Arch. Mech., **64**, 6, pp. 555–579, Warszawa 2012

General steady-state solution and Green's function in orthotropic piezothermoelastic diffusion medium

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The present investigation deals with the study of Green's functions in orthotropic piezothermoelastic diffusion media. With this objective, firstly the two-dimensional general solution in orthotropic piezothermoelastic diffusion media is derived. On the basis of general solution, the Green function for a point heat source and chemical potential source in the interior of semi-infinite orthotropic piezothermoelastic diffusion material is constructed by five newly introduced harmonic functions. The components of displacement, stress, electric displacement, electric potential, temperature change and chemical potential are expressed in terms of elementary functions. Since all the components are expressed in terms of elementary functions, this fact makes them convenient to use. From the present investigation, a special case of interest is also analyzed to depict the effect of diffusion. Resulting quantities are computed numerically and presented graphically to illustrate the effect of diffusion.

Key words: Green's functions, piezothermoelasic diffusion, electric displacement, electric potential, semi-infinite.

1. Introduction

Green's functions or fundamental solutions play an important role in both applied and theoretical studies of solids' physics. Green's functions can be used to construct many analytical solutions of practical problems when boundary conditions are imposed. Green's functions play an important role in the solution of the numerous problems in the mechanics and physics of solids. They are the heart of singular integrals equation method such as the boundary element method which is often used in engineering. For static problems, the general solution is finally expressed in terms of five harmonic functions (more precisely weighted or quasi harmonic functions). Green's functions with applications systematically present various methods of deriving these useful functions. Such a form allows one to use the solution feasibly to solve boundary value problems associated with cracks, defects, inclusions and punches. Many researchers have investigated Green's function for elastic solid in isotropic and anisotropic elastic media, notable among them are Lord Kelvin [1], Freedholm [2], Synge [3], Pan and Chou [4], Deeg [5], Wang [6] and Chen and Lin [7].

Lee and Jiang [8] investigated the boundary integral formulation and two-dimensional fundamental solution for piezoelectric media. Wang and Zheng [9] derived the general solution for three-dimensional problem in piezoelectric media. Ding *et al.* [10] investigated the fundamental solution for piezoelectric media. Ding *et al.* [11] studied the fundamental solution for plane problem of piezoelectric materials.

Piezoelectric ceramics and composites have been extensively used in many engineering applications such as sensors, actuators, intelligent structures etc. when thermal effects are not considered; piezoelectric ceramics and piezoelectric polymers, which are extensively utilized in small structure and intelligent system, all belong to pyroelectric media. Rao and Sunar [12] pointed out that the temperature variation in the piezoelectric media can affect the overall performance of a distributed control system. Therefore, in depth investigation on electro-thermo-mechanical coupling behavior is significant.

The thermal effect is not considered in the above works. Rao and Sunar [12] pointed out the temperature variation in the piezoelectric media. Chen *et al.* [13] derived the general solution for transversely isotropic piezothermoelastic media. Chen *et al.* [14] obtained Green's function of transversely isotropic pyroelectric media with a penny- shaped crack. Hou *et al.* [15] constructed Green's function for a point heat source on the surface of a semi-infinite transversely isotropic pyroelectric media.

Diffusion can be defined as the movement of particles from an area of high concentration to an area of lower concentration until equilibrium is reached. It occurs as a result of second law of thermodynamics which states that the entropy or disorder of any system must always increase with time. Diffusion is important in many life processes. Nowadays, there is a great deal of interest in the study of this phenomena, due to its many application in geophysics and industrial applications. Until recently, thermodiffusion in solids, especially in metals, was considered as a quantity that is independent of body deformation. Practice however indicates that the process of thermodiffusion could have a very considerable influence on the deformation of the body. Thermodiffusion in elastic solid is due to the coupling of temperature, mass diffusion and strain in addition to the exchange of heat and mass with the environment.

Nowacki [16–19] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. This implies infinite speed of propagation of thermoelastic waves. Sherief *et al.* [20] developed the generalized theory of thermoelastic diffusion with one relaxation time which allows finite speeds of propagation of waves. Recently Kumar and Kansal [21] derived the basic equations for generalized thermoelastic diffusion (GL model) and discussed the Lamb waves.

Kumar and Chawla [22] discussed the surface wave propagation in an elastic layer lying over a thermodiffusive elastic half-space with imperfect boundary. Kuang [23] discussed the variational principles for generalized thermodiffusion theory in pyroelectricity. Kumar and Chawla [24] obtained the fundamental solutions for orthotropic thermodiffusive elastic media. Recently Kumar and Chawla [25] derived the Green function for two-dimensional problem in orthotropic thermoelastic diffusion media. However, the important Green's function for two-dimensional problem for a steady point heat source in orthotropic piezothermoelastic diffusion medium has not been discussed so far.

In order to theoretically study surfaces of fairly arbitrary materials for initial and boundary value problems, there is a need for the computational approach because complex geometries can not be treated analytically, in general. So in the present problem, we have developed a formalism to investigate it by using Green's functions.

The Green function for two-dimensional problem in orthotropic piezothermoelastic diffusion medium is investigated in this paper. The adopted approach is to obtain closed form expression for the piezothermoelastic diffusion media by using Green's functions. Green's functions are very useful for the analysis of many problems in the mathematics and physics of piezothermoelastic diffusion solid.

2. Basic equations

The basic governing equations of orthotropic piezothermodiffusive elastic materials can be found in [23]. If all the components are independent coordinate y, this is the so-called plane problem. The constitutive equations in two-dimensional Cartesian coordinate (x, z) can be expressed as

$$\sigma_{xx} = c_{11}\frac{\partial u}{\partial x} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \Phi}{\partial z} - \beta_1 T - b_1\mu, \tag{2.1}$$

$$\sigma_{zz} = c_{13}\frac{\partial u}{\partial x} + c_{33}\frac{\partial w}{\partial z} + e_{33}\frac{\partial \Phi}{\partial z} - \beta_3 T - b_3\mu, \tag{2.2}$$

$$\sigma_{zx} = c_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + e_{15}\frac{\partial \Phi}{\partial x}, \tag{2.3}$$

$$D_x = e_{15}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) - \varepsilon_{11}\frac{\partial \Phi}{\partial x}, \tag{2.4}$$

$$D_z = e_{31}\frac{\partial u}{\partial x} + e_{33}\frac{\partial w}{\partial z} - \varepsilon_{33}\frac{\partial \Phi}{\partial z} + p_3 T, \tag{2.5}$$

where u and w are components of the mechanical displacement in x and z directions, respectively; σ_{ij} and D_i are the components of stress and electric displacement, respectively; β_i and b_i are material constants. Φ, T and μ are electric potential ,temperature increment and chemical potential respectively; $c_{ij}, e_{ij}, \varepsilon_{ij}$, and p_3 are elastic piezoelectric, dielectric, thermal modules, diffusion modules and pyroelectric constants, respectively.

The mechanical, electric, heat equilibrium and mass diffusions equations for static problem, in the absence of body forces, free charges, heat sources and mass diffusive sources are

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{zx}}{\partial z} = 0, \qquad \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = 0, \tag{2.6}$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial \sigma_z}{\partial z} = 0, \tag{2.7}$$

$$\left(\lambda_1 \frac{\partial^2}{\partial^2 x} + \lambda_3 \frac{\partial^2}{\partial^2 z}\right) T = 0, \tag{2.8}$$

$$\left(D_1 \frac{\partial^2}{\partial^2 x} + D_3 \frac{\partial^2}{\partial^2 z}\right) \mu = 0. \tag{2.9}$$

We define the dimensionless quantities:

$$(x', z', u', w') = \frac{\omega_1^*}{v_1}(x, z, u, w), \qquad \sigma'_{ij} = \frac{\sigma_{ij}}{\beta_1 T_0},$$
$$T' = \frac{T}{T_0}, \quad \mu' = \frac{\mu}{v_1^2}, \quad D'_i = \frac{D_i}{\sqrt{\beta_1 T_0}}, \quad H' = \frac{v_1}{T_0 \lambda_1 \omega_1^*} H, \quad P' = \frac{P}{v_1 D_1 \omega_1^*},$$

where

$$v_1^2 = \frac{\beta_1 T_0}{b_1}, \qquad \omega_1^* = \frac{\beta_1 c_{11}}{b_1 \lambda_1}. \tag{2.10}$$

Substituting Eqs. (2.1)–(2.5) into Eqs. (2.6) and (2.7) and applying the dimensionless quantities defined by (2.10) to resulting equations, after suppressing the primes, we obtain

$$\begin{aligned}\left(\frac{\partial^2}{\partial x^2} + \delta_1 \frac{\partial^2}{\partial z^2}\right) u + \left(\delta_2 \frac{\partial^2}{\partial x \partial z}\right) w \\ - e_1 \bar{\varepsilon}_p \frac{\partial^2 \Phi}{\partial x \partial z} - r_1 \left(\frac{\partial}{\partial x}\right) T - q_1 \left(\frac{\partial}{\partial x}\right) \mu = 0,\end{aligned} \tag{2.11}$$

$$\begin{aligned}\left(\delta_2 \frac{\partial^2}{\partial x \partial z}\right) u + \left(\delta_1 \frac{\partial^2}{\partial x^2} + \delta_3 \frac{\partial^2}{\partial z^2}\right) w \\ - \bar{\varepsilon}_p \left(e_2 \frac{\partial^2}{\partial x^2} + \delta_3 \frac{\partial^2}{\partial z^2}\right) \Phi - r_3 \left(\frac{\partial}{\partial z}\right) T - q_3 \left(\frac{\partial}{\partial z}\right) \mu = 0,\end{aligned} \tag{2.12}$$

$$\left(e_1\frac{\partial^2}{\partial x\partial z}\right)u+\left(e_2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)w \\ -\bar{\varepsilon}_q\left(\bar{\varepsilon}\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\varPhi+g_1\left(\frac{\partial}{\partial z}\right)T+h_1\left(\frac{\partial}{\partial z}\right)\mu=0, \tag{2.13}$$

$$\left(\frac{\partial^2}{\partial^2 x}+\bar{\lambda}\frac{\partial^2}{\partial^2 z}\right)T=0, \tag{2.14}$$

$$\left(\frac{\partial^2}{\partial^2 x}+\bar{D}\frac{\partial^2}{\partial^2 z}\right)\mu=0, \tag{2.15}$$

$$(\delta_1,\delta_2,\delta_3)=\frac{1}{c_{11}}\left(c_{44},c_{13}+c_{44},c_{33}\right),$$

$$(e_1,e_2)=\frac{1}{e_{33}}\left(e_{31}+e_{15},e_{15}\right),$$

$$(r_1,r_3,q_1,q_3)=\frac{1}{c_{11}}(\beta_1T_0,\beta_3T_0,b_1v_1^2,b_3v_1^2),$$

$$(\varepsilon,g_1)=\frac{1}{\varepsilon_{33}}(\varepsilon_{11},P_3T_0,b_3v_1^2),$$

$$(\bar{\varepsilon}_p,\bar{\varepsilon}_q)=\frac{1}{v_1}\left(\frac{e_{33}\varPhi_0\omega_1^*}{c_{11}},\frac{\varepsilon_{33}\varPhi_0\omega_1^*}{e_{33}}\right),$$

$$\bar{\lambda}=\frac{\lambda_3}{\lambda_1},\quad \bar{D}=\frac{D_3}{D_1},\quad \varPhi_0=\frac{v_1\beta_1T_0}{\omega^*e_{33}}.$$

Equations (2.11)–(2.15) can be written as

$$D\{u,w,\varPhi,T,\mu\}^t=0, \tag{2.16}$$

where D is the differential operator matrix given by

(2.17)

$$\begin{bmatrix} \frac{\partial^2}{\partial x^2}+\delta_1\frac{\partial^2}{\partial z^2} & \delta_2\frac{\partial^2}{\partial x\partial z} & e_1\bar{\varepsilon}_p\frac{\partial^2}{\partial x\partial z} & -r_1\frac{\partial}{\partial x} & -q_1\frac{\partial}{\partial x} \\ \delta_2\frac{\partial^2}{\partial x\partial z} & \delta_1\frac{\partial^2}{\partial x^2}+\delta_3\frac{\partial^2}{\partial z^2} & \bar{\varepsilon}_p\left(e_2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial x^2}\right)\frac{\partial}{\partial z} & -r_3\frac{\partial}{\partial z} & -q_3\frac{\partial}{\partial z} \\ e_1\frac{\partial^2}{\partial x\partial z} & e_2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial x^2} & -\bar{\varepsilon}_q\left(\bar{\varepsilon}\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right) & g_1\frac{\partial}{\partial z} & h_1\frac{\partial}{\partial z} \\ 0 & 0 & 0 & \left(\frac{\partial^2}{\partial x^2}+\bar{a}\frac{\partial^2}{\partial x^2}\right) & 0 \\ 0 & 0 & 0 & 0 & \left(\frac{\partial^2}{\partial x^2}+\bar{D}\frac{\partial^2}{\partial x^2}\right) \end{bmatrix}.$$

Equation (2.16) is a homogeneous set of differential equations in $u,w,\varPhi,T,\mu$. The general solution according to the operator theory is as follows:

$$u=A_{i1}F,\quad w=A_{i2}F,\quad \varPhi=A_{i3}F, \\ T=A_{i4}F,\quad \mu=A_{i5}F\quad (i=1,2,3,4,5). \tag{2.18}$$

The determinant of the matrix D is given as

$$|D| = \left(a\frac{\partial^6}{\partial z^6} + b\frac{\partial^6}{\partial x^2 \partial z^4} + c\frac{\partial^6}{\partial x^4 \partial z^2} + d\frac{\partial^6}{\partial x^6}\right) \times \left(\frac{\partial^2}{\partial x^2} + \bar{\lambda}\frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2}{\partial x^2} + \bar{D}\frac{\partial^2}{\partial z^2}\right), \tag{2.19}$$

where a, b, c, d are given in Appendix A. The function F in Eq. (2.18) satisfies the following homogeneous equation:

$$|D|F = 0. \tag{2.20}$$

It can be seen that if i was set to 1 or 2 in Eq. (2.18), one can get two sets of general solution with $P = 0$, $T = 0$ and $\mu = 0$, which are actually to those for pure elasticity (see [26]); $i = 1, 2$ and 3 corresponds to the solution for piezoelectricity discussed in [28]; $i = 4$ corresponds to the general solution W_1 with $\mu = 0$ which is identical to that for piezothermoelaticity; $i = 5$ corresponds to the general solution W_2 with $T = 0$.

Due to the linear nature of the piezothermoelastic diffusion theory adopted in this paper, follows the same procedure as used by Li *et el.* [28, 29] with superposing W_1 and W_2, this leads to

$$\begin{aligned}
u &= \left(a_1\frac{\partial^6}{\partial x^6} + b_1\frac{\partial^6}{\partial x^4 \partial z^2} + c_1\frac{\partial^6}{\partial x^2 \partial z^4} + d_1\frac{\partial^6}{\partial z^6}\right)\frac{\partial F}{\partial x},\\
w &= \left(a_2\frac{\partial^6}{\partial x^6} + b_2\frac{\partial^6}{\partial x^4 \partial z^2} + c_2\frac{\partial^6}{\partial x^2 \partial z^4} + d_2\frac{\partial^6}{\partial z^6}\right)\frac{\partial F}{\partial z},\\
\Phi &= \left(a_3\frac{\partial^6}{\partial x^6} + b_3\frac{\partial^6}{\partial x^4 \partial z^2} + c_3\frac{\partial^6}{\partial x^2 \partial z^4} + d_3\frac{\partial^6}{\partial z^6}\right)\frac{\partial F}{\partial z},\\
T &= \left(a_4\frac{\partial^8}{\partial x^8} + b_4\frac{\partial^8}{\partial x^6 \partial z^2} + c_4\frac{\partial^8}{\partial x^4 \partial z^4} + d_4\frac{\partial^8}{\partial x^6 \partial z^2} + l_5\frac{\partial^8}{\partial z^8}\right)F,\\
\mu &= \left(a_5\frac{\partial^8}{\partial x^8} + b_5\frac{\partial^8}{\partial x^6 \partial z^2} + c_5\frac{\partial^8}{\partial x^4 \partial z^4} + d_5\frac{\partial^8}{\partial x^6 \partial z^2} + l_6\frac{\partial^8}{\partial z^8}\right)F,
\end{aligned} \tag{2.21}$$

where the coefficients a_k, b_k, c_k, d_k ($k = 1, 2, 3, 4, 5$) and l_5, l_6 are the expressions given in Appendix B.

The general solution of Eq. (2.16) in terms of F can be rewritten as

$$\prod_{j=1}^{5}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_j^2}\right)F = 0, \tag{2.22}$$

where $z_j = s_j z$, $s_4 = \sqrt{\lambda_1/\lambda_3}$, $s_5 = \sqrt{D_1/D_3}$ and s_j $(j = 1, 2, 3)$ are three roots (with positive real part) of the following algebraic equation:

$$as^6 - bs^4 + cs^2 - d = 0. \tag{2.23}$$

As known from the generalized Almansi theorem [30], the function F can be expressed in terms of five harmonic functions:

$$\begin{aligned}
&1.\ F = F_1 + F_2 + F_3 + F_4 + F_5 && \text{for distinct } s_j\ (j = 1, 2, 3, 4, 5),\\
&2.\ \ F = F_1 + F_2 + F_3 + F_4 + zF_5 && \text{for } s_1 \neq s_2 \neq s_3 \neq s_4 = s_5,\\
&3.\ F = F_1 + F_2 + F_3 + zF_4 + z^2F_5 && \text{for } s_1 \neq s_2 \neq s_3 = s_4 = s_5,\\
&4.\ F = F_1 + F_2 + zF_3 + z^2F_4 + z^3F_4 && \text{for } s_1 \neq s_2 = s_3 = s_4 = s_5,\\
&5.\ F = F_1 + zF_2 + z^2F_3 + z^3F_4 + z^4F_5 && \text{for } s_1 = s_2 = s_3 = s_4 = s_5,
\end{aligned} \tag{2.24}$$

where F_j satisfies the following harmonic equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_j^2}\right)F_j = 0, \qquad (j = 1, 2, 3, 4, 5). \tag{2.25}$$

The general solution for the case of distinct roots, can be derived as follows:

$$\begin{gathered}
u = \sum_{j=1}^{5} p_{1j}\frac{\partial^7 F_j}{\partial x \partial z_j^6}, \qquad w = \sum_{j=1}^{5} s_j p_{2j}\frac{\partial^7 F_j}{\partial z_j^7}, \qquad \Phi = \sum_{j=1}^{5} s_j p_{3j}\frac{\partial^7 F_j}{\partial z_j^7},\\
T = p_{44}\frac{\partial^8 F_4}{\partial z_4^8}, \qquad \mu = p_{55}\frac{\partial^8 F_5}{\partial z_5^8}.
\end{gathered} \tag{2.26}$$

Equation (2.26) can be further simplified by taking

$$p_{1j}\frac{\partial^6 F_j}{\partial z_j^6} = \psi_j. \tag{2.27}$$

Use of (2.27) in Eq. (2.26) yields

$$\begin{gathered}
u = \sum_{j=1}^{5} \frac{\partial \psi_j}{\partial x}, \qquad w = \sum_{j=1}^{5} s_j P_{1j}\frac{\partial \psi_j}{\partial z_j}, \qquad \Phi = \sum_{j=1}^{5} s_j P_{1j}\frac{\partial \psi_j}{\partial z_j},\\
T = P_{34}\frac{\partial^2 \psi_4}{\partial z_4^2}, \qquad C = P_{45}\frac{\partial^2 \psi_5}{\partial z_5^2},
\end{gathered} \tag{2.28}$$

where

$$P_{1j} = p_{2j}/p_{1j}, \qquad P_{2j} = p_{3j}/p_{1j}, \qquad P_{34} = p_{44}/p_{14}, \qquad P_{45} = p_{55}/p_{15}.$$

The function ψ_j satisfies the harmonic equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_j^2}\right)\psi_j = 0, \qquad j = 1, 2, 3, 4, 5. \tag{2.29}$$

Applying the dimensionless quantities defined by (2.7) to Eqs. (2.1)–(2.5), after suppressing the primes, with the aid of (2.28) we obtain

$$\begin{aligned}
\sigma_{xx} &= \sum_{j=1}^{5} (-f_1 + f_2 s_j^2 P_{1j} + f_3 s_j^2 P_{2j} - P_{3j} - f_4 P_{4j}) \frac{\partial^2 \psi_j}{\partial z_j^2}, \\
\sigma_{zz} &= \sum_{j=1}^{5} (-f_2 + f_5 s_j^2 P_{1j} + f_6 s_j^2 P_{2j} - f_7 P_{3j} - f_8 P_{4j}) \frac{\partial^2 \psi_j}{\partial z_j^2}, \\
\sigma_{zx} &= \sum_{j=1}^{5} [f_9(1 + P_{1j}) + f_{10} P_{2j}] s_j \frac{\partial^2 \psi_j}{\partial x \partial z_j}, \\
D_x &= \sum_{j=1}^{5} [l_1(1 + P_{1j}) - n_{10} P_{2j}] s_j \frac{\partial^2 \psi_j}{\partial x \partial z_j}, \\
D_z &= \sum_{j=1}^{5} (-l_2 + l_3 s_j^2 P_{1j} - n_2 s_j^2 P_{2j} + n_3 P_{3j} - n_4 P_{4j}) \frac{\partial^2 \psi_j}{\partial z_j^2},
\end{aligned} \tag{2.30}$$

where

$$P_{31} = P_{32} = P_{33} = P_{35} = 0 \qquad \text{and} \qquad P_{41} = P_{42} = P_{43} = P_{44} = 0,$$

and

$$(f_1, f_2, f_3, f_4, f_5, f_6,) = \frac{1}{\beta_1 T_0}\left(c_{11}, c_{13}, \frac{e_{31}\omega_1^* \Phi_0}{T_0}, b_1\mu_0, c_{33}, \frac{e_{33}\omega_1^* \Phi_0}{T_0}\right),$$

$$f_7 = \frac{\beta_3}{\beta_1}, \quad f_8 = \frac{b_3\mu_0}{b_1 T_0}, \quad f_9 = \frac{c_{44}}{\beta_1 T_0}, \quad f_{10} = \frac{e_{15}\omega_1^* \Phi_0}{\beta_1 T_0 v_1},$$

$$(l_1, l_2.l_3, n_1, n_2, n_3, n_4) = \frac{1}{\sqrt{\beta_1 T_0}}\left(e_{15}, e_{31}, e_{33}, \frac{\varepsilon_{11}\omega_1^* \Phi_0}{v_1}, \frac{\varepsilon_{11}\omega_1^* \Phi_0}{v_1}, P_3 T_0, b_3^* \mu_0\right).$$

Substituting Eq. (2.30) into Eqs. (2.1)–(2.5), with the aid of (2.5) and (2.6) gives

$$
\begin{aligned}
& f_1-(f_2P_{1j}+f_3P_{2j})s_j^2+P_{3j}+f_4P_{4j}=[f_9(1+P_{1j})+f_{10}P_{2j}]s_j^2,\\
& -f_2+(f_5P_{1j}+f_6P_{2j})s_j^2-f_7P_{3j}-f_8P_{4j}=[f_9(1+P_{1j})+f_{10}P_{2j}],\\
& -l_2+(l_3P_{1j}-n_2P_{2j})s_j^2+n_3P_{3j}+n_4P_{4j}=l_1(1+P_{1j}),\\
& (\lambda_1-\lambda_3s_j^2)P_{3j}=0,\\
& (D_1-D_3s_j^2)P_{4j}=0 \qquad (j=1,2,3,4,5).
\end{aligned} \tag{2.31}
$$

By virtue of the above equations, the general solution (2.30) can be simplified as

$$
\begin{gathered}
\sigma_{xx}=-\sum_{j=1}^{5}s_j^2w_{1j}\frac{\partial^2\psi_j}{\partial z_j^2},\quad \sigma_{zz}=\sum_{j=1}^{5}w_{1j}\frac{\partial^2\psi_j}{\partial z_j^2},\quad \sigma_{zx}=\sum_{j=1}^{5}s_jw_{1j}\frac{\partial^2\psi_j}{\partial x\partial z_j},\\
D_x=\sum_{j=1}^{5}s_jw_{2j}\frac{\partial^2\psi_j}{\partial x\partial z_j},\qquad D_z=\sum_{j=1}^{5}w_{2j}\frac{\partial^2\psi_j}{\partial z_j^2},
\end{gathered} \tag{2.32}
$$

where

$$
\begin{aligned}
w_{1j}&=\frac{f_1-(f_2P_{1j}+f_3P_{2j})s_j^2+P_{3j}+f_4P_{4j}}{s_j^2}=f_9(1+P_{1j})+f_{10}P_{2j}\\
&=-f_2+(f_5P_{1j}+f_6P_{2j})s_j^2-f_7P_{3j}-f_8P_{4j},\\
w_{2j}&=-l_2+(l_3P_{1j}-n_2P_{2j})s_j^2+n_3P_{3j}+n_4P_{4j}=l_1(1+P_{1j})-n_1P_{2j}.
\end{aligned} \tag{2.33}
$$

3. Green's function for a point heat source and chemical potential source in the interior of a semi-infinite orthotropic piezothermodiffusion elastic material

As shown in Fig. 1 we consider an orthotropic semi-infinite piezothermodiffusion elastic material $z \geq 0$. A point heat source H and chemical potential source

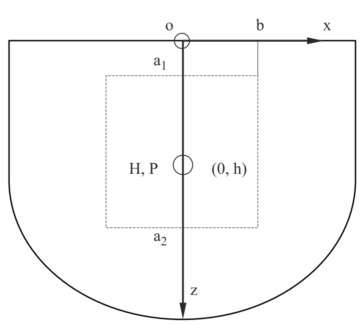

Fig. 1. A semi-infinite piezothermodiffusive elastic plane applied by a point heat source of strength H and chemical potential source of strength P.

P are applied at the point $(0, h)$ in two-dimensional Cartesian coordinate (x, z) and the surface $z = 0$ is free, thermally insulated and impermeable boundary. In Cartesian coordinate system, the general solution given by equation (2.28) and (2.32) in this semi-infinite plane is derived in this section.

In the rest of the paper, following notations are introduced:

$$\begin{aligned} z_j &= s_j z, \qquad h_k = s_k h, \qquad z_{jk} = z_j + h_k, \\ r_{jk} &= \sqrt{x^2 + z_{jk}^2}, \qquad \bar{z}_{jk} = z_j - h_k, \\ \bar{r}_{jk} &= \sqrt{x^2 + \bar{z}_{jk}^2} \qquad (j, k = 1, 2, 3, 4). \end{aligned} \tag{3.1}$$

By virtue of trial and error method, Green's functions in the semi-infinite plane are assumed in the following form:

$$\begin{aligned} \psi_j = {} & A_j \left[\frac{1}{2}(\bar{z}_{jj}^2 - x^2)\left(\log \bar{r}_{jj} - \frac{3}{2}\right) - x\bar{z}_{jj} \tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right)\right] \\ & + \sum_{k=1}^{5} A_{jk} \left[\frac{1}{2}(z_{jk}^2 - x^2)\left(\log r_{jk} - \frac{3}{2}\right) - x z_{jk} \tan^{-1}\left(\frac{x}{z_{jk}}\right)\right], \\ & \qquad j = 1, 2, 3, 4, 5, \end{aligned} \tag{3.2}$$

where A_j and A_{jk} $(j, k = 1, 2, 3, 4, 5)$ are 30 constants to be determined.

The boundary conditions at the surface $z = 0$ are

$$\sigma_{zz} = \sigma_{zx} = 0, \qquad D_z = 0, \qquad \frac{\partial \mu}{\partial z} = 0, \qquad \frac{\partial T}{\partial z} = 0. \tag{3.3}$$

Substituting Eq. (2.30) into Eqs. (2.28) and (2.32), we obtain

$$\begin{aligned} u = {} & \sum_{j=1}^{5} A_j \left[x(\log \bar{r}_{jj} - 1) + \bar{z}_{jj} \tan^{-1} \frac{x}{\bar{z}_{jj}} \right] \\ & - \sum_{j=1}^{5}\sum_{k=1}^{5} A_{jk} \left[x(\log r_{jk} - 1) + z_{jk} \tan^{-1} \frac{x}{z_{jk}} \right], \end{aligned} \tag{3.4a}$$

$$\begin{aligned} w = {} & \sum_{j=1}^{5} s_j P_{1j} A_j \left[\bar{z}_{jj}(\log \bar{r}_{jj} - 1) - x \tan^{-1} \frac{x}{\bar{z}_{jj}} \right] \\ & + \sum_{j=1}^{5}\sum_{k=1}^{5} s_j P_{1j} A_{jk} \left[z_{jk}(\log r_{jk} - 1) - x \tan^{-1} \frac{x}{z_{jk}} \right], \end{aligned} \tag{3.4b}$$

$$\Phi = \sum_{j=1}^{5} s_j P_{1j} A_j \left[\bar{z}_{jj}(\log \bar{r}_{jj} - 1) - x \tan^{-1} \frac{x}{\bar{z}_{jj}}\right] + \sum_{j=1}^{5}\sum_{k=1}^{5} s_j P_{1j} A_{jk} \left[z_{jk}(\log r_{jk} - 1) - x \tan^{-1} \frac{x}{z_{jk}}\right], \tag{3.4c}$$

$$T = P_{34} A_4 \log \bar{r}_{44} + P_{34} \sum_{k=1}^{5} A_{4k} r_{4k}, \tag{3.4d}$$

$$\mu = P_{35} A_5 \log \bar{r}_{55} + P_{45} \sum_{k=1}^{5} A_{5k} r_{5k}, \tag{3.4e}$$

$$\sigma_{xx} = -\sum_{j=1}^{5} s_j^2 w_{1j} A_j \log \bar{r}_{jj} - \sum_{j=1}^{5}\sum_{k=1}^{5} s_j^2 w_{1j} A_{jk} \log r_{jk}, \tag{3.4f}$$

$$\sigma_{zz} = \sum_{j=1}^{5} w_{1j} A_j \log \bar{r}_{jj} + \sum_{j=1}^{5}\sum_{k=1}^{5} w_{1j} A_{jk} \log r_{jk}, \tag{3.4g}$$

$$\sigma_{zx} = -\sum_{j=1}^{5} s_j w_{1j} A_j \tan^{-1} \frac{x}{\bar{z}_{jj}} - \sum_{j=1}^{5}\sum_{k=1}^{5} s_j w_{1j} A_{jk} \tan^{-1} \frac{x}{z_{jk}}, \tag{3.4h}$$

$$D_x = -\sum_{j=1}^{5} s_j w_{2j} A_j \tan^{-1} \frac{x}{\bar{z}_{jj}} - \sum_{j=1}^{5}\sum_{k=1}^{5} s_j w_{2j} A_{jk} \tan^{-1} \frac{x}{z_{jk}}, \tag{3.4i}$$

$$D_z = \sum_{j=1}^{5} w_{2j} A_j \log \bar{r}_{jj} + \sum_{j=1}^{5}\sum_{k=1}^{5} w_{2j} A_{jk} \log\, r_{jk}. \tag{3.4j}$$

Considering the continuity on plane $z = h$ for w, Φ, σ_{zx} and D_x gives the following expressions:

$$\sum_{j=1}^{5} s_j P_{1j} A_j = 0, \tag{3.5}$$

$$\sum_{j=1}^{5} s_j w_{1j} A_j = 0, \tag{3.6}$$

$$\sum_{j=1}^{5} s_j P_{2j} A_j = 0, \tag{3.7}$$

$$\sum_{j=1}^{5} s_j w_{2j} A_j = 0 \tag{3.8}$$

Equations (3.5)–(3.8) can be written in combined form as

$$\sum_{j=1}^{5} s_j P_{mj} A_j = 0, \qquad m = 1, 2, \tag{3.9}$$

$$\sum_{j=1}^{5} s_j w_{mj} A_j = 0. \tag{3.10}$$

Substitution w_{mj} $(m = 1, 2)$ from Eq. (2.33) into (3.10) gives

$$\sum_{j=1}^{5} s_j [f_9(1 + P_{1j}) + f_{10} P_{2j}] A_j = 0, \tag{3.11}$$

$$\sum_{j=1}^{5} s_j [l_1(1 + P_{1j}) - n_1 P_{2j}] A_j = 0, \tag{3.12}$$

By their virtue, Eqs. (3.9), (3.11) and (3.12) can be simplified to one equation

$$\sum_{j=1}^{5} s_j A_j = 0. \tag{3.13}$$

Considering the mechanical, electric and thermal equilibrium as well as chemical potential per unit mass for a rectangle of $a_1 \le z \le a_2$ and $-b \le x \le b$ $(b > 0)$, four equations can be obtained

$$\int_{-b}^{b} [\sigma_{zz}(x, a_2) - \sigma_{zz}(x, a_1)] dx + \int_{0}^{a} [\sigma_{zx}(b, z) - \sigma_{zx}(-b, z)] dz = 0, \tag{3.14a}$$

$$\int_{-b}^{b} [D_z(x, a_2) + D_z(x, a_1)] dx + \int_{0}^{a} [D_x(b, z) - D_x(-b, z)] dz = 0, \tag{3.14b}$$

$$-\bar{\lambda} \int_{-b}^{b} \left[\frac{\partial T}{\partial z}(x, a_2) - \frac{\partial T}{\partial z}(x, a_1)\right] dx - \int_{0}^{a} \left[\frac{\partial T}{\partial x}(b, z) - \frac{\partial T}{\partial x}(-b, z)\right] dz = H, \tag{3.14c}$$

$$-\bar{D} \int_{-b}^{b} \left[\frac{\partial \mu}{\partial z}(x, a_2) - \frac{\partial \mu}{\partial z}(x, a_1)\right] dx - \int_{0}^{a} \left[\frac{\partial \mu}{\partial x}(b, z) - \frac{\partial \mu}{\partial x}(-b, z)\right] dz = P. \tag{3.14d}$$

Some useful integrals are listed as follows:

$$\int \log \bar{r}_{jj} = x(\log \bar{r}_{jj} - 1) + \bar{z}_{jj} \tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right), \tag{3.15a}$$

$$\int \log r_{jk} = x(\log r_{jk} - 1) + z_{jk} \tan^{-1}\left(\frac{x}{z_{jk}}\right), \tag{3.15b}$$

$$\int \tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right) = \frac{1}{s_j}\left(x \log \bar{r}_{jj} + \bar{z}_{jj} \tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right)\right), \tag{3.15c}$$

$$\int \tan^{-1}\left(\frac{x}{z_{jk}}\right) = \frac{1}{s_j}\left(x \log r_{jk} + z_{jk} \tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right)\right), \tag{3.15d}$$

$$\int \frac{\partial T}{\partial z} dx = s_4 P_{34}\left(A_4 \tan^{-1} \frac{x}{\bar{z}_{44}} + \sum_{k=1}^{4} A_{4k} \tan^{-1} \frac{x}{z_{4k}}\right), \tag{3.15e}$$

$$\int \frac{\partial T}{\partial x} dz = -\frac{P_{34}}{s_4}\left(A_4 \tan^{-1} \frac{x}{\bar{z}_{44}} + \sum_{k=1}^{4} A_4 \tan^{-1} \frac{x}{z_4}\right), \tag{3.15f}$$

$$\int \frac{\partial \mu}{\partial z} dx = A_j s_j P_{2j} \tan^{-1} \frac{x}{\bar{z}_{jj}} + \sum_{k=1}^{4} A_{jk} s_j P_{2j} \tan^{-1} \frac{x}{z_{jk}}, \tag{3.15g}$$

$$\int \frac{\partial \mu}{\partial x} dz = \frac{A_j}{s_j} P_{2j} \tan^{-1} \frac{x}{\bar{z}_{jj}} - \sum_{k=1}^{4} \frac{A_{jk}}{s_j} P_{2j} \tan^{-1} \frac{x}{z_{jk}}. \tag{3.15h}$$

It is noticed that the integrals (3.15f, h) are not continuous at $z = h$; hence, the following expression should be used:

$$\begin{aligned} \int_{a_1}^{a_2} \frac{\partial T}{\partial x} dz &= \int_{a_1}^{h^-} \frac{\partial T}{\partial x} dz + \int_{h^+}^{a_2} \frac{\partial T}{\partial x} dz, \\ \int_{a_1}^{a_2} \frac{\partial \mu}{\partial x} dz &= \int_{a_1}^{h^-} \frac{\partial \mu}{\partial x} dz + \int_{h^+}^{a_2} \frac{\partial \mu}{\partial x} dz. \end{aligned} \tag{3.16}$$

Substituting Eq. (3.4) into Eqs. (3.14a, b) and using the integrals (3.15a, b), yields

$$\sum_{j=1}^{5} w_{mj} A_j I_1 + \sum_{j=1}^{5} w_{mj} \sum_{k=1}^{5} A_{jk} I_2 = 0 \qquad (m = 1, 2), \tag{3.17}$$

where

$$
(3.18)\qquad
\begin{aligned}
I_1 = {} & \left[\left(x(\log \bar{r}_{jj} - 1) + \bar{z}_{jj}\tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right)\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b} \\
& - \left[\left(x\log \bar{r}_{jj} + \bar{z}_{jj}\tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right)\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=a_2}, \\
I_2 = {} & \left[\left(x(\log r_{jk} - 1) + z_{jk}\tan^{-1}\left(\frac{x}{z_{jk}}\right)\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b} \\
& - \left[\left(x\log r_{jk} + z_{jk}\tan^{-1}\left(\frac{x}{z_{jk}}\right)\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=a_2}.
\end{aligned}
$$

To simplify, we obtain $I_1 = 0$ and $I_2 = 0$, i.e., Eqs. (3.17) and (3.14a, b) are satisfied automatically.

Using Eq. (3.4d) in Eq. (3.14c), and using the integrals (3.15e, f) with the aid of (3.16a) and $s_4 = \sqrt{\lambda_1/\lambda_3}$ in the resulting equation, we obtain

$$
(3.19)\qquad A_4 I_3 + \sum_{k=1}^{4} A_{4k} I_4 = \frac{H}{P_{34}\sqrt{\lambda_3/\lambda_1}},
$$

where

$$
(3.20)\qquad
\begin{aligned}
I_3 = {} & -\left[\left(\tan^{-1}\left(\frac{x}{\bar{z}_{44}}\right)\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b} - \left[\left(\tan^{-1}\left(\frac{x}{\bar{z}_{44}}\right)\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=h^-} \\
& + \left[\left(\tan^{-1}\left(\frac{x}{\bar{z}_{44}}\right)\right)_{x=-b}^{x=b}\right]_{z=h^+}^{z=a_2}, \\
I_2 = {} & \left[\left(\tan^{-1}\left(\frac{x}{z_{4k}}\right)\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=a_2} - \left[\left(\tan^{-1}\left(\frac{x}{z_{4k}}\right)\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b}.
\end{aligned}
$$

When solving Eqs. (3.20, we obtain $I_3 = -2\pi$ and $I_4 = 0$.

Thus, A_4 can be determined from Eqs. (3.19) and (3.20), as follows:

$$
(3.21)\qquad A_4 = -\frac{H}{2\pi P_{34}\sqrt{\lambda_3/\lambda_1}}.
$$

Substituting the value of μ from Eq. (3.4e) into Eq. (3.14d), and using the integrals (3.15g, h) with the aid of (3.16d) and $s_5 = \sqrt{D_1/D_3}$ in the resulting equation, we obtain

$$A_5 I_3 + \sum_{k=1}^{5} A_{5k} I_4 = \frac{P}{P_{45}\sqrt{D_3/D_1}}, \tag{3.22}$$

where

$$\begin{aligned} I_5 = &-\left[\left(\tan^{-1}\left(\frac{x}{\bar{z}_{55}}\right)\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b} - \left[\left(\tan^{-1}\left(\frac{x}{\bar{z}_{55}}\right)\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=h^-} \\ &+ \left[\left(\tan^{-1}\left(\frac{x}{\bar{z}_{55}}\right)\right)_{x=-b}^{x=b}\right]_{z=h^+}^{z=a_2} = -2\pi, \\ I_2 = &\left[\left(\tan^{-1}\left(\frac{x}{z_{5k}}\right)\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=a_2} - \left[\left(\tan^{-1}\left(\frac{x}{z_{5k}}\right)\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b} = 0. \end{aligned} \tag{3.23}$$

Thus, A_5 can be determined from Eqs. (3.22) and (3.23), as follows:

$$A_5 = -\frac{P}{2\pi P_{45}\sqrt{D_3/D_1}}. \tag{3.24}$$

At the surface $z = 0$, Eq. (2.16) reduces to

$$\begin{gathered} z_j = 0, \qquad h_k = s_k h, \qquad z_{jk} = h_k, \\ r_{jk} = \sqrt{x^2 + h_k^2}, \qquad \bar{z}_{jk} = -h_k, \qquad \bar{r}_{jk} = \sqrt{x^2 + h_k^2}. \end{gathered} \tag{3.25}$$

Substituting Eq. (3.4) into boundary conditions (3.3) and with the aid of $s_4 = \sqrt{\lambda_1/\lambda_3}$, $s_5 = \sqrt{D_1/D_3}$ and Eq. (3.25), we obtain

$$-s_j w_{1j} A_j + \sum_{k=1}^{5} s_k w_{1k} A_{kj} = 0, \tag{3.26}$$

$$w_{mj} A_j + \sum_{k=1}^{5} w_{mk} A_{kj} = 0, \qquad j = 1, 2, 3, 4, 5, \tag{3.27}$$

$$A_4 - A_{44} = 0, \qquad A_{4k} = 0, \qquad m = 1, 2, \tag{3.28}$$

$$A_5 - A_{55} = 0, \qquad A_{5k} = 0, \qquad k = 1, 2, 3, 4. \tag{3.29}$$

Thus, 30 constants A_j and A_{jk} $(j, k = 1, 2, 3, 4, 5)$ can be determined by 30 equations including Eqs. (3.9), (3.13), (3.21), (3.24) and (3.26)–(3.29).

4. Special case

In the absence of chemical potential per unit mass, Eqs. (3.4) reduce to

$$
\begin{aligned}
\text{(4.1a)} \qquad u = {} & \sum_{j=1}^{4} A_j \left[x(\log \bar{r}_{jj} - 1) + \bar{z}_{jj} \tan^{-1} \frac{x}{\bar{z}_{jj}} \right] \\
& - \sum_{j=1}^{4} \sum_{k=1}^{4} A_{jk} \left[x(\log r_{jk} - 1) + z_{jk} \tan^{-1} \frac{x}{z_{jk}} \right],
\end{aligned}
$$

$$
\begin{aligned}
\text{(4.1b)} \qquad w = {} & \sum_{j=1}^{4} s_j P_{1j} A_j \left[\bar{z}_{jj}(\log \bar{r}_{jj} - 1) - x \tan^{-1} \frac{x}{\bar{z}_{jj}} \right] \\
& + \sum_{j=1}^{4} \sum_{k=1}^{4} s_j P_{1j} A_{jk} \left[z_{jk}(\log r_{jk} - 1) - x \tan^{-1} \frac{x}{z_{jk}} \right],
\end{aligned}
$$

$$
\begin{aligned}
\text{(4.1c)} \qquad \Phi = {} & \sum_{j=1}^{4} s_j P_{1j} A_j \left[\bar{z}_{jj}(\log \bar{r}_{jj} - 1) - x \tan^{-1} \frac{x}{\bar{z}_{jj}} \right] \\
& + \sum_{j=1}^{4} \sum_{k=1}^{4} s_j P_{1j} A_{jk} \left[z_{jk}(\log r_{jk} - 1) - x \tan^{-1} \frac{x}{z_{jk}} \right],
\end{aligned}
$$

$$
\text{(4.1d)} \qquad T = P_{34} A_4 \log \bar{r}_{44} + P_{34} \sum_{k=1}^{4} A_{4k*} r_{4k},
$$

$$
\text{(4.1e)} \qquad \sigma_{xx} = -\sum_{j=1}^{4} s_j^2 w_{1j} A_j \log \bar{r}_{jj} - \sum_{j=1}^{4} \sum_{k=1}^{4} s_j^2 w_{1j} A_{jk} \log r_{jk},
$$

$$
\text{(4.1f)} \qquad \sigma_{zz} = \sum_{j=1}^{4} w_{1j} A_j \log \bar{r}_{jj} + \sum_{j=1}^{4} \sum_{k=1}^{4} w_{1j} A_{jk} \log r_{jk},
$$

$$
\text{(4.1g)} \qquad \sigma_{zx} = -\sum_{j=1}^{4} s_j w_{1j} A_j \tan^{-1} \frac{x}{\bar{z}_{jj}} - \sum_{j=1}^{4} \sum_{k=1}^{4} s_j w_{1j} A_{jk} \tan^{-1} \frac{x}{z_{jk}},
$$

$$
\text{(4.1h)} \qquad D_x = -\sum_{j=1}^{4} s_j w_{2j} A_j \tan^{-1} \frac{x}{\bar{z}_{jj}} - \sum_{j=1}^{4} \sum_{k=1}^{4} s_j w_{2j} A_{jk} \tan^{-1} \frac{x}{z_{jk}},
$$

$$
\text{(4.1i)} \qquad D_z = \sum_{j=1}^{4} w_{2j} A_j \log \bar{r}_{jj} + \sum_{j=1}^{4} \sum_{k=1}^{4} w_{2j} A_{jk} \log r_{jk},
$$

$z_j = s_j z$, $s_4 = \sqrt{\lambda_1/\lambda_3}$, and s_j $(j = 1, 2, 3)$ are three roots (with positive real part) of Eq. (2.23).

Considering the continuity on plane $z = h$ for w, Φ, σ_{zx} and D_x using Eqs. (4.1) with the aid of $s_4 = \sqrt{\lambda_1/\lambda_3}$ and $A_4 = -H/(2\pi P_{34}\sqrt{\lambda_3/\lambda_1})$ gives the following expressions in the absence of diffusion:

$$\sum_{j=1}^{4} s_j P_{mj} A_j = 0, \qquad m = 1, 2, \tag{4.2}$$

$$\sum_{j=1}^{4} s_j A_j = 0, \tag{4.3}$$

$$- s_j w_{1j} A_j + \sum_{k=1}^{4} s_k w_{1k} A_{kj} = 0, \tag{4.4}$$

$$w_{mj} A_j + \sum_{k=1}^{4} w_{mk} A_{kj} = 0, \qquad j = 1, 2, 3, 4, \tag{4.5}$$

$$A_4 - A_{44} = 0, \qquad A_{4k} = 0, \qquad m = 1, 2k = 1, 2, 3. \tag{4.6}$$

Twenty constants A_j and A_{jk} $(j, k = 1, 2, 3, 4)$ can be determined by 20 equations including Eqs. (4.2)–(4.6) by using the method of Cramer's rule.

The above results are similar to the ones obtained by XIONG *et al.* [31].

5. Numerical results and discussion

In order to determine the constants A_j, A_{jk} $(j, k = 1, 2, 3, 4, 5)$, the method of Cramer's rule has been used to solve the system of non-homogeneous equations. We have used the MATLAB 7.04 software to compute the values of A_j, A_{jk} $(j, k = 1, 2, 3, 4, 5)$ for computer program.

The material chosen for numerical calculations is cadmium selenide (CdSe), which is orthotropic material. The physical data for piezothermoelastic as given in Sharma [32] are

$$\mathrm{c}_{11} = 74.1 \times 10^9\ \mathrm{Nm}^{-2}, \quad \mathrm{c}_{12} = 45.2 \times 10^9\ \mathrm{Nm}^{-2}, \quad \mathrm{c}_{13} = 39.3 \times 10^9\ \mathrm{Nm}^{-2},$$
$$\mathrm{c}_{33} = 83.6 \times 10^9\ \mathrm{Nm}^{-2}, \quad \mathrm{c}_{44} = 13.2 \times 10^9\ \mathrm{Nm}^{-2},$$
$$T_0 = 298\ \mathrm{K}, \quad \beta_1 = 6.21 \times 10^5\ \mathrm{C}^2/\mathrm{Nm}^2, \quad \beta_3 = 5.51 \times 10^5\ \mathrm{C}^2/\mathrm{Nm}^2,$$
$$\lambda_1 = 9\ \mathrm{Wm}^{-1}\mathrm{K}^{-1}, \quad \lambda_3 = 7\ \mathrm{Wm}^{-1}\mathrm{K}^{-1},$$
$$e_{13} = -0.160 \times 10^{-3}\ \mathrm{cm}^{-2}, \quad e_{33} = 34 \times 10^{-3}\ \mathrm{cm}^{-2},$$
$$e_{15} = -0.138 \times 10^{-3}\ \mathrm{cm}^{-2}, \quad \varepsilon_{11} = 8.26 \times 10^{-11}\ \mathrm{Nm}^{-2}/\mathrm{K},$$
$$\varepsilon_{33} = 9.03 \times 10^{-11}\ \mathrm{Nm}^{-2}/\mathrm{K}, \quad p_3 = -2.9 \times 10^{-6}\ \mathrm{cm}^{-2}/K,$$
$$D_1 = 0.15\ \mathrm{Cm}^{-2}, \quad D_3 = 0.25\ \mathrm{Cm}^{-2}.$$

Behaviour of components of stress, electric potential, temperature change and chemical potential per unit mass

Figures 2–4 show variation of components of stress (σ_{33}, σ_{31}), electric potential (Φ), temperature change (T) and chemical potential per unit mass (μ) with respect to distance x. The without center symbol lines correspond to piezothermoelastic (PTE) and the centre symbols on these lines correspond to piezothermoelastic diffusion (PTDE).

Figure 2 shows that the values of normal stress (σ_{33}) increase for both cases of PTE and PTDE. It is noticed that for smaller values of x, the values of σ_{33} for the case of PTE remain larger (in comparison to PTDE), but for higher values of x reverse behavior occurs.

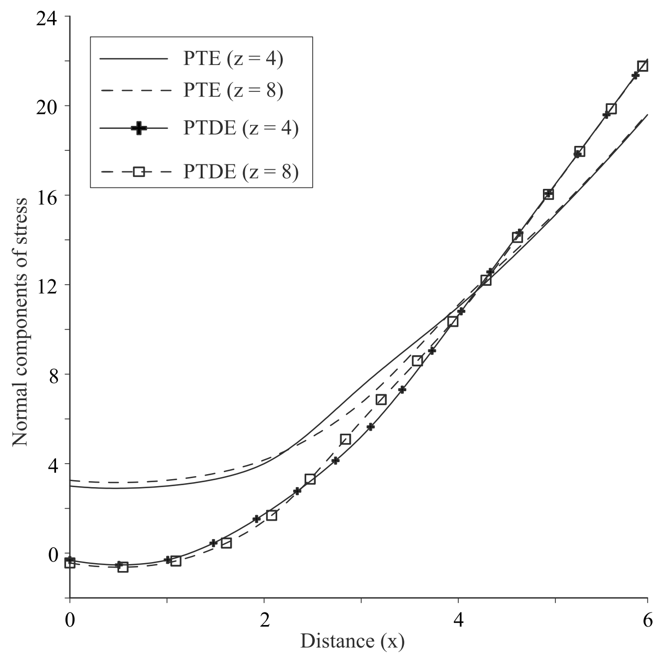

Fig. 2. Variation of normal components of stress (σ_{33}) w.r.t. distance (x).

Figure 3 shows that for smaller values of x, the values of tangential stress (σ_{31}) for the cases of PTE ($z = 5$) and PTDE ($Z = 5$) increase, but for higher values of x, they decrease whereas for the cases PTE ($z = 10$) and PTDE ($z = 10$), the values of σ_{31} decrease initially, but for higher values of x, they increase. It is noticed that the values of σ_{31} in case of PTDE remain more (in comparison with PTE).

Figure 4 shows that for smaller values of x, the values of electric potential (Φ) slightly decrease for both cases of PTE and PTDE, but for higher values

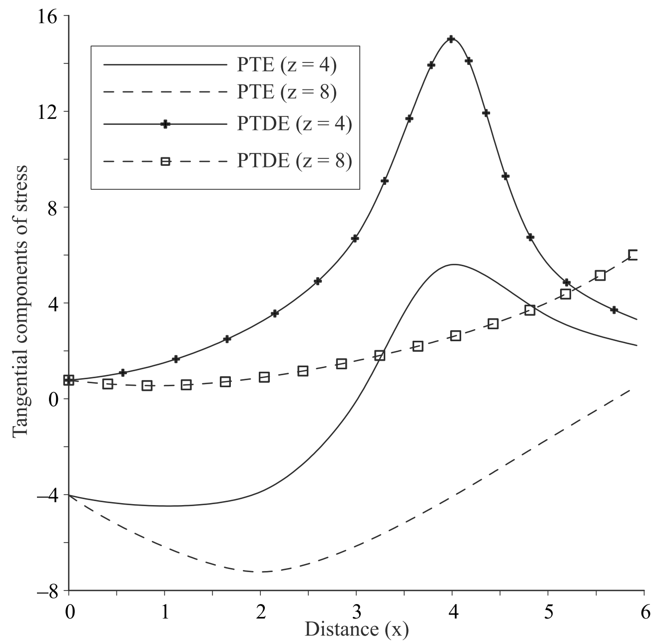

Fig. 3. Variation of tangential components of stress (σ_{31}) w.r.t. distance (x).

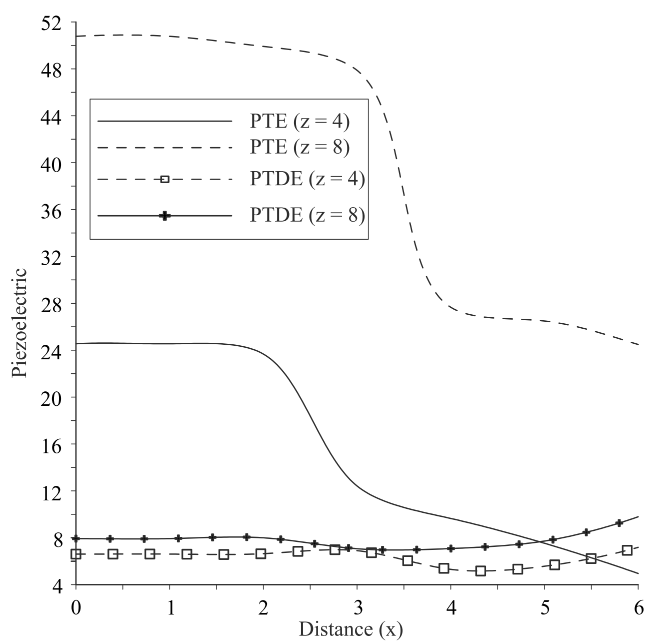

Fig. 4. Variation of electric potential w.r.t. distance (x).

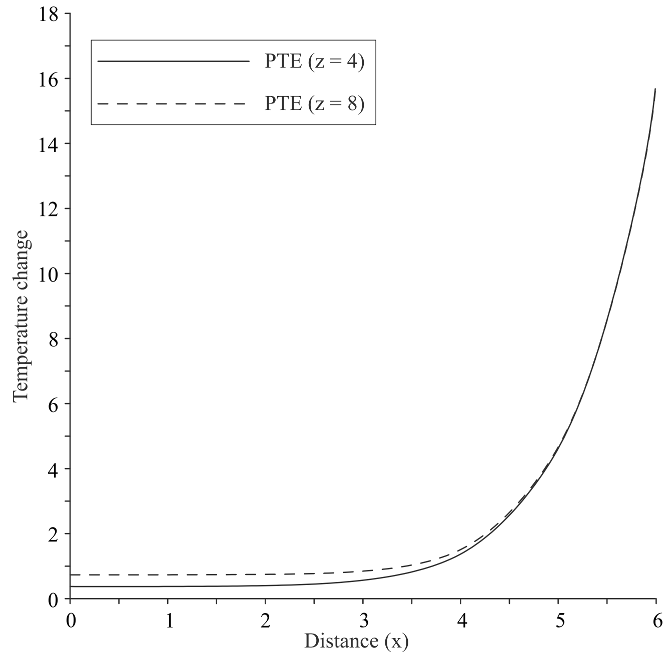

Fig. 5. Variation of temperature change w.r.t. x.

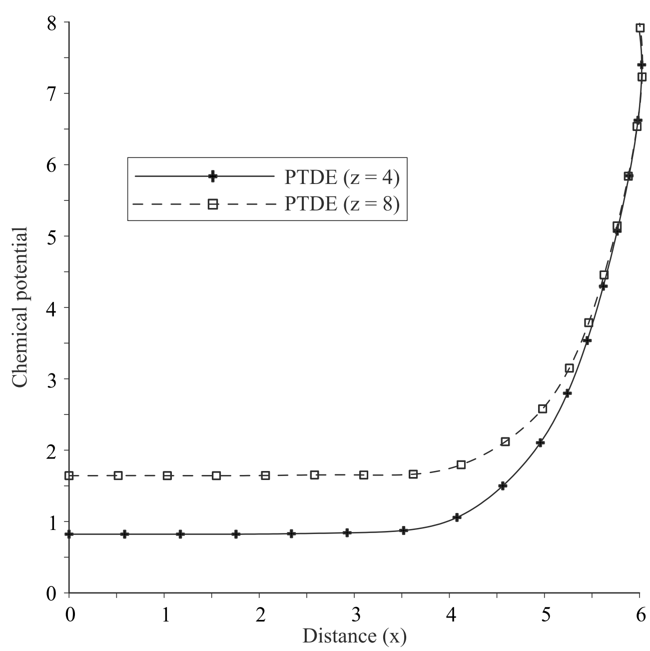

Fig. 6. Variation of chemical potential w.r.t. x.

of x, the values of Φ in case of PTE ($z = 5$, $z = 10$) decrease, whereas for the case of PTDE ($z = 5$, $z = 10$), they slightly increase. It is noticed that the values of Φ in case of PTE ($z = 10$) remain more (in comparison with PTE ($z = 5$), PTDE ($z = 5$, $z = 10$)).

Figure 5 shows that the values of temperature change (T) increase for both cases PTE ($z = 5$, $z = 10$) for comparison, it is noticed that the values of T in case of PTE ($z = 10$) remain more (in comparison with PTE $z = 5$) for smaller values of x, but for higher values of x, the values of T in case of PTE ($z = 5$) remain more, but there is minor difference in both values.

Figure 6 shows that the values of chemical potential (μ) increase for both cases of PTDE ($z = 5$, $z = 10$) and for comparison it is noticed that the values of μ remain more in case of PTDE ($z = 10$) (in comparison with $z = 5$) for smaller values of x, but for higher values of x reverse behavior occurs.

6. Conclusion

Green's functions have become a fundamental mathematical technique for solving boundary value problems. Most treatments however focus on its theory and classical applications in physics rather than the practical means of finding Green's functions for application in science and engineering. Green's function for two-dimensional problem in orthotropic piezothermoelastic diffusion medium has been derived. With this objective, the two-dimensional general solution in orthotropic piezothermodiffusion elastic medium has been derived at first. Based on the obtained two-dimensional general solution, Green' functions for a point heat source and chemical potential source in the interior of semi-infinite orthotropic piezothermodiffusion elastic plane is constructed by five newly introduced harmonic functions. The components of displacement, stress, electric displacement, electric potential, temperature change and chemical potential are expressed in terms of elementary functions. Since all the components are expressed in terms of elementary functions, this makes them convenient to use. The components of displacement, electric potential, temperature change and chemical potential are computed numerically and depicted graphically. From the present investigation, a special case of interest is also deduced to depict the effect of diffusion.

Acknowledgement

Vijay Chawla is thankful to Kurukshetra University, Kurukshetra for financial support in terms of University Research Scholarship.

Appendix A

$$
\begin{aligned}
a &= -\delta_1(\varepsilon_p + \delta_3),\\
b &= \varepsilon_q(\delta_2^2 - \delta_3) - \delta_1(\delta_1\varepsilon_q + \delta_3\bar{\varepsilon}) - 2\delta_1\varepsilon_p e_2 + 2\delta_2\varepsilon_p e_1 - \varepsilon_p(1 + e_1\delta_3\varepsilon_p),\\
c &= e_1\varepsilon_p(e_2\delta_2 - \delta_1 e_1) - \varepsilon_q(\delta_1 + \delta_3\bar{\varepsilon}) - 2\varepsilon_p e_2 - \delta_1(\delta_1\varepsilon_q\bar{\varepsilon} - \varepsilon_p e_2^2)\\
&\quad + \delta_2(\delta_2\varepsilon_q\bar{\varepsilon} + e_1 e_2\varepsilon_p),\\
d &= -(\varepsilon_p e_2^2 + \varepsilon_q\bar{\varepsilon}\delta_1).
\end{aligned}
$$

Appendix B

$$
\begin{aligned}
a_1 &= -q_1(\varepsilon_q\delta_1\bar{\varepsilon} + \varepsilon_p e_2^2),\\
b_1 &= \delta_2(\varepsilon_q q_3\bar{\varepsilon} - \varepsilon_p h_1 e_2) + \varepsilon_p e_1(\delta_1 h_1 + e_2 q_3) - \varepsilon_q q_1(\delta_1 + \delta_3\bar{\varepsilon})\\
&\quad - 2q_1 e_2\varepsilon_p - \bar{a}q_1(\varepsilon_q\delta_1\bar{\varepsilon} + \varepsilon_p e_2^2)\\
&\quad + \delta_2(r_3\varepsilon_q\bar{\varepsilon} - \varepsilon_p g_1 e_2) + \varepsilon_p e_1(g_1\delta_1 + e_2 r_3)\\
&\quad - \varepsilon_q r_1(\delta_1 + \delta_3\bar{\varepsilon}) - 2r_1 r_2\varepsilon_p - \bar{D}\varepsilon_p r_1(\delta_1\bar{\varepsilon} + e_2^2),\\
c_1 &= \delta_2(r_3\varepsilon_q - \varepsilon_p g_1) + e_1\varepsilon_p(g_1\delta_3 + r_3) - r_1(\varepsilon_q\delta_3 - \varepsilon_p)\\
&\quad + \bar{D}[\delta_2\varepsilon_q(r_3\bar{\varepsilon} - g_1 e_2) + \varepsilon_p e_1(\delta_1 g_1 + e_2 r_3) - r_1(\varepsilon_q\delta_3 - \varepsilon_p)\\
&\quad + \delta_2(q_3\varepsilon_q - h_1\varepsilon_p) + 2e_1\varepsilon_p h_1\delta_3 - q_1(\varepsilon_q\delta_3 + \varepsilon_p) + e_1 q_3\varepsilon_p]\\
&\quad + a[\delta_2(q_3\varepsilon_p\bar{\varepsilon} - \varepsilon_p h_1 e_2)\\
&\quad + \varepsilon_p e_1(\delta_1 h_1 + q_3 e_2) - 2q_1\varepsilon_p e_2 - \varepsilon_q(q_1\delta_1 + \delta_3 q\bar{\varepsilon})],\\
d_1 &= \bar{D}[\delta_2(\varepsilon_q r_3 - \varepsilon_p g_1) + \varepsilon_p e_1(g_1\delta_3 + r_3) - r_1(\varepsilon_q\delta_3 + \varepsilon_p)]\\
&\quad + \bar{a}[\delta_2(\varepsilon_q q_3\bar{\varepsilon} - \varepsilon_p h_1 e_2) + \varepsilon_p e_1(\delta_1 h_1 + e_2 q_3) - \varepsilon_q q_1(\delta_1 + \delta_3\bar{\varepsilon}) - 2q_1\varepsilon_p e_2,\\
a_2 &= \varepsilon_p e_2(g_1 + e_1 r_1) + \varepsilon_q\bar{\varepsilon}(r_1\delta_2 - r_3) + \varepsilon_p(h_1 e_2 - q_3\bar{\varepsilon}) + q_1(\varepsilon_q\delta_2\bar{\varepsilon} + e_1 e_2\varepsilon_p),\\
b_2 &= \bar{D}[\varepsilon_p e_2(g_1 + e_1 r_1) + \varepsilon_q\bar{\varepsilon}(r_1\delta_2 - r_3)] + \varepsilon_p g_1(1 + \delta_1 e_2)\\
&\quad - \varepsilon_q r_3(1 + \delta_1\bar{\varepsilon}) - e_1(g_1\delta_2\varepsilon_p - e_1 r_3) + r_1(\varepsilon_q\delta_2 + e_1\varepsilon_p)\\
&\quad + \bar{a}[\varepsilon_p(h_1 e_2 - q_3\bar{\varepsilon}) + q_1(\varepsilon_q\delta_2 + e_1 e_2\varepsilon_p)] + \varepsilon_p h_1(1 + \delta_1 e_2)\\
&\quad - q_3(\varepsilon_p - \varepsilon_q\delta_1\bar{\varepsilon}) - e_1\varepsilon_p(h_1\delta_2 - e_1 q_3) + q_1(\varepsilon_q\delta_2 + e_1\varepsilon_p)\\
c_2 &= \bar{D}[\varepsilon_p g_1(1 + \delta_1 e_2) - \varepsilon_q r_3(1 + \delta_1\bar{\varepsilon})] + \delta_2(\varepsilon_q r_1 - g_1 e_1\varepsilon_p)\\
&\quad + e_1(\varepsilon_p r_1 - e_1 r_3)] + \delta_1(\varepsilon_p g_1 - \varepsilon_q r_3) + \bar{a}[\varepsilon_p h_1(1 + \delta_1 e_2) - q_3(\varepsilon_p + \varepsilon_q\delta_1\bar{\varepsilon}),\\
&\quad - e_1\varepsilon_p(\delta_2 h_1 + e_1 q_3) + q_1(\varepsilon_p e_1^2 + \varepsilon_q\delta_2)] + \varepsilon_p\delta_1(h_1 - q_3),\\
d_2 &= \bar{D}\delta_1(\varepsilon_p g_1 - \varepsilon_q r_3) + \bar{a}\delta_1(\varepsilon_p h_1 - \varepsilon_q q_3),
\end{aligned}
$$

$$a_3 = e_2(r_1\delta_2 - r_3) - \delta_1(g_1 + e_1r_1) + e_2(q_1\delta_2 - q_3) - \delta_1(h_1 + q_1e_1),$$

$$\begin{aligned} b_3 = \bar{D}[&e_2(r_1\delta_2 - r_3) - \delta_1(g_1 + e_1r_1)]\delta_2(\delta_2g_1 + r_3e_1) + r_1(\delta_2 - \delta_3e_1) \\ &+ g_1(\delta_3 + \delta_1^2) + r_3(1 + \delta_1e_2) + a[e_2(\delta_2q_1 - q_3) - \delta_1(h_1 + q_1e_1)] \\ &+ h_1(\delta_2^2 - \delta_3) + q_3(\delta_2e_1 - 1) - \delta_1(\delta_1h_1 + q_3e_2) + q_1(\delta_2 - \delta_3e_1), \end{aligned}$$

$$\begin{aligned} c_3 = \bar{D}[&\delta_2(\delta_2g_1 + r_3e_1) + r_1(\delta_2 - \delta_3e_1) - g_1(\delta_3 + \delta_1^2) - r_3(1 + \delta_1e_2)] \\ &- \delta_1(r_3 + \delta_3g_1) + a[h_1(\delta_2^2 - \delta_3) - q_3(1 + \delta_1e_2) - \delta_1^2h_1 + q_3e_1\delta_2 \\ &+ q_1(\delta_2 - \delta_3e_1)] - \delta_1(\delta_3h_1 + q_3), \end{aligned}$$

$$d_3 = -\bar{D}\delta_1(r_3 + \delta_3g_1) - \bar{a}\delta_1(\delta_3h_1 + q_3),$$

$$a_4 = d, \quad b_4 = c + \bar{a}D, \quad c_4 = b + c\bar{D}, \quad d_4 = a + b\bar{D}, \quad l_5 = a\bar{D},$$

$$a_5 = d, \quad b_5 = c + \bar{a}d, \quad c_5 = b + \bar{a}c, \quad d_5 = a + \bar{a}b, \quad l_6 = a\bar{a}.$$

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Received April 14, 2012; revised version September 29, 2012.