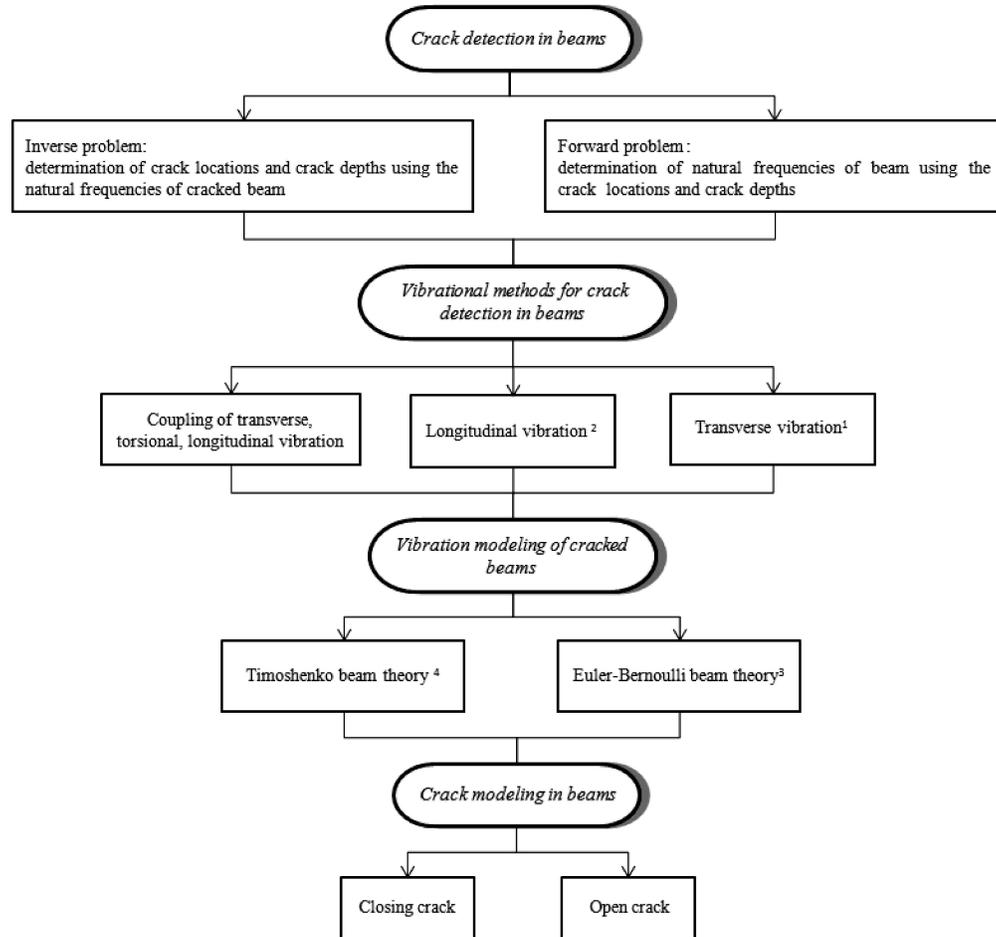


FIG. 1. Damage detection methods in structures.



<sup>1</sup> References regarding transverse vibration of cracked beams: [1–4, 11–20, 22–26, 28–37, 39–40].

<sup>2</sup> References regarding longitudinal vibration of cracked beams in addition to transverse vibration: [21, 27, 38].

<sup>3</sup> References regarding vibration of cracked beams using Euler–Bernoulli beam theory: [1–10, 14–40].

<sup>4</sup> References regarding vibration of cracked beams using Timoshenko beam theory: [11–13].

FIG. 2. Crack detection in beam like structures.

open crack to neglect the non-linearity effects due to a crack closure [15]. However, MATVEEV [16] studied the dynamic characteristics of a cantilever Euler–Bernoulli beam with a closing edge transverse crack. The presence of multiple cracks in a beam causes the dynamic response to become more involved in comparison to a single-crack beam. Therefore, the majority of the studies conducted in the past considered crack detection in beams with a single crack [11–14, 16–28]. However, several studies have recently considered multiple-crack beams [3, 4, 15, 26, 29–34, 39]. SEKHAR [4] summarized different studies on

double- and multi-crack and noted the identification methods in vibrating structures such as beams, rotors, pipes, etc. The propagation of a crack in a beam would reduce the local stiffness at the crack location [34]. DIMAROGONAS [6] suggested the use of an equivalent rotational spring at the crack location. Therefore, in the majority of the studies, the local stiffness in a cracked beam under flexural vibration was modeled as a massless rotational spring [3, 11, 12, 15, 18–26, 28–33, 36, 39, 40].

There are also different methods for solving the forward and inverse problems. For example, LEE [30] solved the forward problem using the finite element method (FEM) and inverse problem using the Newton–Raphson method. MAITI [33] considered crack detection by transfer matrix method (TMM). ZHANG *et al.* [37] considered the detection of multiple cracks in stepped cantilever beam combining wavelet analysis with TMM. At first, they found the crack location by peaks of wavelet coefficients and then they identified the crack depth by transform matrix. ROSALES *et al.* [20] presented a solution for the inverse problem with a power series technique and the use of artificial neural networks. However, in most of the presented methods in the literature, it is necessary to know the number of cracks that occurred a priori in the structure to be able to detect them.

In this article, a crack detection algorithm is presented in order to detect simultaneously the unknown number of cracks that occurred in the beam and their specifications (location and severity). The accomplishment of the proposed technique is that it can easily distinguish among different multiple-crack situations. For example, if the measured data are extracted from a multiple-crack situation and we trick the algorithm to identify a single-crack event, our adaptable algorithm detects the actual number of cracks as it should do. Furthermore, the algorithm could detect any number of cracks available if we do not have enough information about the actual number of cracks.

## 2. Formulation

In this section, the formulation of multiple-crack detection in Euler–Bernoulli beams using natural frequencies is presented. Following, a crack detection algorithm is presented in order to solve the common restriction in the multiple-crack detection problems that is the identification of the actual number of cracks developed in the beam.

### 2.1. Multiple-crack formulation

As shown in Fig. 3, consider a prismatic Euler–Bernoulli beam, including  $m$  normal and open cracks at the normalized locations,  $\beta_1, \beta_2, \dots, \beta_i, \dots, \beta_m$ , i.e.,  $\beta_i = x_i/L$ ,  $i = 1, 2, \dots, m$ ,  $0 < \beta < 1$ .

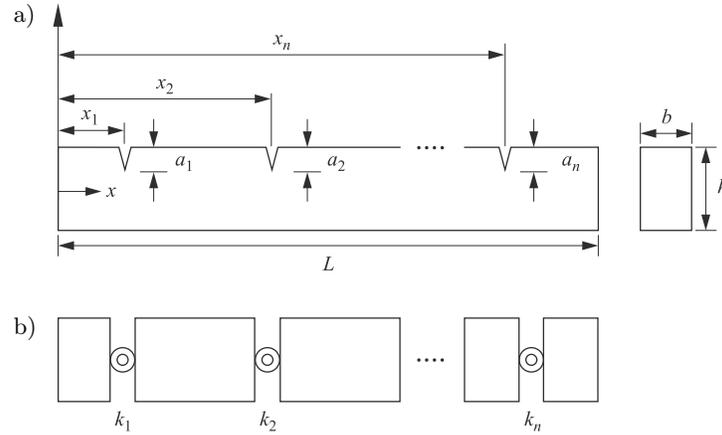


FIG. 3. a) An actual multiple-crack beam, b) cracks modeled by equivalent rotational springs.

Each crack can be modeled as a massless rotational spring with the stiffness of  $k_i$ , which can be written as follows [10]:

$$(2.1) \quad k_i = \frac{Ebh^2}{72\pi f(\gamma_i)},$$

where  $f(\gamma)$  is the correction function as follows:

$$(2.2) \quad f(\gamma) = 0.6384\gamma^2 - 1.035\gamma^3 + 3.7201\gamma^4 - 5.1774\gamma^5 \\ + 7.553\gamma^6 - 7.3324\gamma^7 + 2.4909\gamma^8.$$

Using Rayleigh's quotient for healthy and cracked beams and some simplifications, one can obtain a relation between the natural frequencies of healthy and cracked beams and the crack parameters, as follows:

$$(2.3) \quad \frac{\Delta\omega_n}{\omega_n} \cong \frac{\sum_{i=1}^m \frac{M_{i,n}^2}{2k_i}}{2U_n},$$

Equation (2.3) is the principal and well-known equation, which is used in crack detection problems. In order to use Eq. (2.3) in our crack detection algorithm, the equation is rewritten in the following form [39]:

$$(2.4) \quad \frac{\Delta\omega_n}{\omega_n} = \psi_n \sum_{i=1}^m \frac{\phi_n''^2(\beta_i)}{k_i}, \quad n = 1, 2, \dots, N,$$

where the crack-independent modal parameter  $\psi_n$  is defined as

$$(2.5) \quad \psi_n \equiv \frac{EI}{2L \int_0^1 [\phi_n''(\beta)]^2 d\beta}.$$

In general, as the natural frequencies and mode shapes of the healthy beam are obtained from a theoretical solution, they slightly differ from natural frequencies of cracked beam which are extracted from either experiments or finite element solutions. Without considering this point, the errors of crack detection problem may increase.

Therefore, we may adjust the beam Young’s modulus in each mode, in order to equalize the measured (simulated) and analytical natural frequencies of the uncracked beam in the corresponding mode. In this way, which is called “zero setting” [40], there would be a possibility to have different Young’s modulus in each mode for just one uncracked beam, as follows:

$$(2.6) \quad E_n = \left( \frac{\omega_{\text{measured}}}{\omega_{\text{analytical}}}_n \right)^2 \times E.$$

Based on this technique, we may adjust the beam Young’s modulus in each mode, in order to equalize the measured and analytical natural frequencies of the uncracked beam in the corresponding mode. In this way, there is a possibility that we have different Young’s modulus in each mode for just one uncracked beam.

Therefore, we need to modify the  $\psi_n$  parameters obtained from Eq. (2.5), as:

$$(2.7) \quad \psi_n \equiv \frac{E_n I}{2L \int_0^1 [\phi_n''(\beta)]^2 d\beta}.$$

The unknown parameters in the common crack detection problem are the normalized spatial coordinates  $\beta_i$ , and  $\gamma_i$ ,  $i = 1, 2, \dots, m$ , if we would have a probable situation of  $m$ -crack identification. Thus, for obtaining  $2m$  unknown parameters mentioned above, we need  $2m$  independent algebraic equations to be solved simultaneously. We provide these equations by measuring  $2m$  natural frequencies of the intact and cracked beams. Finally, by using the basic equations (2.4) and (2.5), we transform the solution of a nonlinear optimization problem into the solution of  $2m$  simultaneous nonlinear algebraic equations, as follows:

$$(2.8) \quad \left\{ \frac{\Delta\omega_j}{\omega_j} \right\}_{2m \times 1} = \text{diag} (\psi_j)_{2m \times 2m} [\phi_{ji}]_{2m \times m} \left\{ \frac{1}{k_i} \right\}_{m \times 1},$$

where

$$(2.9) \quad \phi_{ji} = \phi_j''^2(\beta_i), \quad j = 1, 2, \dots, 2m, \quad i = 1, 2, \dots, m.$$

As an example, for triple-crack damage identification, localization and quantification,  $m = 3$ , we need to solve the following system of algebraic equations:

$$(2.10) \quad \left\{ \begin{array}{c} \frac{\Delta\omega_1}{\omega_1} \\ \frac{\Delta\omega_2}{\omega_2} \\ \frac{\Delta\omega_3}{\omega_3} \\ \frac{\Delta\omega_4}{\omega_4} \\ \frac{\Delta\omega_5}{\omega_5} \\ \frac{\Delta\omega_6}{\omega_6} \end{array} \right\} = \begin{bmatrix} \psi_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_6 \end{bmatrix} \begin{bmatrix} \phi_1''(\beta_1) & \phi_1''(\beta_2) & \phi_1''(\beta_3) \\ \phi_2''(\beta_1) & \phi_2''(\beta_2) & \phi_2''(\beta_3) \\ \phi_3''(\beta_1) & \phi_3''(\beta_2) & \phi_3''(\beta_3) \\ \phi_4''(\beta_1) & \phi_4''(\beta_2) & \phi_4''(\beta_3) \\ \phi_5''(\beta_1) & \phi_5''(\beta_2) & \phi_5''(\beta_3) \\ \phi_6''(\beta_1) & \phi_6''(\beta_2) & \phi_6''(\beta_3) \end{bmatrix} \left\{ \begin{array}{c} \frac{1}{k_1} \\ \frac{1}{k_2} \\ \frac{1}{k_3} \end{array} \right\},$$

Therefore, we obtain the unknown parameters  $\beta_i$  and  $k_i$ , and knowing  $k_i$  parameters, we can easily obtain  $\gamma_i$  parameters.

## 2.2. Unknown number of crack identification algorithm

In this section, we present a damage detection algorithm to predict the number of cracks incurred in the beam in addition to the detection problem of crack parameters. The anticipation of the number of cracks at the first step of the solution procedure may seem as a limitation. Furthermore, if assumed that the number of cracks would be many more than the actual one, the procedure of solution algorithm and checking the results would be more complicated. It is necessary to mention that we do not need to know how many cracks exist in order to execute our algorithm successfully. Generally, we formulate Eq. (2.8) for an  $m$ -crack identification challenge; however, our algorithm could have enough flexibility and adaptability to detect the actual number of  $m$  cracks that developed, from zero to a maximum number of  $m$  cracks. This fact authorizes us to develop only a general relationship, and to run the computational task only once, while our numerical procedure enables us to predict the actual number of cracks that developed.

In the presented algorithm, we use the following idea. We start our procedure by considering the least number of cracks that we could expect. Then, if we obtain a “zero” for the crack depth in the results, it means that our primary assumption about the number of cracks should be more than that of the actual one. Thus, we could solve the problem correctly, although we could reduce our primary assumption and resolve the problem. However, if we do not obtain a zero for the crack depth in the results, it could mean that our predicted number of cracks might be false and the results could be unreliable. Therefore, we should increase the number of cracks and solve the problem again, until we would obtain at least one “zero” in the depth of the cracks.

Note that we could start our algorithm by considering a large number of cracks in the beam, therefore, we could obtain zeros for some crack depths and

the actual number of cracks would be obtained. However, this would cause more complicated and time-consuming computations as well as more errors. Therefore, the algorithm suggests starting the procedure considering the least expected number of the cracks. This algorithm can be programmed easily. In the following examples, the use of presented algorithm in the crack detection problem is shown.

### 3. Numerical examples

In this section, we present some numerical examples in order to validate our presented algorithm. The examples are provided for single- and double-crack beams.

#### 3.1. Example 1: single-crack beam

In this example, we utilize a prismatic rectangular cross-section beam with the properties listed in Table 1. To prove our assertion, in this example, we set our default for a double-crack beam, while, in simulated data, there exists only one crack. Thus, we organize our algorithm for a double-crack detection situation, as follows:

$$(3.1) \quad \left\{ \begin{array}{c} \frac{\Delta\omega_1}{\omega_1} \\ \frac{\Delta\omega_2}{\omega_2} \\ \frac{\Delta\omega_3}{\omega_3} \\ \frac{\Delta\omega_4}{\omega_4} \end{array} \right\} = \begin{bmatrix} \psi_1 & 0 & 0 & 0 \\ 0 & \psi_2 & 0 & 0 \\ 0 & 0 & \psi_3 & 0 \\ 0 & 0 & 0 & \psi_4 \end{bmatrix} \begin{bmatrix} \phi_1''(\beta_1) & \phi_1''(\beta_2) \\ \phi_2''(\beta_1) & \phi_2''(\beta_2) \\ \phi_3''(\beta_1) & \phi_3''(\beta_2) \\ \phi_4''(\beta_1) & \phi_4''(\beta_2) \end{bmatrix} \left\{ \begin{array}{c} \frac{1}{k_1} \\ \frac{1}{k_2} \end{array} \right\}.$$

**Table 1. Geometrical and material properties.**

Length, $L$ [m]	Height×width, $b \times h$ [m <sup>2</sup> ]	Young's modulus, $E$ [Pa]	Poisson's ratio, $\nu$	Mass density, $\rho$ [kg/m <sup>3</sup> ]
0.24	$0.02 \times 0.012$	$2.1 \times 10^{11}$	0.3	7860

We need to know four natural frequencies of the intact and cracked beams for solving Eq. (3.1). For modeling and free vibration analysis of the cracked beam, we have used an FEM processing software package. We have modeled a top surface crack, as a 0.5 mm wide V-notch, in a cross-section normal to the beam axis. We have utilized an eight-node 3D solid FE with very fine mesh around the crack. We list the resulting natural frequencies of the single-crack beam in Table 2.

**Table 2.** Natural frequencies obtained by FE for a single-crack cantilever beam.

Case no.	Crack location and size		Natural frequencies (Hz)			
	$\beta$	$\gamma$	$\omega_1$	$\omega_2$	$\omega_3$	$\omega_4$
Uncracked beam	Analytical		289.92	1816.93	5087.45	9969.38
	FE		288.25	1751.3	4688.1	8670.8
1	0.6	0.3	286.16	1642.2	4518.4	8595.4
2	0.3	0.2	282.03	1741.7	4577.6	8625.6

We obtain the required vibration mode shapes of the intact beam and correct them using Eq. (2.6). As it can be seen, the system of equations is nonlinear. To solve the system of equations (3.1), we implement an m.file in MATLAB, based on the fsolve function [39]. As this method depends on an initial guess to solve the set of equations, it may introduce a local minimum as the result rather than a global minimum. Therefore, we divide the solution domain  $(\beta_i, \gamma_i)$  into several subdomains and choose an initial guess from each subdomain. In this way we obtain a set of initial guesses that covers the entire solution domain. The equations are solved using the obtained initial guesses and the best solution is chosen as the result. The outputs of this code are the best solution satisfying Eq. (3.1).

If the  $i$ -th normalized crack size ( $\gamma_i$ ) is less than 0.05, we assume there is no crack in the  $i$ -th location. By enforcing a “zero” for both normalized parameters of  $\gamma_i$  and  $\beta_i$  corresponding to the ignorable crack, we resolve Eq. (3.1) and we use the final results. For the presented cracked beam, i.e.,  $\beta = 0.6$  and  $\gamma = 0.3$ , we obtain the following results for the first run of the algorithm:

$$\begin{aligned}\beta_1 &= 0.605, & \gamma_1 &= 0.281, \\ \beta_2 &= 0.450, & \gamma_2 &= 0.029.\end{aligned}$$

As we may observe, the normalized crack depth is less than 0.05, i.e.,  $\gamma_2 = 0.029$ . Accordingly, as mentioned previously, we set  $\beta_2 = 0$  and  $\gamma_2 = 0$ , resolve equations and obtain the following results:

$$\beta_1 = 0.600, \quad \gamma_1 = 0.281.$$

Regarding the obtained results, the percent relative errors of damage parameters by our algorithm are:

$$\% \text{ error}(\beta) = 0.0, \quad \% \text{ error}(\gamma) = -1.9.$$

Another example is solved and tabulated in Table 3 as the case No. 2 for a single-crack beam. Note that as we use four natural frequencies in our crack detection procedure, we are in a position to believe that we may not encounter the challenge of non-uniqueness results.

**Table 3.** Comparison of predicted and simulated crack locations and sizes for the single-crack cantilever beam from Table 2.

Case no.	Step 1		Results		% error( $\beta$ )	% error( $\gamma$ )
1	$\beta_1 = 0.605$ $\beta_2 = 0.450$	$\gamma_1 = 0.281$ $\gamma_2 = 0.029$	$\beta_1 = 0.600$	$\gamma_1 = 0.281$	0.0	-1.9
2	$\beta_1 = 0.310$ $\beta_2 = 0.310$	$\gamma_1 = 0.191$ $\gamma_2 = 0.001$	$\beta_1 = 0.308$	$\gamma_1 = 0.191$	0.8	-0.9

**3.2. Example 2: double-crack beam**

In this example, we utilize the same beam as in Example 1. For instance, consider the case No. 1 from Table 4. Two cracks are modeled in the beam with the following specifications:

$$\begin{aligned}\beta_1 &= 0.2, & \gamma_1 &= 0.3, \\ \beta_2 &= 0.4, & \gamma_2 &= 0.2.\end{aligned}$$

The first six natural frequencies of this beam are shown in Table 4. In this example, we consider that our beam contains two cracks as the initial guess. Thus, we organize our algorithm for a double-crack detection situation as in Eq. (3.1).

Solving the set of equations, we obtain the following results:

$$\begin{aligned}\beta_1 &= 0.212, & \gamma_1 &= 0.277, \\ \beta_2 &= 0.404, & \gamma_2 &= 0.194.\end{aligned}$$

As it can be seen, there is no “zero” in our results for cracks depth. Therefore, in this section we cannot conclude that our beam is a double-crack beam.

To solve this problem, we continue our algorithm assuming that the beam contains three cracks. Solving Eq. (2.10) with this assumption we obtain the following results:

$$\begin{aligned}\beta_1 &= 0.212, & \gamma_1 &= 0.277, \\ \beta_2 &= 0.402, & \gamma_2 &= 0.188, \\ \beta_3 &= 0.5822, & \gamma_3 &= 0.038.\end{aligned}$$

It can be seen that one of the crack depths, i.e., 0.038, is less than 0.05 and can be considered as zero. Therefore, we can conclude that our beam has two cracks with the following specifications:

$$\begin{aligned}\beta_1 &= 0.212, & \gamma_1 &= 0.277, \\ \beta_2 &= 0.404, & \gamma_2 &= 0.194.\end{aligned}$$

**Table 4. Natural frequencies obtained by FE for a double-crack cantilever beam.**

Case no.	Crack location and size				Natural frequencies (Hz)					
	$\beta_1$	$\gamma_1$	$\beta_2$	$\omega_2$	$\omega_1$	$\omega_2$	$\omega_3$	$\omega_4$	$\omega_5$	$\omega_6$
Uncracked beam	Analytical				289.92	1816.93	5087.45	9969.38	16 480	22 613
	FE				288.25	1751.3	4688.1	8670.8	13 441	18 770
1	0.2	0.3	0.4	0.2	263.92	1713.1	4521.6	8224.6	12 796	18 720
2	0.15	0.2	0.35	0.25	269.78	1705.3	4531.7	8575.5	12 752	18 223

And the prediction errors are as follows:

$$\begin{aligned} \% \text{ error}(\beta_1) &= 1.2, & \% \text{ error}(\gamma_1) &= -2.3, \\ \% \text{ error}(\beta_2) &= 0.4, & \% \text{ error}(\gamma_2) &= -0.6. \end{aligned}$$

Another example is solved and presented in Table 5 as the case No. 2 for a double-crack beam.

We can start our algorithm by assuming that a large number of cracks occurred in the beam and then eliminate the zero depth cracks from our results; however, this assumption will considerably increase the CPU time needed to solve the algorithm.

**Table 5. Comparison of predicted and simulated crack locations and sizes for the double-crack cantilever beam from Table 4.**

Case no.	Step 1	Step 2	Results	% error( $\beta$ )	% error( $\gamma$ )
1	$\beta_1=0.212, \gamma_1=0.277$	$\beta_1=0.212, \gamma_1=0.277$	$\beta_1=0.212, \gamma_1=0.277$	( $\beta_1$ ): 1.2	( $\gamma_1$ ): -2.3
	$\beta_2=0.404, \gamma_2=0.194$	$\beta_2=0.402, \gamma_2=0.188$	$\beta_2=0.404, \gamma_2=0.194$	( $\beta_2$ ): 0.4	( $\gamma_2$ ): -0.6
		$\beta_3=0.5822, \gamma_3=0.038$			
2	$\beta_1=0.16, \gamma_1=0.184$	$\beta_1=0.161, \gamma_1=0.186$	$\beta_1=0.16, \gamma_1=0.184$	( $\beta_1$ ): 1	( $\gamma_1$ ): -1.6
	$\beta_2=0.357, \gamma_2=0.242$	$\beta_2=0.163, \gamma_2=0.014$	$\beta_2=0.357, \gamma_2=0.242$	( $\beta_2$ ): 0.7	( $\gamma_2$ ): -0.8
		$\beta_3=0.365, \gamma_3=0.253$			

#### 4. Conclusion

In this article, a simple method for detecting, localizing and quantifying multiple cracks in beams using natural frequencies was presented. We modeled cracks

as rotational springs and demonstrated a relationship among natural frequencies, crack locations and depths. The main advantage of presented algorithm was that it could detect any number of cracks available if we could not have enough information about the actual number of cracks. Furthermore, the algorithm was adaptable to detect actual number of cracks that incurred despite the fact that we misguided it by a wrong assumption about the number of cracks. The presented crack detection algorithm is feasible for computer programming. In the presented examples, the algorithm could find the actual number of cracks successfully with an arbitrary initial guess about that number. Furthermore, the algorithm could detect crack locations and depths with acceptable accuracies.

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