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Electro-elastic coupled fields of the general line source in infinite multilayered piezoelectric medium

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Electro-elastic coupled fields excited by a general line source in a multilayered anisotropic piezoelectric medium are expressed in an analytical form. The general line source represents a combination of a straight 4D dislocation with the force and charge, distributed along the same line. The results are obtained in the form of well-convergent Fourier integrals. They can be considered as Green's functions describing electro-elastic fields created in the given medium by arbitrary 2D bulk distributions of dislocations, forces, charges and electro-potential. The analysis is accomplished in terms of the propagator matrix which is equally applicable to both the stratified and graded layered media.

Key words: piezoelectricity, anizotropy, multilayers, line source.

1. Introduction

The modern materials are often designed for a definite function in particular devices with optimized efficiency. Requirements of nano-technology call now for media not only anisotropic but also inhomogeneous. The artificial materials produced by methods of molecular epitaxy are often composed of different atomic layers. The moduli of such multilayered media are continuous functions of one coordinate. This can provide new important physical properties of solids, but their description becomes very non-trivial and requires new methods and approaches. In this paper we shall meet the problem just of this kind, considering electro-elastic coupled fields excited in an arbitrary layered piezoelectric medium of unrestricted anisotropy by a general line source formed by a 4D dislocation and/or line distributions of load and charge.

In fact, the description of the dislocation field in an arbitrary anisotropic medium, even when it is homogeneous and purely elastic, is not an easy problem. The progress here was attained not so long ago (see for a review [1, 2]). The first results for homogeneous unbounded piezoelectrics were obtained in [3–5]. They were related to a description of 2D electro-elastic fields of straight dislocations. Much later these results were extended for bounded and piece-wise homogeneous piezoelectric media in the series [6–11]. The new approach to a more general theory of arbitrary curved dislocations in unbounded piezoelectric media was developed in paper [12], where most of fundamental results of dislocation theory for elastic media were adopted for piezoelectrics.

In this paper we return to the 2D electro-elastic fields of a straight dislocation (to be exact, of a general line source including the dislocation) in an infinite anisotropic but also *inhomogeneous* piezoelectric medium which has material moduli arbitrarily dependent on one coordinate. Our theoretical approach will be based on the extension of the propagator formalism developed in [13–15] for purely elastic multilayers. We have partially used this extension in a recent paper [11] devoted to dislocation fields in the 3-layered piezoelectric structure formed by the layer between two substrates. Earlier dislocation fields in piezoelectric [16] and piezoelectric-piezomagnetic [17, 18] inhomogeneous media have been also obtained for the other type of 1D inhomogeneity related to a dependence of material moduli on the polar angle.

2. Statement of the problem

Consider an infinite piezoelectric medium of unrestricted anisotropy which is inhomogeneous along one direction specified by the unit vector $\mathbf{n}$. We choose the y axis along $\mathbf{n}$, so that the material moduli of the medium are functions of this coordinate. Let us denote $c_{ijkl}(y)$ the elastic moduli tensor, $e_{ijk}(y)$ for the piezoelectric moduli and $\varepsilon_{ij}(y)$ for the permittivity tensor (Fig. 1). Thus, in the Cartesian plane xz all these moduli are constant.

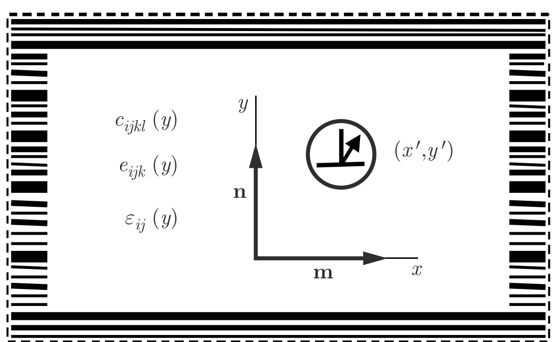

Fig. 1. The multilayered piezoelectric medium, its material tensors and the general line source in the Cartesian coordinate system.

Let us choose the general line source parallel to the z axes and passing through the point (x', y'). The source consists of the dislocation with Burgers vector $\mathbf{b}$, its electrostatic analog characterized by the potential jump $\Delta\varphi$, the line force $\mathbf{f}$ and the electric charge q(both per unit length of the source). The electrostatic dislocation is defined [5, 12] by the jump of the potential φ at an arbitrary surface spanned by the dislocation line. In our case we shall choose for such a surface the semi-plane parallel to xz: $y = y'$, $x > x'$. Following [12], it is convenient to consider a combined 4D dislocation with the Burgers 4-vector

$$\mathbf{B} = \begin{pmatrix} \mathbf{b} \\ \Delta\varphi \end{pmatrix}. \tag{2.1}$$

In Kroener's manner [19] such a dislocation can be completely defined by the "plastic distortion"

$$U^0_{lK} = -n_l B_K H(x - x')\delta(y - y'), \tag{2.2}$$

where n_l is the component of the unit vector $\mathbf{n} \parallel y$ orthogonal to the layers, $H(x)$ is the Heaviside step function and $\delta(y)$ is the Dirac delta function.

In the same 4D space the fields of the mechanical displacements $\mathbf{u}$ and the electric potential φ, altogether form the generalized "displacement" 4-vector

$$\mathbf{U} = \begin{pmatrix} \mathbf{u} \\ \varphi \end{pmatrix}. \tag{2.3}$$

Similarly, the force $\mathbf{f}$ and the charge q densities form the generalized "force" 4-vector

$$\mathbf{F} = \begin{pmatrix} \mathbf{f} \\ -q \end{pmatrix}. \tag{2.4}$$

Combinations of the mechanical stress tensor $\boldsymbol{\sigma}$ with the electric displacement vector $\mathbf{D}$ and the elastic distortion tensor $\boldsymbol{\beta}$ with the electric field $\mathbf{E}$ give, respectively, generalized "elastic" stresses ($\boldsymbol{\Sigma}$) and "elastic" distortions ($\mathbf{U}^{el}$):

$$\mathit{\Sigma}_{iJ} = \begin{cases} \sigma_{ij}, & J = j = 1,2,3; \\ D_i, & J = 4. \end{cases} \qquad U^{el}_{lK} = \begin{cases} \beta_{lk}, & K = k = 1,2,3; \\ -E_l, & K = 4; \end{cases} \tag{2.5}$$

which are represented as non-square 3×4 matrices ($i, l = 1, 2, 3$; $J, K = 1, \ldots, 4$).

The relation between the above matrices is given by the generalized Hooke's law

$$\mathit{\Sigma}_{iJ} = C_{iJKl} U^{el}_{lK}, \tag{2.6}$$

where by definition

$$C_{iJKl}(y) = \begin{cases} c_{ijkl}, & J, K = j, k = 1, 2, 3, \\ e_{lij}, & J = j = 1, 2, 3,\ K = 4, \\ e_{ikl}, & J = 4,\ K = k = 1, 2, 3, \\ -\varepsilon_{il}, & J = 4,\ K = 4, \end{cases} \tag{2.7}$$

is the extended moduli tensor, being in our case a function of y. It is easily checked that the combination of Eqs. (2.5)–(2.7) gives the standard constitutive relations

$$\sigma_{ij} = c_{ijlk}\beta_{kl} - e_{lij}E_l, \qquad D_j = e_{jlk}\beta_{kl} + \varepsilon_{jl}E_l. \tag{2.8}$$

And vice versa, the combination of the equilibrium and Maxwell's equations,

$$\operatorname{div}\boldsymbol{\sigma} = -\mathbf{f}\delta(x-x')\delta(y-y'), \qquad \operatorname{div}\mathbf{D} = q\delta(x-x')\delta(y-y'), \tag{2.9}$$

gives the extended equilibrium equation

$$\operatorname{div}\boldsymbol{\Sigma} = -\mathbf{F}\delta(x-x')\delta(y-y'). \tag{2.10}$$

For further applications it is convenient to rewrite Eq. (2.10) in the form

$$\operatorname{div}(\boldsymbol{\Sigma} + \boldsymbol{\Sigma}^0) = 0, \tag{2.11}$$

where the tensor $\boldsymbol{\Sigma}^0$ is defined by a dyad with the components

$$\Sigma^0_{iJ} = (m_i F_J)H(x-x')\delta(y-y'), \qquad i = 1,2,3, \quad J = 1,\ldots,4, \tag{2.12}$$

and m_i is the component of the unit vector $\mathbf{m} \parallel x$ (Fig. 1).

Here we must recall that in presence of the dislocation (2.2), being a part of the general line source, the elastic distortion U^{el}_{lK} is not equal to a gradient of the displacement vector U_K. One should now distinguish between elastic (U^{el}_{lK}), plastic (U^0_{lK}) and total ($U^t_{lK} = U_{K,l}$) distortions, which are related to each other through the Kroener-like [19] equation

$$U^t_{lK} \equiv U_{K,l} = U^{el}_{lK} + U^0_{lK}. \tag{2.13}$$

Substituting (2.6) into (2.10) with bearing in mind (2.13), we come to the differential equation of the 2nd order with respect to the 4-vector $\mathbf{U}$ defined by (2.3).

3. The Stroh–Barnett–Lothe alternative approach: 8D formalism

However, as it was demonstrated in [13–18], there is a much more promising way of solving a plane elastic problem in a one-dimensionally inhomogeneous medium. Instead of considering the system of 4 differential equations of the 2nd order, it is more convenient to transform the problem to 8 differential equations of the 1st order. This way was first indicated by Stroh [20] for a pure elastic medium, and then its extension for piezoelectrics was given by Barnett and Lothe [5]. Here we shall follow our approach in [11]. Let us add to the displacement field $\mathbf{U}(x,y)$ the additional unknown 4-vector field $\mathbf{V}(x,y)$ defined by:

$$\partial_x \mathbf{V} = -n\mathbf{\Sigma}, \qquad \partial_y \mathbf{V} = (\mathbf{\Sigma} + \mathbf{\Sigma}^0). \tag{3.1}$$

With these substitutions the equilibrium equation (2.11) is satisfied automatically and the "Hooke's" law (2.6) can be rearranged into the following 8D relation:

$$\left(\mathbf{I}\frac{\partial}{\partial y} - \mathbf{N}(y)\frac{\partial}{\partial x}\right)\boldsymbol{\eta}(x,y) = -\mathbf{g}H(x-x')\delta(y-y'), \tag{3.2}$$

which is just the required differential equation with respect to the new unknown 8-vector

$$\boldsymbol{\eta}(x,y) = \begin{pmatrix}\mathbf{U}(x,y)\\ \mathbf{V}(x,y)\end{pmatrix}. \tag{3.3}$$

In (3.2) $\mathbf{g}$ is the strength 8-vector of the general line source,

$$\mathbf{g} = \begin{pmatrix}\mathbf{B}\\ -\mathbf{F}\end{pmatrix}, \tag{3.4}$$

$\mathbf{I}$ is the identity 8×8 matrix (below we shall use the same notation for a 4×4 identity matrix) and $\mathbf{N}$ is the 8×8 matrix

$$\mathbf{N}(y) = -\begin{pmatrix}(nn)^{-1}(nm) & (nn)^{-1}\\ (mn)(nn)^{-1}(nm)-(mm) & (mn)(nn)^{-1}\end{pmatrix}, \tag{3.5}$$

where the 4×4 matrices (nn), (mn), (nm) and (mm) are defined by convolutions of the type $(ab)_{JK} = a_i C_{iJKl} b_l$ formed by the extended moduli tensor C_{iJKl} (2.7) with the unit vectors $\mathbf{m}$ and $\mathbf{n}$ (Fig. 1).

After the Fourier transformation in Eq. (3.2) with respect to x (i.e. along the layers)

$$\boldsymbol{\eta}(x,y) = \int_{-\infty}^{\infty} dk\, \exp(ikx)\boldsymbol{\eta}(k,y), \quad H(x-x') = \frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{dk}{k}\exp[ik(x-x')], \tag{3.6}$$

one obtains the ordinary differential equation

$$\left(\mathbf{I}\frac{\partial}{\partial y} - ik\mathbf{N}(y)\right)\boldsymbol{\eta}(k,y) = -\mathbf{g}\frac{\exp(-ikx')}{2\pi ik}\delta(y-y'). \tag{3.7}$$

We note that for a convergence of the Fourier expansion $(3.6)_2$ of the Heaviside function, we have given to k an infinitesimally small imaginary part:

$$k = \operatorname{Re} k - i\varepsilon, \qquad \varepsilon \to +0, \tag{3.8}$$

which will be implied in all further calculations.

4. The propagator formalism

We start the analysis of Eq. (3.7) from its homogeneous form

$$\frac{\partial}{\partial y}\boldsymbol{\eta}(k,y) = ik\mathbf{N}(y)\boldsymbol{\eta}(k,y). \tag{4.1}$$

This type of equations is very well known both in the linear algebra [21, 22] and in physics [23]. For instance, the time-dependent Schroedinger equation in quantum mechanics [23] belongs just to this particular type. Such equations are usually solved in terms of a propagator (or transfer) matrix. It is evident that Eq. (4.1) is satisfied by the function

$$\boldsymbol{\eta}(k,y) = \mathbf{W}^{(k)}(y\,|\,y^0)\mathbf{X}^{(k)}, \tag{4.2}$$

where $\mathbf{X}^{(k)}$ is an arbitrary 8-vector and $\mathbf{W}^{(k)}(y\,|\,y^0)$ is the propagator 8×8 matrix defined by

$$\mathbf{W}^{(k)}(y\,|\,y^0) = \mathrm{Ord}\exp\left(ik\int\limits_{y^0}^{y} dt\mathbf{N}(t)\right). \tag{4.3}$$

This matrix contains the ordering (Ord) operator which arranges the integrands of multiple integrals arising after expansion of the exponent in (4.3), so that the products of non-commuting matrices $\mathbf{N}(t_m)\mathbf{N}(t_{m-1})\ldots\mathbf{N}(t_1)$ would contain arguments putting in definite order: $t_m > t_{m-1} > \cdots > t_1$ for $y > y^0$ or, vise versa, $t_m < t_{m-1} < \cdots < t_1$ for $y < y^0$. Below, for a reference point y^0 we shall choose $y^0 = y'$.

Let us introduce the eigenvectors ($\zeta_{k\alpha}$) and eigenvalues ($\tau_{k\alpha}$) of the propagator matrix (4.3):

$$\mathbf{W}^{(k)}(y\,|\,y')\zeta_{k\alpha} = \tau_{k\alpha}\zeta_{k\alpha}, \qquad \alpha = 1,\ldots,8. \tag{4.4}$$

Basing on the arguments similar to those in [15] for purely elastic multilayers, one can show that

$$\tau_{k\alpha} = \exp[ikp_{k\alpha}(y-y')], \qquad p_{-k\alpha} = p_{k\alpha}, \tag{4.5}$$

where the set of parameters $p_{k\alpha}$ forms four pairs of complex conjugates and may be numbered as

$$[p_{k\alpha}]^* = p_{k\alpha+4}, \qquad \operatorname{Im} p_{k\alpha} > 0, \qquad \operatorname{Im} p_{k\alpha+4} < 0, \qquad \alpha = 1,\ldots,4. \tag{4.6}$$

The eigenvectors $\zeta_{k\alpha}$ also have the property

$$[\zeta_{-k\alpha}]^* = \zeta_{k\alpha+4}, \qquad \alpha = 1,\ldots,4. \tag{4.7}$$

In addition, apart from some exclusive cases of degeneracy which are beyond this consideration, these eigenvectors are orthogonal and complete:

$$\zeta_{k\alpha} \cdot \mathbf{T}\zeta_{-k\beta} = \delta_{\alpha\beta}, \quad \sum_{\alpha=1}^{8} \zeta_{k\alpha} \otimes \mathbf{T}\zeta_{-k\alpha} = \mathbf{I}, \quad \mathbf{T} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{pmatrix}, \tag{4.8}$$

where $\otimes$ is the dyadic product and 4×4 blocks $\mathbf{0}$ and $\mathbf{I}$ are zero and unit matrices in R^4.

A spectral decomposition of the propagator matrix has the form:

$$\mathbf{W}^{(k)}(y\,|\,y') = \sum_{\alpha=1}^{8} \zeta_{k\alpha} \otimes \mathbf{T}\zeta_{-k\alpha} \exp[ikp_{k\alpha}(y-y')], \tag{4.9}$$

which can be further conveniently decomposed into two submatrices

$$\mathbf{W}^{(k)}(y\,|\,y') = \mathbf{w}^{(k)}(y\,|\,y') + [\mathbf{w}^{(-k)}(y\,|\,y')]^*, \tag{4.10}$$

where in view of (4.9) and $(4.5)_2$–(4.7) one has

$$\mathbf{w}^{(k)}(y\,|\,y') = \sum_{\alpha=1}^{4} \zeta_{k\alpha} \otimes \mathbf{T}\zeta_{-k\alpha} \exp[ikp_{k\alpha}(y-y')]. \tag{4.11}$$

By their definition, both the submatrices in the right-hand side of (4.10) manifest quite different behavior at $\pm\infty$ limits of the product $k(y-y')$:

$$\lim_{k(y-y')\to\infty} \mathbf{w}^{(k)}(y\,|\,y') = 0, \qquad \lim_{k(y-y')\to\infty} [\mathbf{w}^{(-k)}(y\,|\,y')]^* = \infty, \tag{4.12}$$

$$\lim_{k(y-y')\to-\infty} \mathbf{w}^{(k)}(y\,|\,y') = \infty, \qquad \lim_{k(y-y')\to-\infty} [\mathbf{w}^{(-k)}(y\,|\,y')]^* = 0. \tag{4.13}$$

Thus, a general solution of homogeneous Eq. (4.1) can be expressed in the form

$$\boldsymbol{\eta}_{\text{gen}}(k,y) = \mathbf{W}^{(k)}(y\,|\,y')\mathbf{C}^{(k)}, \tag{4.14}$$

where $\mathbf{C}^{(k)}$ is the unknown 8-vector which will be found below from the requirement of convergence of the Fourier integral and natural conditions at infinities $y \to \pm\infty$.

5. Finding the electro-elastic fields of the general line source

Now it is time to return to initial inhomogeneous equation (3.7). Its complete solution is formed by a sum of general solution (4.14) and any partial solution of Eq. (3.7). For the latter, one can choose the function

$$\boldsymbol{\eta}_{\text{part}}(k,y) = -\frac{\exp(-ikx')}{2\pi ik} H(y-y')\mathbf{W}^{(k)}(y\,|\,y')\mathbf{g}, \tag{5.1}$$

where we have taken into account the identity $\mathbf{W}^{(k)}(y' \,|\, y') = \mathbf{I}$. Combining (4.14) and (5.1) it is convenient to renormalize the 8-vector $\mathbf{C}^{(k)}$ in accordance with the relation

$$\mathbf{C}^{(k)} = \mathbf{c}^{(k)} \frac{\exp(-ikx')}{2\pi ik}. \tag{5.2}$$

The result is

$$\boldsymbol{\eta}(k, y) = \frac{\exp(-ikx')}{2\pi ik} \mathbf{W}^{(k)}(y \,|\, y')[\mathbf{c}^{(k)} - \mathbf{g}H(y - y')]. \tag{5.3}$$

In order to avoid divergences of the Fourier integral $(3.6)_1$ due to an exponential growth of the functions $\mathbf{w}^{(k)}$ and $[\mathbf{w}^{(-k)}]^*$ at $k \to \pm\infty$, Eqs. $(4.12)_2$ and $(4.13)_1$, one has to choose the appropriate 8-vector $\mathbf{c}^{(k)}$ providing finite magnitudes of the integrand at both infinities $y \to \pm\infty$.

We shall suppose that the considered inhomogeneous medium is characterized by definite limits of the eigenvectors and eigenvalues of the propagator matrix at $y \to \pm\infty$:

$$\lim_{y \to \pm\infty} \zeta_{k\alpha}(y \,|\, y') = \zeta^{\pm}_{k\alpha}, \qquad \lim_{y \to \pm\infty} p_{k\alpha}(y \,|\, y') = p^{\pm}_{k\alpha}, \qquad \alpha = 1, \ldots, 8. \tag{5.4}$$

Let us now decompose the unknown 8-vector $\mathbf{c}^{(k)}$ into its projections along the eigenvectors of the complete set $\{\zeta^{-}_{k\alpha}\}$:

$$\mathbf{c}^{(k)} = \sum_{\alpha=1}^{8} c_{k\alpha} \zeta^{-}_{k\alpha}. \tag{5.5}$$

In these terms, bearing in mind the radical difference in limiting properties of the two submatrices of the decomposition (4.10), the limit of function (5.3) at $y \to \infty$ acquires the form

$$\begin{aligned} &\boldsymbol{\eta}(k, y \to -\infty) \\ &\quad = \frac{\exp(-ikx')}{2\pi ik} \left\{ \sum_{\alpha=1}^{4} c_{k\alpha} \zeta^{-}_{k\alpha} \exp(ikp^{-}_{k\alpha} y) + \sum_{\alpha=5}^{8} c_{k\alpha} \zeta^{-}_{k\alpha} \exp(ikp^{-}_{k\alpha} y) \right\}. \end{aligned} \tag{5.6}$$

By Eq. (5.6), with accepted rule (4.6) of numeration of eigenvalues, we come to the following requirements with respect to the unknown coefficients $c_{k\alpha}$ in (5.5):

$$c_{k\alpha} = \begin{cases} 0, & k > 0, \ \alpha = 1, \ldots, 4 \\ 0, & k < 0, \ \alpha = 5, \ldots, 8. \end{cases} \tag{5.7}$$

In order to find the nonvanishing components $c_{k\alpha}$, consider the opposite limit of the function (5.3):

$$\boldsymbol{\eta}(k, y \to \infty) = \frac{\exp(-ikx')}{2\pi ik}\left\{\sum_{\alpha=1}^{4} c'_{k\alpha}\zeta^{+}_{k\alpha}\exp(ikp^{+}_{k\alpha}y) + \sum_{\alpha=5}^{8} c'_{k\alpha}\zeta^{+}_{k\alpha}\exp(ikp^{+}_{k\alpha}y)\right\}, \tag{5.8}$$

where the new notation is introduced

$$c'_{k\alpha} = \sum_{\gamma=1}^{8} M^{(k)}_{\alpha\gamma}(c_{k\gamma} - g_{k\gamma}), \qquad M^{(k)}_{\alpha\gamma} = \zeta^{+}_{-k\alpha}\cdot \mathbf{T}\zeta^{-}_{k\gamma}, \qquad g_{k\gamma} = \zeta^{-}_{-k\gamma}\cdot \mathbf{T}\mathbf{g}. \tag{5.9}$$

Due to the same rule (4.6), the finiteness of (5.8) is guaranteed if

$$c'_{k\alpha} = \begin{cases} 0, & k > 0,\ \alpha = 1, \dots, 4 \\ 0, & k < 0,\ \alpha = 5, \dots, 8. \end{cases} \tag{5.10}$$

Combining (5.9) with (5.10) and keeping in mind (5.7), one obtains the two systems of equations with respect to the nonvanishing components $c_{k\alpha}$:

$$\sum_{\gamma=1}^{4} M^{(k)}_{\alpha\gamma}c_{k\gamma} = \sum_{\gamma=1}^{8} M^{(k)}_{\alpha\gamma}g_{k\gamma}, \qquad k < 0,\ \alpha = 1, \dots, 4, \tag{5.11}$$

$$\sum_{\gamma=5}^{8} M^{(k)}_{\alpha\gamma}c_{k\gamma} = \sum_{\gamma=1}^{8} M^{(k)}_{\alpha\gamma}g_{k\gamma}, \qquad k > 0,\ \alpha = 5, \dots, 8. \tag{5.12}$$

Let us now express in (5.11), (5.12) the sets $\{c_{k\gamma}\}$ and $\{g_{k\gamma}\}$ as 8-component vectors, each in the block form of a pair of 4-vectors:

$$\begin{aligned} \{c_{k1}, \dots, c_{k4}; c_{k5}, \dots, c_{k8}\}^{\mathrm{T}} &= (\mathbf{c}^{(1)}_k; \mathbf{c}^{(2)}_k)^{\mathrm{T}}, \\ \{g_{k1}, \dots, g_{k4}; g_{k5}, \dots, g_{k8}\}^{\mathrm{T}} &= (\mathbf{g}^{(1)}_k; \mathbf{g}^{(2)}_k)^{\mathrm{T}}, \end{aligned} \tag{5.13}$$

where the superscripts T stand for transposition. And decompose 8×8 matrix $\mathbf{M}_k = \{M^{(k)}_{\alpha\gamma}\}$ also in the block form:

$$\mathbf{M}_k = \begin{pmatrix} \mathbf{M}^{11}_k & \mathbf{M}^{12}_k \\ \mathbf{M}^{21}_k & \mathbf{M}^{22}_k \end{pmatrix} \equiv \begin{pmatrix} \{\zeta^{+}_{-k\alpha}\cdot \mathbf{T}\zeta^{-}_{k\gamma}\} & \{\zeta^{+}_{-k\alpha}\cdot \mathbf{T}\zeta^{-*}_{-k\gamma}\} \\ \{\zeta^{+*}_{k\alpha}\cdot \mathbf{T}\zeta^{-}_{k\gamma}\} & \{\zeta^{+*}_{k\alpha}\cdot \mathbf{T}\zeta^{-*}_{-k\gamma}\} \end{pmatrix}, \qquad \alpha, \gamma = 1, \dots, 4, \tag{5.14}$$

where we have made use of the property (4.7). We draw attention to the following useful properties of the introduced 4×4 block matrices:

$$\mathbf{M}_k^{11} = (\mathbf{M}_{-k}^{22})^*, \qquad \mathbf{M}_k^{12} = (\mathbf{M}_{-k}^{21})^*, \tag{5.15}$$

which follow from Eq. (4.7).

In terms of (5.13), (5.14), system (5.11), (5.12) acquires the form

$$\mathbf{M}_k^{11}\mathbf{c}_k^{(1)} = \mathbf{M}_k^{11}\mathbf{g}_k^{(1)} + \mathbf{M}_k^{12}\mathbf{g}_k^{(2)}, \qquad k<0; \tag{5.16}$$

$$\mathbf{M}_k^{22}\mathbf{c}_k^{(2)} = \mathbf{M}_k^{21}\mathbf{g}_k^{(1)} + \mathbf{M}_k^{22}\mathbf{g}_k^{(2)}, \qquad k>0. \tag{5.17}$$

These equations together with (5.7) are solved by

$$\mathbf{c}_k^{(1)} = [\mathbf{g}_k^{(1)} + (\mathbf{M}_k^{11})^{-1}\mathbf{M}_k^{12}\mathbf{g}_k^{(2)}]H(-k), \tag{5.18}$$

$$\mathbf{c}_k^{(2)} = [\mathbf{g}_k^{(2)} + (\mathbf{M}_k^{22})^{-1}\mathbf{M}_k^{21}\mathbf{g}_k^{(1)}]H(k). \tag{5.19}$$

The found solutions are conveniently combined to the initial scalar coefficients

$$c_{k\alpha} = \left(g_{k\alpha} + \sum_{\gamma=1}^{8} m_{\alpha\gamma}^{(k)} g_{k\gamma}\right) H(-k\,\mathrm{Im}\,p_{k\alpha}^-), \qquad \alpha = 1,\ldots,8, \tag{5.20}$$

where the new 8×8 matrix $\mathbf{m}_k = \{m_{\alpha\gamma}^{(k)}\}$ is introduced:

$$\mathbf{m}_k = \begin{pmatrix} \mathbf{0} & (\mathbf{M}_k^{11})^{-1}\mathbf{M}_k^{12} \\ (\mathbf{M}_k^{22})^{-1}\mathbf{M}_k^{21} & \mathbf{0} \end{pmatrix}. \tag{5.21}$$

Substituting (5.20) with $(5.9)_3$ into (5.5), one obtains

$$\mathbf{c}^{(k)} = \sum_{\beta,\gamma=1}^{8} \zeta_{k\beta}^- (\zeta_{-k\gamma}^- \cdot \mathbf{Tg})(\delta_{\beta\gamma} + m_{\beta\gamma}^{(k)}) H(-k\,\mathrm{Im}\,p_{k\beta}^-), \tag{5.22}$$

which together with (5.3), (4.9) and $(3.19)_1$ leads to the final solution

$$\boldsymbol{\eta}(x,y) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{dk}{k} \exp[ik(x-x')]\mathbf{W}^{(k)}(y\,|\,y') \tag{5.23}$$

$$\times \sum_{\beta,\gamma=1}^{8} \zeta_{k\beta}^- (\zeta_{-k\gamma}^- \cdot \mathbf{Tg})\big\{\delta_{\beta\gamma} H[k(y-y')\mathrm{Im}\,p_{k\beta}^-]\,\mathrm{sgn}(y'-y) + m_{\beta\gamma}^{(k)} H(-k\mathrm{Im}\,p_{k\beta}^-)\big\}.$$

Here we have made use of the identity

$$H(a) - H(b) = H(-ab)\,\mathrm{sgn}(a). \tag{5.24}$$

The found solution (5.23) can be transformed to a more compact form. Let us introduce the new 8×8 matrices

$$
\mathbf{u}_k = \sum_{\beta=1}^{4} \zeta^-_{k\beta} \otimes \mathbf{T}\zeta^-_{-k\beta},
\qquad\qquad (5.25)
$$
$$
\mathbf{v}_k = \sum_{\beta,\gamma=1}^{4} \zeta^{-*}_{-k\beta} \otimes [(\mathbf{M}^{22}_k)^{-1}\mathbf{M}^{21}_k]_{\beta\gamma}\mathbf{T}\zeta^-_{k\gamma}.
$$

In these terms, basing on Eqs. (4.7) and (5.15), one can express the second line of (5.23) in the form

$$
(5.26)\qquad \sum_{\beta,\gamma=1}^{8} \zeta^-_{k\beta}(\zeta^-_{-k\gamma}\cdot \mathbf{Tg}) \times \big\{\delta_{\beta\gamma} H[k(y-y')\operatorname{Im} p^-_{k\beta}]\operatorname{sgn}(y'-y) - m^{(k)}_{\beta\gamma} H(-k \operatorname{Im} p^-_{k\beta})\big\}
$$
$$
= \big(\operatorname{sgn}(y'-y)\{\mathbf{u}^H_k[k(y-y')] + \mathbf{u}^*_{-k}H[-k(y-y')]\} + \mathbf{v}^H_k(k) + \mathbf{v}^*_{-k}H(-k)\big)\mathbf{g}.
$$

Accordingly, Eq. (5.23) is transformed to

$$
(5.27)\qquad \boldsymbol{\eta}(x,y) = \frac{1}{\pi}\operatorname{Im}\int_0^\infty \frac{dk}{k}\exp[ik(x-x')]\mathbf{W}^{(k)}(y\,|\,y')(\mathbf{I}H(y'-y) - \mathbf{u}^+_k\mathbf{v}_k)g,
$$

where the identity $\mathbf{u}_k + \mathbf{u}^*_{-k} = \mathbf{I}$ by $(4.8)_2$ was taken into account.

Constructing the above solution, we have chosen the set $\{c_{k\alpha}\}$ in such a way that the integral in (5.23) would be well convergent at its both infinite limits of integration, for the point y situated at any of infinities $y = \pm\infty$. One can check that convergence retains also for any intermediate positions of y. Here we omit a bulky but straightforward analysis, which would be analogous to that in [14] for pure elasticity with a similar conclusion: the integrand in (5.23) is exponentially small at $k \to \pm\infty$ for any y, y' if all summations are accomplished before integration.

But at $k \to 0$ the same integral is logarithmically divergent, even with our implied imaginary addition $-i\varepsilon$ to $\operatorname{Re} k$ in (3.8). This divergence leads to an addition of the term $\sim \ln\varepsilon$ tending to infinity but independent of coordinates. Fortunately, such divergence has no physical consequences because the function $\boldsymbol{\eta}(x,y)$ is sort of a potential defined to any constant, even infinite. Only derivatives of the function $\boldsymbol{\eta}(x,y)$, Eqs. (2.13) and (3.1), determine physically measurable values. Formally, each such derivative provides in the integrands of (5.22) and (5.27) the additional factor k which excludes the divergence at $k \to 0$. This problem is very well known in the theory of dislocations for purely elastic media, both homogeneous [2] and layered [13].

6. Limiting transition to a homogeneous medium

Let us consider the transition from the electro-elastic field (5.27) of the general line source in an arbitrary layered piezoelectric medium to the case of a homogeneous solid. When the matrix $\mathbf{N}$ becomes independent of coordinates, the propagator (4.3) takes the form

$$\mathbf{W}^{(k)}(y \mid y') = \exp[ik\mathbf{N}(y-y')] = \sum_{\alpha=1}^{8} \xi_\alpha \otimes \mathbf{T}\xi_\alpha \exp[ikp_\alpha(y-y')], \tag{6.1}$$

where ξ_α and p_α are the eigenvectors and eigenvalues of the matrix $\mathbf{N}$. Comparing (6.1) with (4.9) one concludes that for homogeneity everywhere, including infinities $y \to \pm\infty$, there must be:

$$\zeta_{k\alpha} = \xi_\alpha, \qquad p_{k\alpha} = p_\alpha. \tag{6.2}$$

Consequently, due to the orthogonality relation $(4.8)_1$, the non-diagonal block-matrices in (5.14)–(5.18) vanish: $\mathbf{M}^{12} = \mathbf{M}^{21} = 0$. Therefore the matrix $\mathbf{m}_k$ (5.20) is an identical zero, which means that also $m^{(k)}_{\alpha\gamma} = 0$ in (5.21) and (5.22). In the same way, the matrices $\mathbf{v}_k$ and $\mathbf{v}^*_{-k}$ in (5.25)–(5.27) also must vanish. Thus, one easily obtains, instead of (5.27),

$$\boldsymbol{\eta}(x,y) = -\frac{1}{\pi} \operatorname{Im} \sum_{\alpha=1}^{4} \xi_\alpha(\xi_\alpha \cdot \mathbf{Tg}) \int_0^\infty \frac{dk}{k} \exp\{\pm ik[x - x' + p_\alpha(y-y')]\}, \tag{6.3}$$

where by $(4.6)_2$ we imply that $\operatorname{Im} p_\alpha > 0$ and the notation $\pm = \operatorname{sgn}(y-y')$ is accepted. Now the integral in (6.3) can be explicitly taken. Denote

$$J_\pm(t) = \int_0^\infty \frac{dk}{k} \exp(\pm ikt), \tag{6.4}$$

where $t = x - x' + p_\alpha(y-y')$. One can easily obtain the result that the derivative of $J_\pm(t)$ is equal to $J_\pm(t) = -1/t$. Therefore, to an arbitrary constant one has $J_\pm(t) = -\ln t$. This gives the expression

$$\boldsymbol{\eta}(x,y) = \frac{1}{\pi} \operatorname{Im} \sum_{\alpha=1}^{4} \xi_\alpha(\xi_\alpha \cdot \mathbf{Tg}) \ln[x - x' + p_\alpha(y-y')], \tag{6.5}$$

which is equivalent to the classical results of the dislocation theory obtained for homogeneous media, purely elastic [1, 2] or piezoelectric [5, 12].

7. Physical fields

In accordance with (2.13) and (3.1), the found potential field $\boldsymbol{\eta}(x,y)$ (5.27) allows one to express all electro-elastic physical fields excited in the considered layered medium in the form:

$$\mathbf{U}^{el} = \left(\mathbf{m}\frac{\partial}{\partial x} + \mathbf{n}\frac{\partial}{\partial y}\right) \otimes \mathbf{U}, \tag{7.1}$$

$$\boldsymbol{\Sigma} = \left(\mathbf{m}\frac{\partial}{\partial y} - \mathbf{n}\frac{\partial}{\partial x}\right) \otimes \mathbf{V}. \tag{7.2}$$

Substituting here the decomposed potential field $\boldsymbol{\eta}(x,y) = [\mathbf{U}(x,y), \mathbf{V}(x,y)]^{\mathrm{T}}$ (5.27), we obtain

$$\begin{pmatrix} \mathbf{U}^{el}(x,y) \\ \boldsymbol{\Sigma}(x,y) \end{pmatrix} = \frac{1}{\pi}\mathrm{Re}\int_0^\infty dk \sum_{\alpha=1}^{8} \begin{pmatrix} (\mathbf{m} + p_{k\alpha}\mathbf{n}) \otimes \mathbf{U}_{k\alpha}(y\,|\,y') \\ (p_{k\alpha}\mathbf{m} - \mathbf{n}) \otimes \mathbf{V}_{k\alpha}(y\,|\,y') \end{pmatrix} \\ \times \exp\{ik[x - x' + p_{k\alpha}(y - y')]\}\{\zeta_{k\alpha}(y\,|\,y') \cdot [\mathbf{T}(\mathbf{I}H(y'-y) - \mathbf{u}_k + \mathbf{v}_k)]\mathbf{g}\}. \tag{7.3}$$

Basing on Eq. (5.23) one can get an alternative form for this relation:

$$\begin{pmatrix} \mathbf{U}^{el}(x,y) \\ \boldsymbol{\Sigma}(x,y) \end{pmatrix} = \frac{1}{2\pi}\int_{-\infty}^\infty dk \sum_{\alpha,\beta,\gamma=1}^{8} \begin{pmatrix} (\mathbf{m} + p_{k\alpha}\mathbf{n}) \otimes \mathbf{U}_{k\alpha}(y\,|\,y') \\ (p_{k\alpha}\mathbf{m} - \mathbf{n}) \otimes \mathbf{V}_{k\alpha}(y\,|\,y') \end{pmatrix} \\ \times \exp\{ik[x - x' + p_{k\alpha}(y - y')]\}(\zeta_{-k\alpha}(y\,|\,y') \cdot \mathbf{T}\zeta^{-}_{k\beta})(\zeta^{-}_{-k\gamma} \cdot \mathbf{T}\mathbf{g}) \\ \times \{\mp\delta_{\beta\gamma}H(\pm k\,\mathrm{Im}\,p^{-}_{k\beta}) - m^{(k)}_{\beta\gamma}H(-k\,\mathrm{Im}\,p^{-}_{k\beta})\}, \tag{7.4}$$

where for brevity we again introduce $\pm = \mathrm{sgn}(y - y')$ and $\mp = \mathrm{sgn}(y' - y)$. The integrals in (7.3), (7.4) are well convergent with no singularities of the integrands anywhere. But we suppose here that at $x > x'$, the coordinate y may be arbitrarily close to y', however $y \neq y'$. In this case the values U^0_{lK} (2.2) and Σ^0_{iJ} (2.12) automatically vanish and the relations (2.13) and (3.1) are simplified, which is reflected in Eqs. (7.1), (7.2).

The 8D eigenvectors $\boldsymbol{\zeta}_{k\alpha}$ are decomposed in (7.3), (7.4) into 4-vectors: $\boldsymbol{\zeta}_{k\alpha} = [\mathbf{U}_{k\alpha}, \mathbf{V}_{k\alpha}]^{\mathrm{T}}$. In addition, let us also decompose each of these 4-vectors into a 3-vector and a scalar:

$$\mathbf{U}_{k\alpha} = [\mathbf{A}_{k\alpha}, \varphi_{k\alpha}]^{\mathrm{T}}, \qquad \mathbf{V}_{k\alpha} = [\mathbf{L}_{k\alpha}, D_{k\alpha}]^{\mathrm{T}}. \tag{7.5}$$

In these terms, the convolution involving in (5.23) the source strength $\mathbf{g}$, can be expressed as

$$\zeta^-_{-k\gamma}\cdot\mathbf{Tg} = -\mathbf{U}^-_{-k\gamma}\cdot\mathbf{F} + \mathbf{V}^-_{-k\gamma}\cdot\mathbf{B} = -\mathbf{A}^-_{-k\gamma}\cdot\mathbf{f} + \varphi^-_{-k\gamma}q + \mathbf{L}^-_{-k\gamma}\cdot\mathbf{b} + D^-_{-k\gamma}\Delta\varphi. \tag{7.6}$$

Certainly, all four components of the source strength are independent and can be free varied in (7.6). For instance, the electric field $\mathbf{E}$ excited by the ordinary 3D dislocation coming through the point (x', y') with the Burgers vector $\mathbf{b}$ in the considered layered medium, is equal

$$\mathbf{E}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk \sum_{\alpha,\beta,\gamma=1}^{8} [\mathbf{m} + \mathbf{n}p_{k\alpha}(y\,|\,y')]\varphi_{k\alpha}(y\,|\,y') \times \exp\{ik[x - x' + p_{k\alpha}(y - y')]\}(\zeta_{-k\alpha}(y\,|\,y')\cdot\mathbf{T}\zeta^-_{k\beta})(\mathbf{L}^-_{-k\gamma}\cdot\mathbf{b}) \times \{\mp\delta_{\beta\gamma}H(\pm k\,\mathrm{Im}\,p^-_{k\beta}) - m^{(k)}_{\beta\gamma}H(-k\,\mathrm{Im}\,p^-_{k\beta})\}\cdot \tag{7.7}$$

The generalized stress Σ_{iJ} given by (7.2) does not contain the component Σ_{zJ}. The latter can be found from the Hooke's law (2.6). However, in some cases it is sufficient to know only the above two components of the stress tensor. In particular, the corresponding components of an ordinary mechanical stress tensor $\boldsymbol{\sigma}$ excited by the same 3D dislocation and given by Eq. (7.4), have the form

$$\mathbf{m}\boldsymbol{\sigma}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk \sum_{\alpha,\beta,\gamma=1}^{8} p_{k\alpha}(y\,|\,y')\mathbf{L}_{k\alpha}(y\,|\,y') \times \exp\{ik[x - x' + p_{k\alpha}(y - y')]\}(\zeta_{-k\alpha}(y\,|\,y')\cdot\mathbf{T}\zeta^-_{k\beta})(\mathbf{L}^-_{-k\gamma}\cdot\mathbf{b}) \times \{\mp\delta_{\beta\gamma}H(\pm k\,\mathrm{Im}\,p^-_{k\beta}) - m^{(k)}_{\beta\gamma}H(-k\,\mathrm{Im}\,p^-_{k\beta})\}, \tag{7.8}$$

$$\mathbf{n}\boldsymbol{\sigma}(x,y) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} dk \sum_{\alpha,\beta,\gamma=1}^{8} \mathbf{L}_{k\alpha}(y\,|\,y')\exp\{ik[x - x' + p_{k\alpha}(y - y')]\} \times (\zeta_{-k\alpha}(y\,|\,y')\cdot\mathbf{T}\zeta^-_{k\beta})(\mathbf{L}^-_{-k\gamma}\cdot b)\{\mp\delta_{\beta\gamma}H(\pm k\,\mathrm{Im}\,p^-_{k\beta}) - m^{(k)}_{\beta\gamma}H(-k\,\mathrm{Im}\,p^-_{k\beta})\}. \tag{7.9}$$

In the same way one can find the contributions to physical fields from all the considered source types described by Eq. (7.6). General solutions (7.3), (7.4) may also be used as Green's functions for a determination of fields of continuously distributed bulk sources of the same type. If, instead of the line 8D source $\mathbf{g}$, there is a given distribution $\mathbf{g}(x,y)$ of sources, one should just substitute this distribution into the found expression (7.3) or (7.4) as a function $\mathbf{g}(x',y')$ and make additional integration with respect to x' and y'. For instance, the 2D force distribution $\mathbf{f}(x,y)$ excite in the considered layered piezoelectric medium the electroelastic field

$$\begin{pmatrix} \mathbf{U}^{el}(x,y) \\ \mathbf{\Sigma}(x,y) \end{pmatrix} = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} dx' dy' \int\limits_{-\infty}^{\infty} dk \sum_{\alpha,\beta,\gamma=1}^{8} \begin{pmatrix} (\mathbf{m} + p_{k\alpha}\mathbf{n}) \otimes \mathbf{U}_{k\alpha}(y \,|\, y') \\ (p_{k\alpha}\mathbf{m} - \mathbf{n}) \otimes \mathbf{V}_{k\alpha}(y \,|\, y') \end{pmatrix}$$
$$\times \exp\{ik[x - x' + p_{k\alpha}(y - y')]\}(\zeta_{-k\alpha}(y \,|\, y') \cdot \mathbf{T}\zeta^{-}_{k\beta})(A^{-}_{-k\gamma} \cdot f(x',y'))$$
$$\times \{\mp\delta_{\beta\gamma} H(\pm k \operatorname{Im} p^{-}_{k\beta}) - m^{(k)}_{\beta\gamma} H(-k \operatorname{Im} p^{-}_{k\beta})\}, \tag{7.10}$$

where the given force distribution $\mathbf{f}(x, y)$ must satisfy the standard conditions of self-equilibrium.

8. Conclusions

The found solution is general in a sense that it is applicable to any layered medium, graded or stratified, but under important limitation: the eigenvectors and eigenvalues of the propagator matrix $\mathbf{W}^{(k)}(y \,|\, y')$ must have definite limits at $y \to \pm\infty$. The description is implicit: all the obtained analytical expressions for electroelastic fields of the general line source are given in terms of those eigenvectors and eigenvalues, which are supposed to be additionally computed as functions of y. This is the price which has to be paid for an analytical solution of such a complex problem, when an arbitrary anisotropy of the medium is accompanied by its arbitrary layering, and elasticity is coupled with an electric polarization.

Fortunately, for modern computers determination of eigenvectors and eigenvalues of any matrices is quite a standard problem. Finding the propagator matrix itself is also a routine procedure but only for the case of stratified (piece-vise homogeneous) media. A propagator matrix for graded (continuously inhomogeneous) multilayers is reduced to an ordering operator (matricant [21, 22]) and its explicit determination is less trivial. However, the latter problem was recently widely studied and new efficient numerical approaches have been developed [24].

The other important aspect is associated with possible degeneracies of the propagator matrix, its eigenvectors and eigenvalues, due to symmetry of the medium. In this paper we ignored such a possibility supposing that unrestricted anisotropy of the medium excludes any degeneracy. On the other hand, as was recently indicated in [25], such degeneracies really arise in cases of high symmetry of the medium, and for some symmetric boundary problems could strongly influence the convergence of Fourier integrals. So, they need a special care at numerical analysis.

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