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Modeling of the propagation and evolution of nonlinear waves in a wave train

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A theoretical approach is applied to predict the propagation and evolution of nonlinear water waves in a wave train. A semi-analytical solution was derived by applying an eigenfunction expansion method. The solution is applied to study the evolution of nonlinear waves in a wave train and the formation of freak waves. The analysis focuses on the changes of wave profile and wave spectrum due to the interaction of wave components in a wave train. The results indicate that for waves of very low steepness, the changes of wave profile and wave spectrum are of secondary importance and weakly nonlinear wave theories can be applied to describe wave propagation in a wave train. For waves of low and moderate steepness, the nonlinear terms in the free-surface boundary conditions are becoming more and more important and weakly nonlinear wave theories cannot be applied to describe substantial changes in wave profile. A train of basically sinusoidal waves may drastically change its form within a relatively short distance from its original position and freak waves are often formed. The interaction between waves in a wave train and significant wave evolution has substantial effects on a wave spectrum. A train of initially very narrow-banded spectrum changes its simple one-peak spectrum to a broad-banded and often multi-peak spectrum in a fairly short period of time. The analysis shows that these phenomena cannot be described properly by the nonlinear Schrödinger equation or its modifications. Laboratory experiments were conducted in a wave flume to verify theoretical approaches. The free-surface elevation recorded by a system of wave gauges was compared with the results provided by the semi-analytical solution. Theoretical results are in a fairly good agreement with experimental data. A reasonable agreement between theoretical results and experimental data is observed, even for complex changes of long wave trains.

Key words: nonlinear waves, wave instability, wave evolution, initial conditions.

1. Introduction

The prediction of the propagation and transformation of nonlinear water waves is an interesting and ambitious/challenging problem from a theoretical point of view. A reliable prediction of free-surface elevation also is one of the main tasks of coastal and offshore engineers and is of fundamental importance in modeling of a sea state.

Modeling of the propagation and transformation of water waves is not a trivial task. This is because real water waves in nature are nonlinear. In order to describe the process of wave propagation and transformation with sufficient accuracy, nonlinear terms in the free-surface boundary conditions cannot be omitted in the modeling process, which complicates a solution procedure. Additional difficulties arise from a need to apply the boundary conditions on the free surface which is unknown and is a part of a final solution [31]. Despite the difficulties, many valuable nonlinear solutions were achieved, especially solutions for waves of permanent form [31, 16, 24, 19].

The nonlinear solutions achieved for waves of permanent form provided insight into the physics of water waves and many processes related with the propagation of nonlinear waves. These models helped to obtain solutions of practical importance by incorporating in the modeling of transient and irregular features of water waves. Various features of water waves of practical importance have been successfully described by applying numerical techniques. Application of numerical methods enabled a significant progress in modeling of the propagation and transformation of real water waves in two recent decades [29, 1, 18, 20, 11]. Another group of approaches applied to describe transient features of water waves are semi-analytical techniques based on perturbation expansions and spectral methods [30, 32, 8, 28, 4]. The derived models give qualitatively correct results for a wide range of wave parameters, and provide insight into the complex physics of transient nonlinear wave propagation and transformation of water waves in a wave train. An attractive alternative to these approaches is to transform boundary conditions to more convenient forms by applying Taylor series [6, 5, 22, 2, 26, 10]. Comparisons of theoretical results with experimental data indicate that these alternative approaches may give qualitatively and quantitatively correct results even for steep wave events.

A real challenge in modeling of the water waves is a satisfactory prediction of the propagation and transformation of water waves in a wave train of realistic broad-banded frequency spectrum. This is because the process of wave propagation in a wave train of broad-banded spectrum is often unstable and a satisfactory prediction of the propagation and transformation of water waves in a wave train is not a trivial task. The weakly nonlinear Schrödinger equation and its modifications, which are often applied to illustrate wave instability phenomena, cannot be used to predict the transformation of waves in a wave train because they give only qualitatively correct evolution and are basically applicable only to narrow-banded wave processes [4]. There have been several attempts to predict the propagation of waves in a wave train of broad-banded spectrum, however, only qualitatively correct evolution was achieved [15]. The difficulties in a reasonable prediction of wave evolution in a wave train is related with a delicate nature of wave instability phenomena, sensitivity to initial conditions, sensitivity to solution techniques, etc., which makes the modeling of this phenomena a difficult task. On the other hand, the ability to predict wave evolution in a wave train is of significant practical importance because instability may lead to resonant interactions of wave components in a wave train, formation of extreme waves, substantial changes of wave spectrum etc., and understanding of these processes is of vital interest to scientist and engineers [3, 33, 14].

In this work, a theoretical approach is applied to study the propagation and evolution of nonlinear water waves. First, a semi-analytical solution is derived within the Eulerian description to predict the propagation of nonlinear water waves in a wave train. The solution is applied to analyze the effect of wave modulation and wave steepness on the propagation and evolution of nonlinear water waves and the formation of freak waves. Then, laboratory experiments are conducted in the wave flume. Finally, theoretical results are compared with experimental data and conclusions are specified.

2. Theoretical formulation

2.1. Statement of problem

We consider the propagation of nonlinear water waves in a periodic uniform wave train and their evolution due to initial conditions and wave instabilities. A right-hand Cartesian coordinate system is selected such that the xy plane is horizontal and coincides with the undisturbed free surface and z points vertically upwards (Fig. 1).

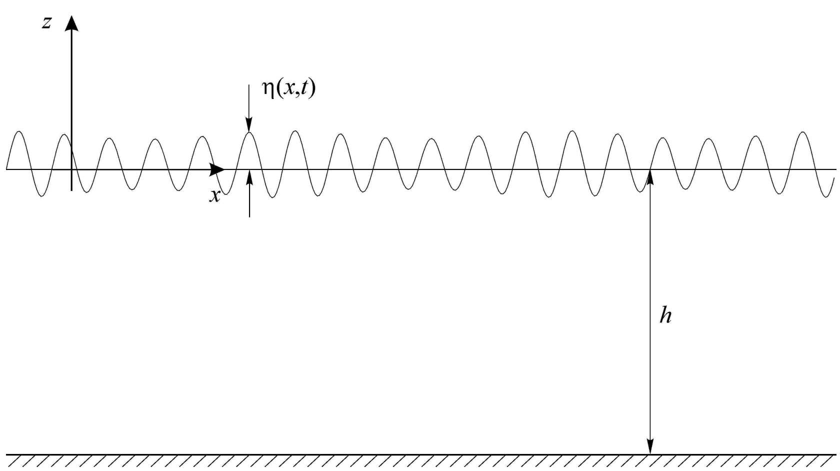

Fig. 1. Definitions sketch and coordinate systems.

It is assumed that:

- The fluid is inviscid and incompressible.
- The fluid motion is irrotational.
- The sea bottom is impervious.

According to the assumptions, the velocity vector, $\mathbf{V}(x,z,t)$, has a potential $\varPhi(x,z,t)$, such that $\mathbf{V} = \nabla\varPhi$ and the Bernoulli equation is

$$\varPhi_t + \frac{1}{\rho}P + gz + \frac{1}{2}|\nabla\varPhi|^2 = 0 \tag{2.1}$$

where ρ is the fluid density, P is the pressure,g is the acceleration due to gravity.

The velocity potential, $\varPhi(x,z,t)$, satisfies the Laplace equation

$$\nabla^2\varPhi = 0 \tag*{(2.2)$_1$}$$

at the free surface, the velocity potential, $\varPhi(x,z,t)$, has to satisfy the kinematic boundary condition

$$\eta_t + \varPhi_x\eta_x - \varPhi_z = 0, \qquad z = \eta(x,t) \tag*{(2.2)$_2$}$$

and the dynamic boundary condition

$$\varPhi_t + g\eta + \frac{1}{2}|\nabla\varPhi|^2 = 0, \qquad z = \eta(x,t); \tag*{(2.2)$_3$}$$

at the sea bottom the following boundary condition must be satisfied:

$$\varPhi_z = 0, \qquad z = -h. \tag*{(2.2)$_4$}$$

Moreover, the velocity potential must satisfy boundary conditions at infinity and initial conditions [31, 16].

A solution of the boundary-value problem, (2.2), is not a trivial task. It is difficult to find the velocity potential which satisfies the free-surface boundary conditions, because the boundary conditions contain nonlinear terms. Additional difficulties arise from a need to apply the boundary conditions on the free surface which is unknown and is a part of a final solution [31]. In order to achieve a solution, the kinematic free-surface boundary condition and the dynamic free-surface boundary condition are often expanded into a Taylor series about the mean position

$$\begin{aligned}
\sum_{n=0}^{\infty} \frac{\eta^n}{n!}\frac{\partial^n}{\partial z^n}(\eta_t + \varPhi_x\eta_x - \varPhi_z) &= 0, \qquad z = 0,\\
\sum_{n=0}^{\infty} \frac{\eta^n}{n!}\frac{\partial^n}{\partial z^n}\left(\varPhi_t + g\eta + \frac{1}{2}|\nabla\varPhi|^2\right) &= 0, \qquad z = 0
\end{aligned} \tag{2.3}$$

which usually helps to obtain a solution [7, 12, 25].

By expanding the kinematic free-surface boundary condition and the dynamic free-surface boundary condition in a Taylor series and collecting terms up to the third order in wave amplitude, one obtains the following boundary value problem:

$$\begin{aligned}
&\nabla^2 \Phi = 0,\\
&\eta_t + \Phi_x \eta_x - \Phi_z + \eta \Phi_{xz} \eta_x - \eta \Phi_{zz} - \frac{1}{2}\eta^2 \Phi_{zzz} = 0, \quad z = 0,\\
&\Phi_t + g\eta + \frac{1}{2}(\Phi_x^2 + \Phi_z^2)\\
&\qquad + \eta \Phi_{zt} + \eta(\Phi_x \Phi_{xz} + \Phi_z \Phi_{zz}) + \frac{1}{2}\eta^2 \Phi_{zzt} = 0, \quad z = 0,\\
&\Phi_z = 0, \quad z = -h.
\end{aligned} \tag{2.4}$$

Moreover, the velocity potential is required to be periodic in space.

Solution technique

It is convenient to seek a solution by applying an eigenfunction expansion method that is a recognized method applied in mathematics and theoretical physics [13, 21]. This method has been shown to be an efficient technique in the modeling of the propagation and transformation of nonlinear waves [9, 26]. Accordingly, the free-surface elevation, η, and the velocity potential, Φ, are sought in the following form:

$$\eta = \eta_0 + \sum_{n=1} (a_n \cos \lambda_n x + b_n \sin \lambda_n x), \tag{2.5$_1$}$$

$$\Phi = \Phi_0 + \sum_{n=1} \frac{\cosh \lambda_n (z+h)}{\cosh \lambda_n h} (A_n \cos \lambda_n x + B_n \sin \lambda_n x), \tag{2.5$_2$}$$

where η_0 , Φ_0 are known functions related with imposed initial conditions, and

$$\lambda_n = \frac{2\pi(n-1)}{b}, \tag{2.5$_3$}$$

in which b is the length of a sector over which the solution is assumed to be periodic.

The solution in the form of the eigenfunction expansions, (2.5), satisfies the Laplace equation and the bottom boundary condition. A time-stepping procedure is applied to satisfy the remaining boundary conditions, $(2.4)_2$ and $(2.4)_3$ and to determine the unknown coefficients of the eigenfunction expansions. Accordingly, in order to satisfy $(2.4)_2$ and $(2.4)_3$ and to determine the unknown coefficients of the eigenfunction expansions, the free-surface elevation, η, and the velocity potential, Φ, are discretized in time and the Adams–Bashford–Moulton predictor-corrector method is applied [23]. The Adams–Bashford–Moulton method enables prediction of the value of a function f from its time derivatives f', and in the present approach the Adams–Bashford predictor

$$(2.6)_1 \qquad f_{n+1} = f_n + \frac{\Delta t}{24}[55f'_n - 59f'_{n-1} + 37f'_{n-2} - 9f'_{n-3}]$$

is combined with the Adams–Moulton corrector

$$(2.6)_2 \qquad f_{n+1} = f_n + \frac{\Delta t}{24}[9f'_{n+1} + 19f'_n - 5f'_{n-1} + f'_{n-2}]$$

and with the free-surface boundary conditions, $(2.4)_2$ and $(2.4)_3$, to predict the free-surface elevation η and the velocity potential Φ at a new time step. Then, a Fourier transform is applied to determine the coefficients a_n, b_n, and A_n, B_n of the eigenfunction expansions. Because the free-surface boundary conditions are nonlinear, the solution procedure requires iterations at each time step. Usually, from three to five iterations are required to determine the free-surface elevation, η, and the velocity potential, Φ, with an accuracy sufficient for typical applications. The application of eigenfunction expansions and a Fourier transform makes the solution procedure a very efficient technique and enables to obtain results even for large spatial or time domains.

Initial conditions

In order to proceed and apply the derived solution, initial values of the free-surface elevation and the velocity potential should be specified along the sector over which the solution is assumed to be periodic. However, the space distribution of the free-surface elevation and the velocity potential are usually not available. Typical wave data available in coastal and offshore engineering consist of wave records from waverider buoys or stationary wave gauges. Thus, it is first necessary to apply a Fourier transform of a recorded time series, and a wave theory to obtain an initial space distribution of the free-surface elevation and the velocity potential required in the present approach. Accordingly, the following formulas are applied to provide initial conditions ($t < 0$)

$$(2.7)_1 \qquad \Phi_0 = \sum_n \frac{g}{\omega_n} \frac{\cosh k_n(z+h)}{\cosh k_n h} \times \{a_{0n} \sin[k_n(x-x_0) - \omega_n t] + b_{0n} \cos[k_n(x-x_0) - \omega_n t]\},$$

$$(2.7)_2 \qquad \eta_0 = \sum_n \{a_{0n} \cos[k_n(x-x_0) - \omega_n t] - b_{0n} \sin[k_n(x-x_0) - \omega_n t]\},$$

provided that the following dispersion equation is satisfied:

$$(2.7)_3 \qquad \frac{\omega_n^2}{g} = k_n \tanh k_n h,$$

where a_{0n} and b_{0n}, are the amplitudes arising from the Fourier transform of a free-surface elevation recorded at x_0, ω_n is the wave frequency ($\omega_n = 2\pi/T_n$) and k_n is the corresponding wave number ($k_n = 2\pi/L_n$).

The progress in the development of radar and satellite techniques may soon resolve the problem of initial values for modeling of the transient water waves. Moreover, the development of new non-invasive measurement techniques, including PIV, acoustic instruments, etc., may soon provide initial condition with sufficient accuracy. In some cases, including laboratory verification of theoretical results, it is already possible to determine initial values for the modeling of transient water waves [26, 27].

3. Results

Wave evolution

The solution derived to describe the propagation of nonlinear water waves was applied to investigate the effect of wave frequencies and wave steepness on the propagation and evolution of water waves in a modulated uniform wave train. The evolution of water waves is predicted for deep-water wave trains of different modulation lengths. Moreover, additional calculations are conducted to investigate the effect of wave steepness on wave evolution and on the formation of freak-type waves. An initial space distribution of the free-surface elevation and the velocity potential is determined by applying (2.7). This enables us to impose initial and boundary conditions in a simple manner to facilitate analysis of results and derivation of conclusions.

The results are analyzed with the emphases on the effect of modulation lengths and wave steepness on the propagation and evolution of waves in a wave train. The attention is paid to the wave profile, the wave spectrum, and the changes of wave profile and wave spectrum due to the nonlinear interaction of wave components in the wave train. Moreover, attention is paid to conditions for which a freak wave is formed in a wave train. The nonlinear interaction of wave components in a wave train is believed to be one of the potential sources of the formation of freak waves.

The model is first applied to predict the propagation and transformation of a wave train for $N = 4$ waves in the modulated wave train segment. The results in Fig. 2 show the profiles of free-surface elevation for the carrier wave of amplitude A and wave number k, satisfying the following wave steepness criteria $Ak = 0.10$, $Ak = 0.12$, and $Ak = 0.14$ at time $t/T = 1$, 20, 40, 60, 80 and 100.

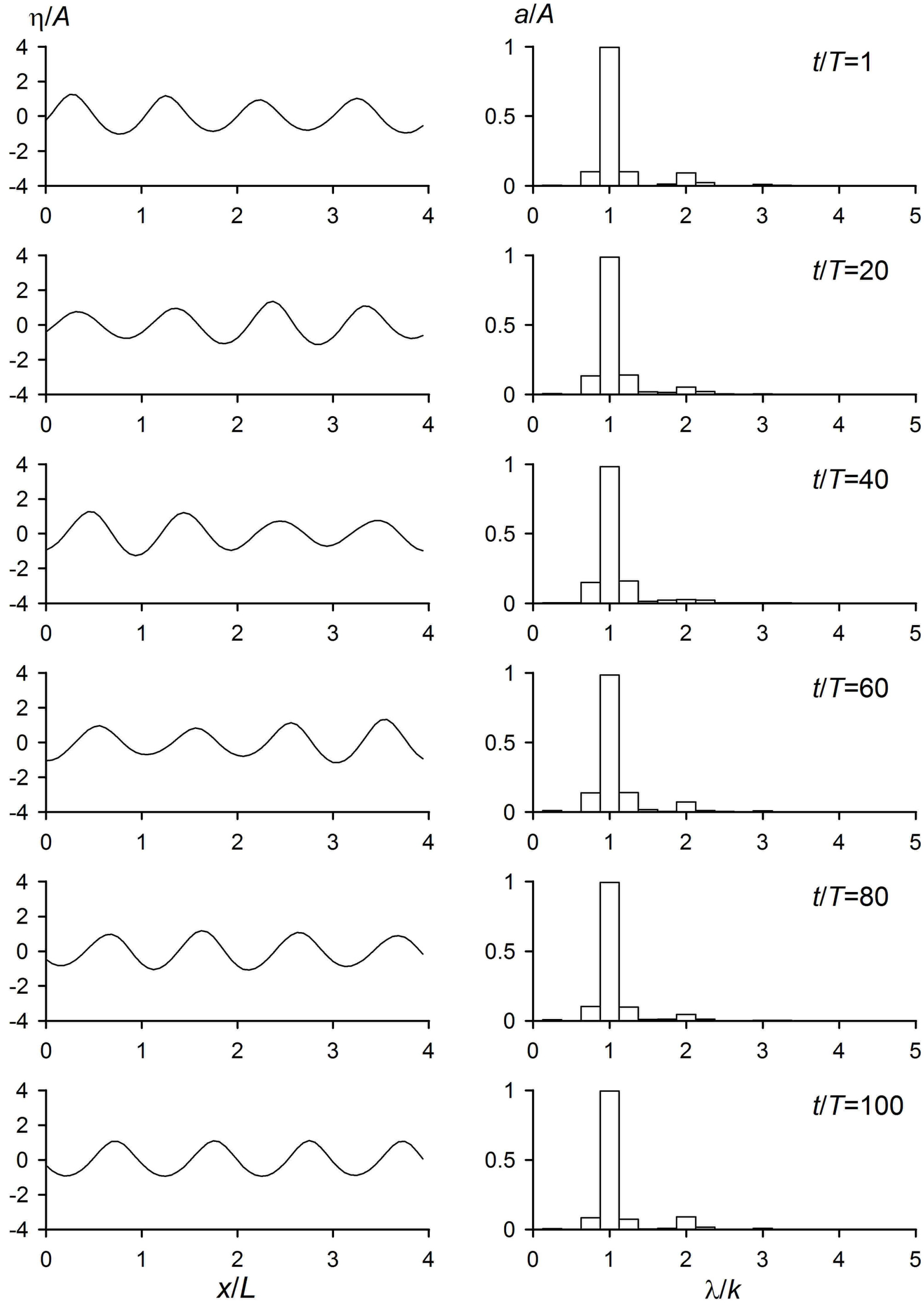

Fig. 2a. Evolution of a wave train for $Ak = 0.10$ and $N = 4$.

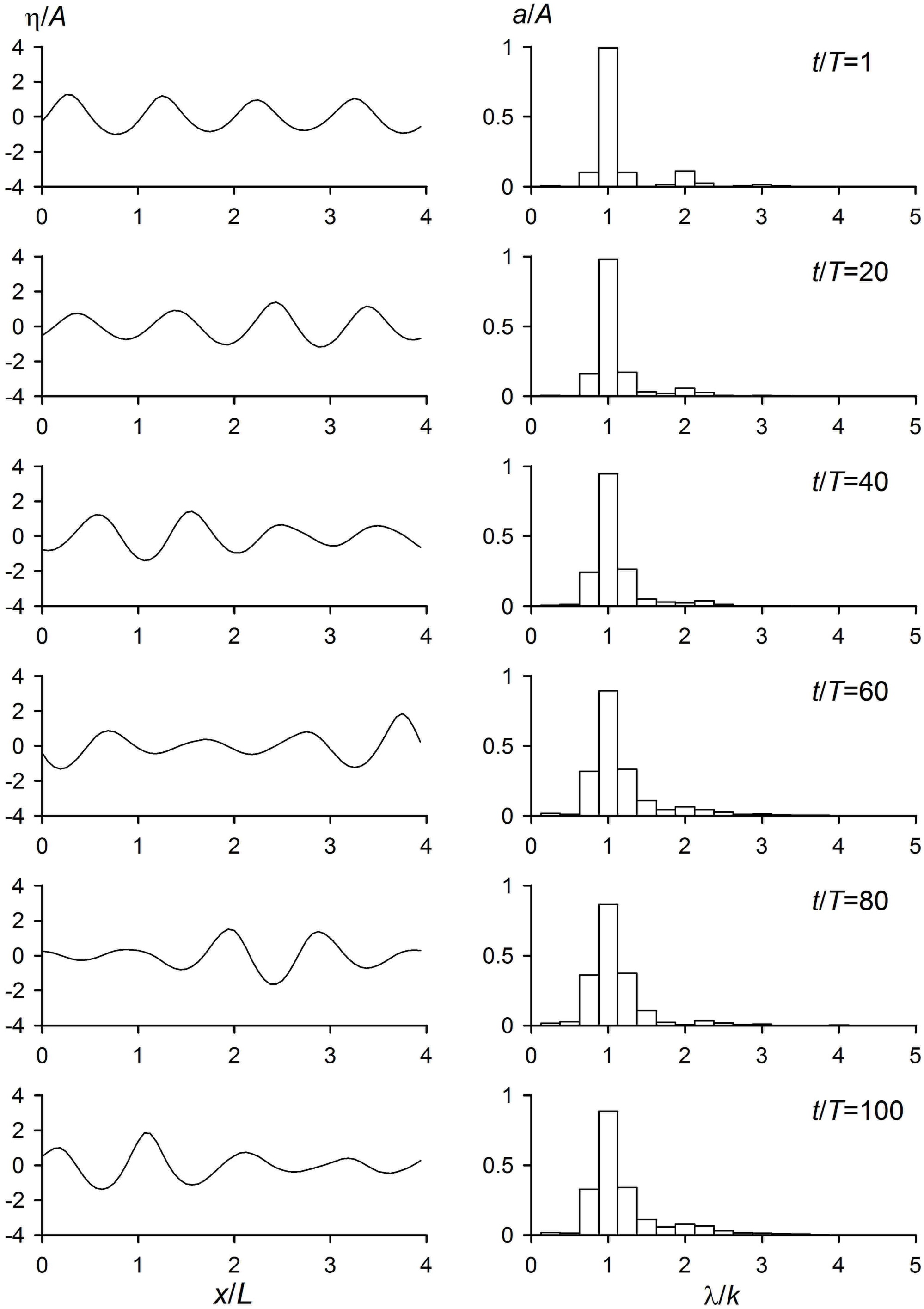

Fig. 2b. Evolution of a wave train for $Ak = 0.12$ and $N = 4$.

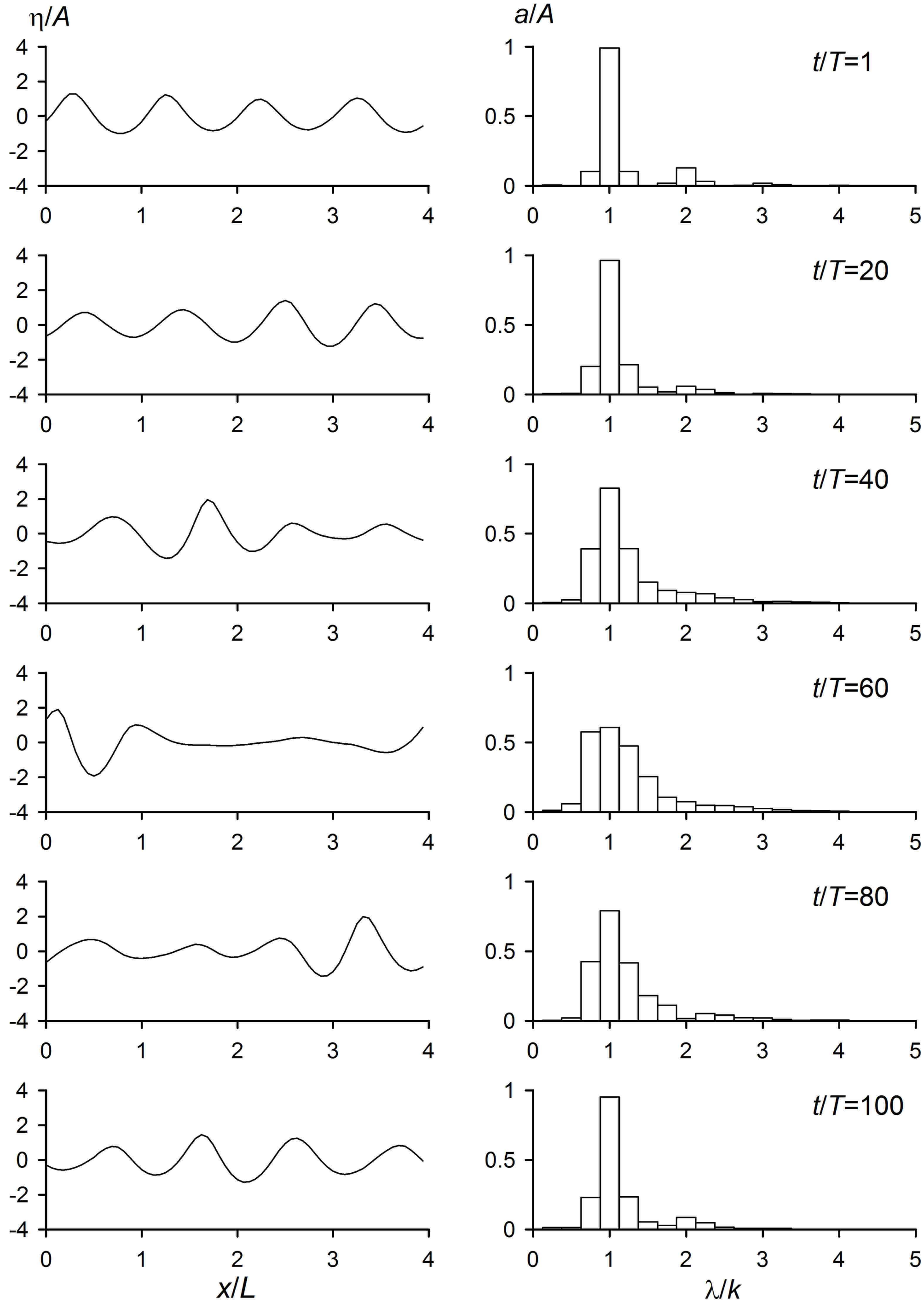

Fig. 2c. Evolution of a wave train for $Ak = 0.14$ and $N = 4$.

The plots in Fig. 2 also show the amplitudes of wave components corresponding to the predicted free-surface elevation. Supplementary information obtained by applying a Fourier analysis is helpful in the analysis of the interaction of waves in a wave train and the wave evolution process.

The results in Fig. 2 show that for waves of very low steepness, the nonlinear effects and the changes of wave profile and wave spectrum are of secondary importance. Accordingly, linear or weakly nonlinear wave theories can be applied to describe wave propagation in a wave train. For waves of higher steepness, the interaction between waves in a wave train is becoming more and more important. The results show that a train of basically sinusoidal waves may drastically change its form within a relatively short distance from its original position. The interaction between waves in a wave train and significant evolution of wave profile has substantial effects on the wave spectrum. A train of initially very narrow-banded spectrum changes its simple one-peak spectrum to a broad-banded spectrum in a fairly short period of time.

The features of wave propagation and evolution in a wave train can be further demonstrated and investigated by increasing the number of waves in the modulated wave train segment. The outcome of calculations conducted for five and six waves in the modulated wave train segments are presented in Fig. 3 and Fig. 4, respectively. The results in Fig. 3 and Fig. 4 show the profiles of free-surface elevation for $Ak = 0.10$, $Ak = 0.12$, and $Ak = 0.14$ at selected propagation phases. The plots in Fig. 3 and Fig. 4 also show the amplitudes of wave components corresponding to the predicted free-surface elevation.

The results in Figs. 3 and 4 show a significant effect of the number of waves in the modulated wave train segments on the evolution of waves in a wave train. The analysis shows that wave instabilities increase with increasing number of waves in the modulated wave train segments and then they decrease. This is consistent with available experimental results. It is worth to note that the solution of the nonlinear Schrödinger equation or its modifications cannot predict wave evolution with sufficient accuracy because of their limited practical applicability range. An analysis shows that the wave evolution is a very sensitive process and solutions derived for weakly nonlinear waves cannot describe this process with sufficient accuracy. The results in Figs. 3 and 4 also show that the interaction between waves in a wave train has substantial effects on a wave spectrum. A train of initially very narrow-banded spectrum changes its simple one-peak spectrum to a broad-banded or multi-peak spectrum. For sufficiently steep initial waves the evolution of waves in a wave train is accompanied by a decrease in frequency observed in the form of a shift of a wave spectrum towards low frequencies. However, shifts of a wave spectrum towards low frequencies are temporary and are part of a complex recurrence phenomenon observed in the evolution of a wave train.

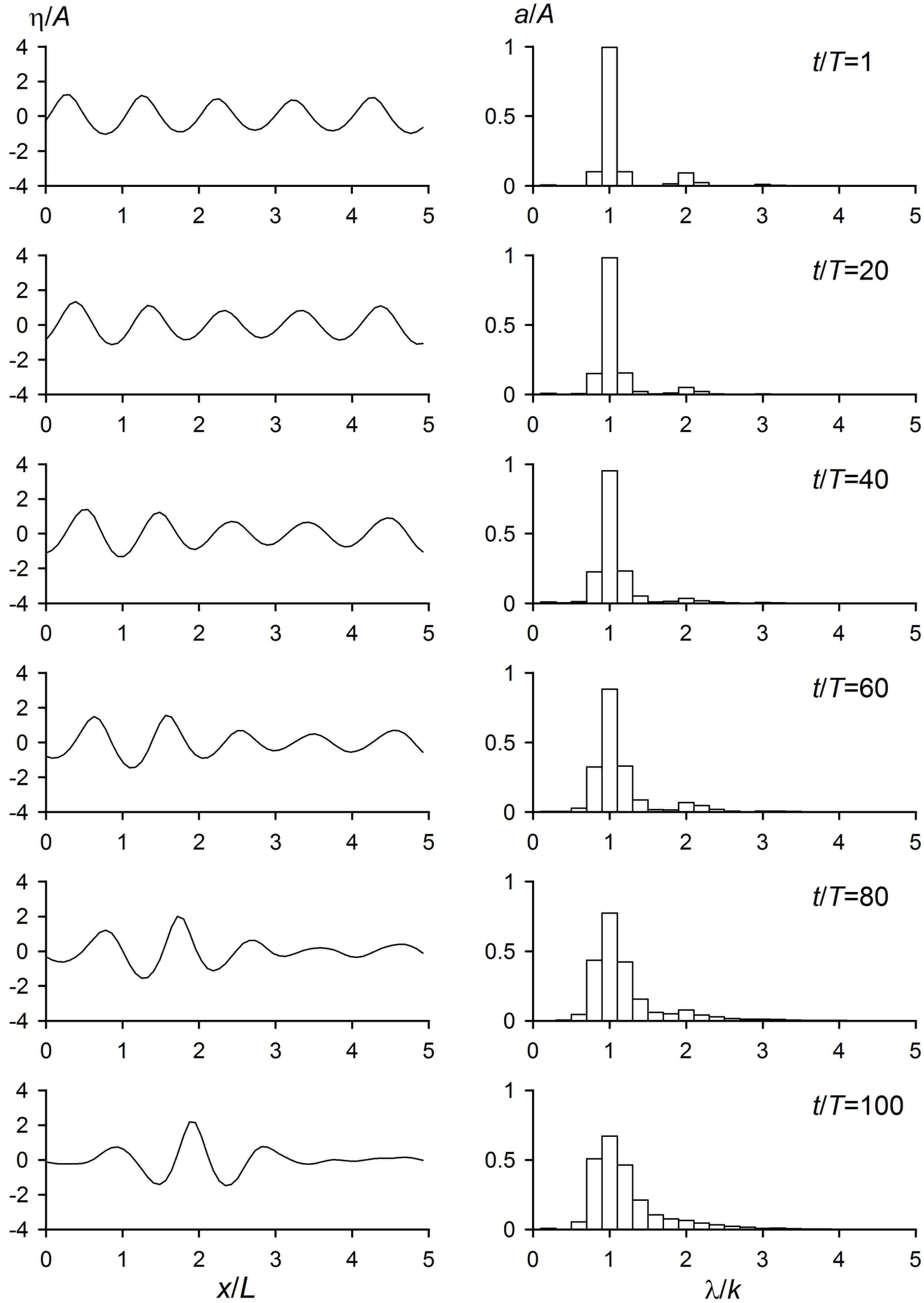

FIG. 3a. Evolution of a wave train for $Ak = 0.10$ and $N = 5$.

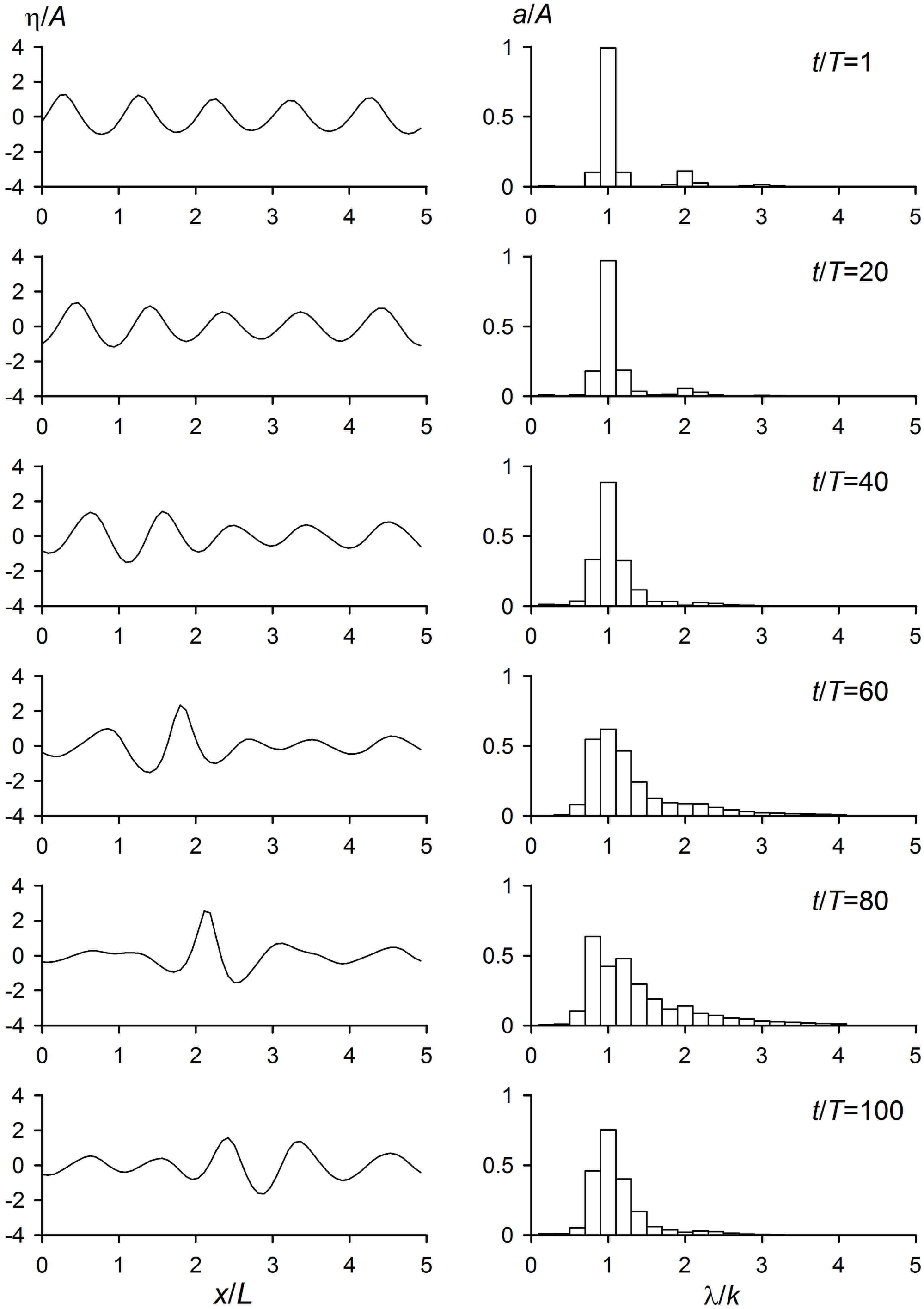

Fig. 3b. Evolution of a wave train for $Ak = 0.12$ and $N = 5$.

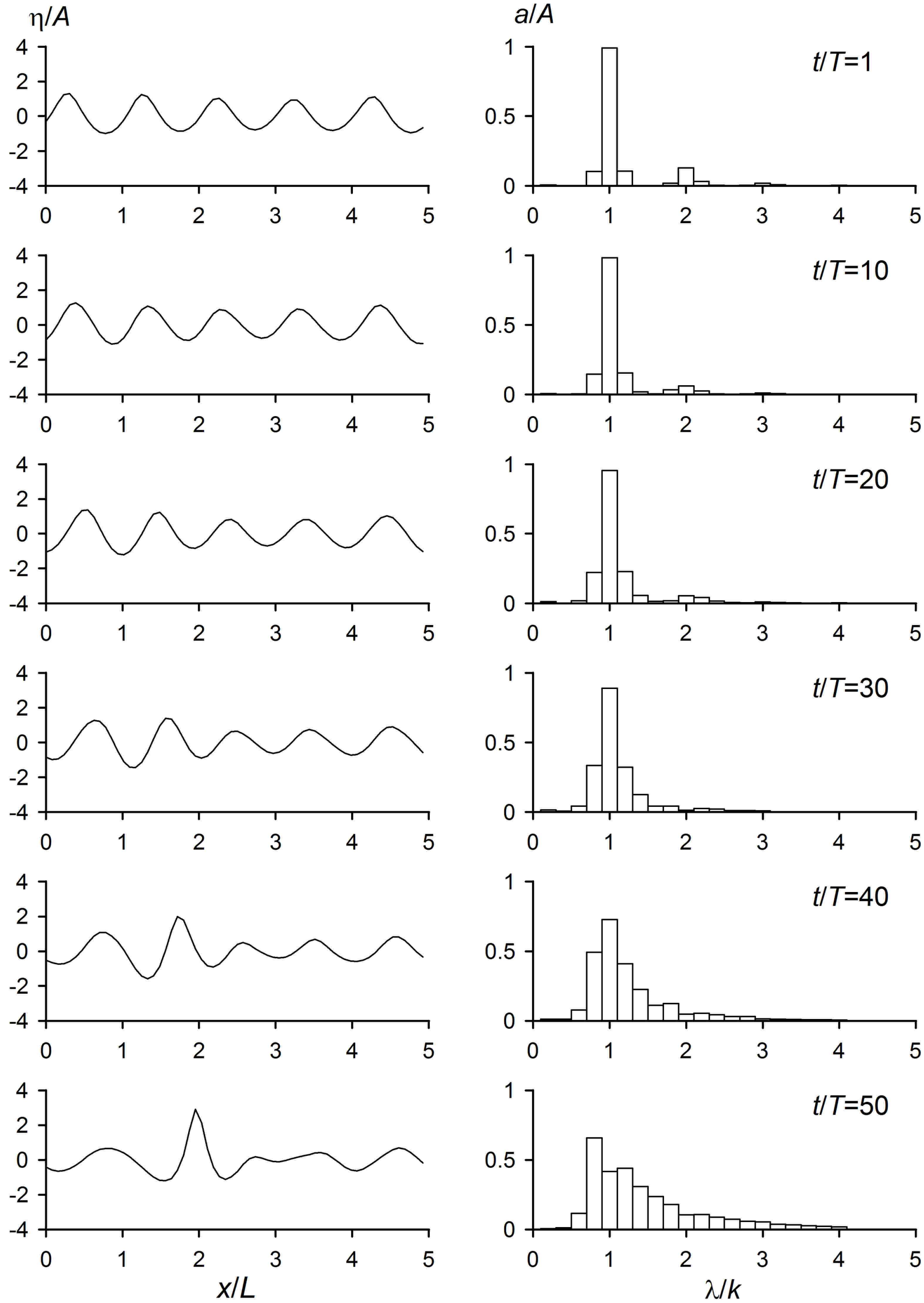

Fig. 3c. Evolution of a wave train for $Ak = 0.14$ and $N = 5$.

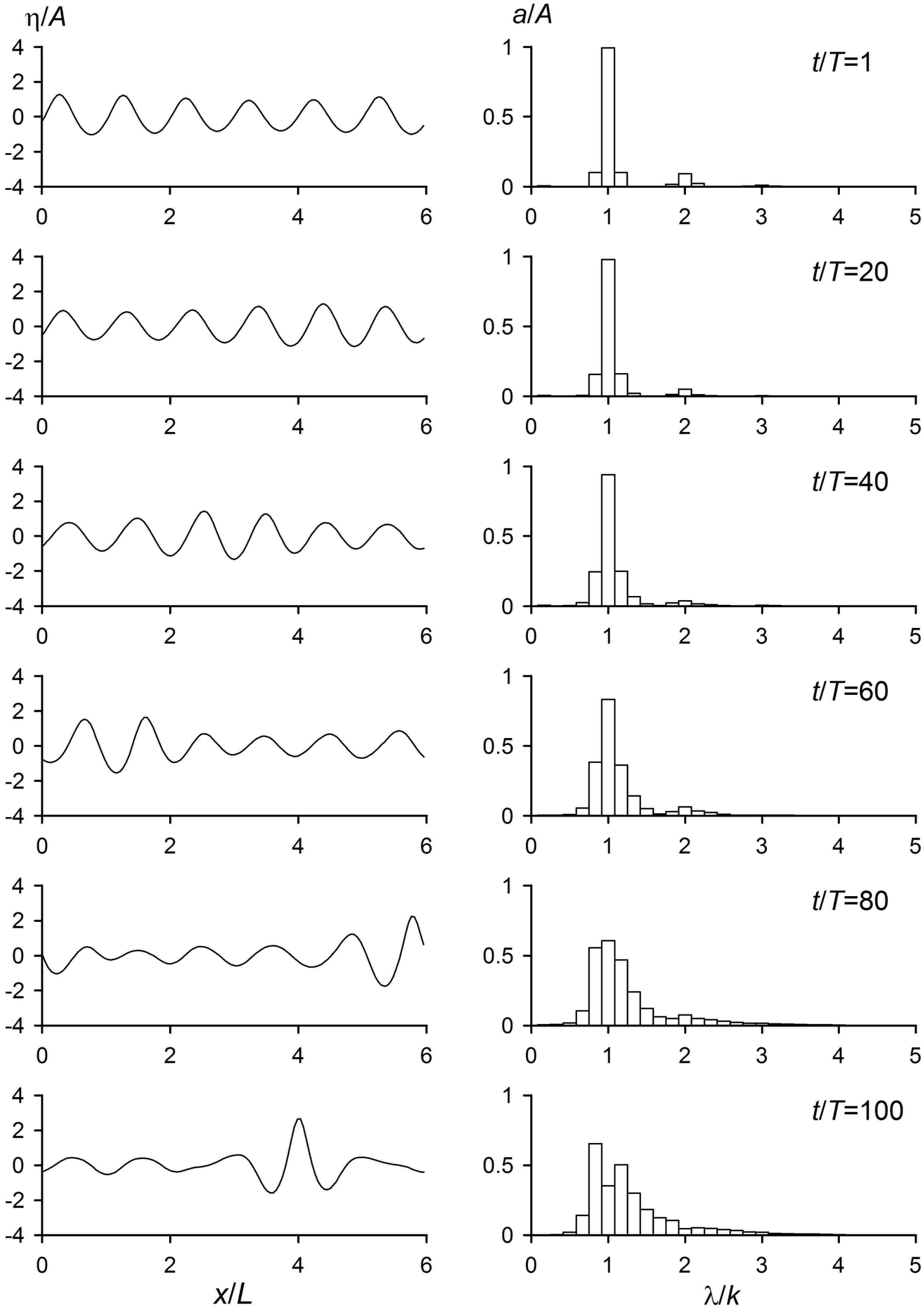

Fig. 4a. Evolution of a wave train for $Ak = 0.10$ and $N = 6$.

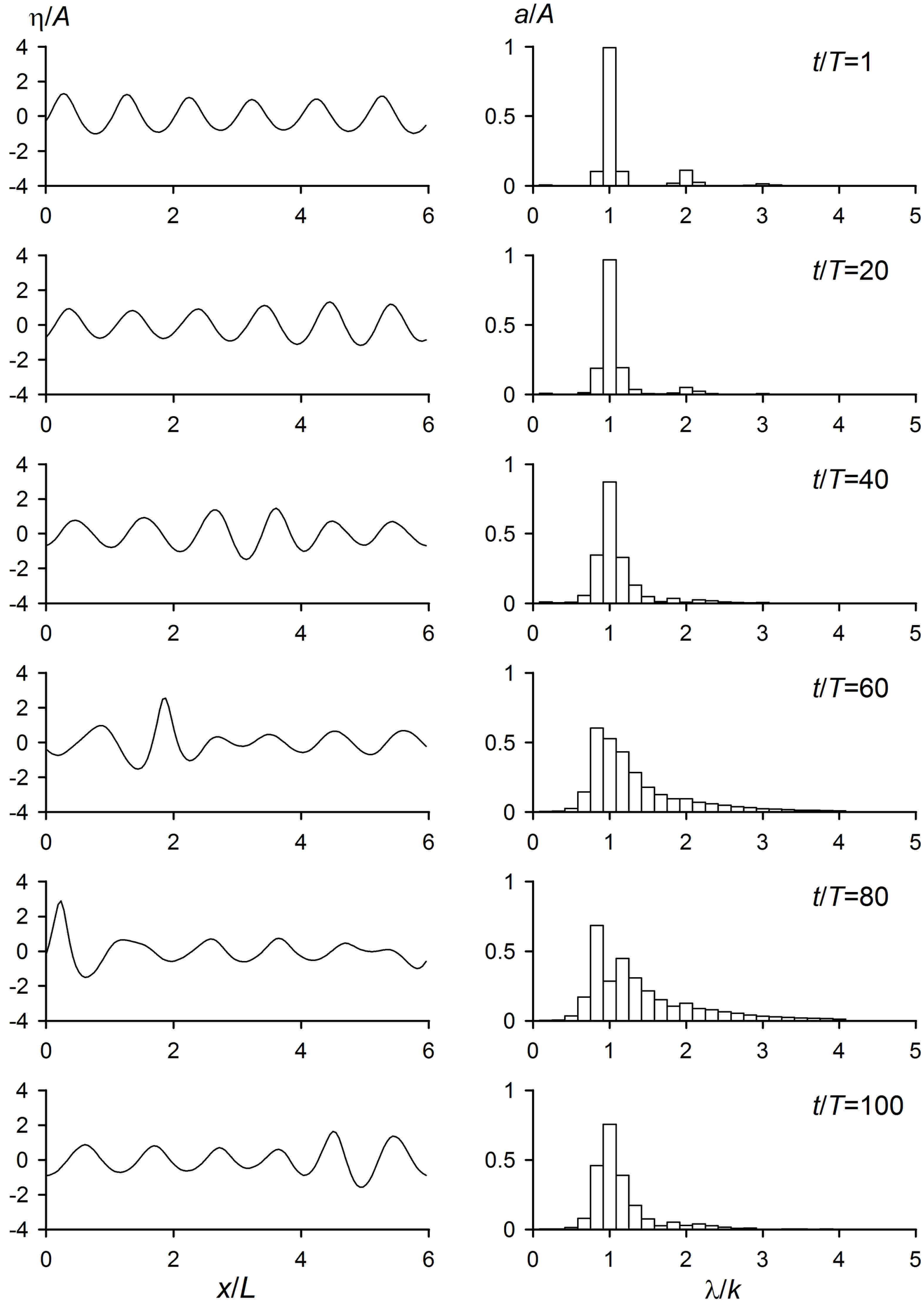

Fig. 4b. Evolution of a wave train for $Ak = 0.12$ and $N = 6$.

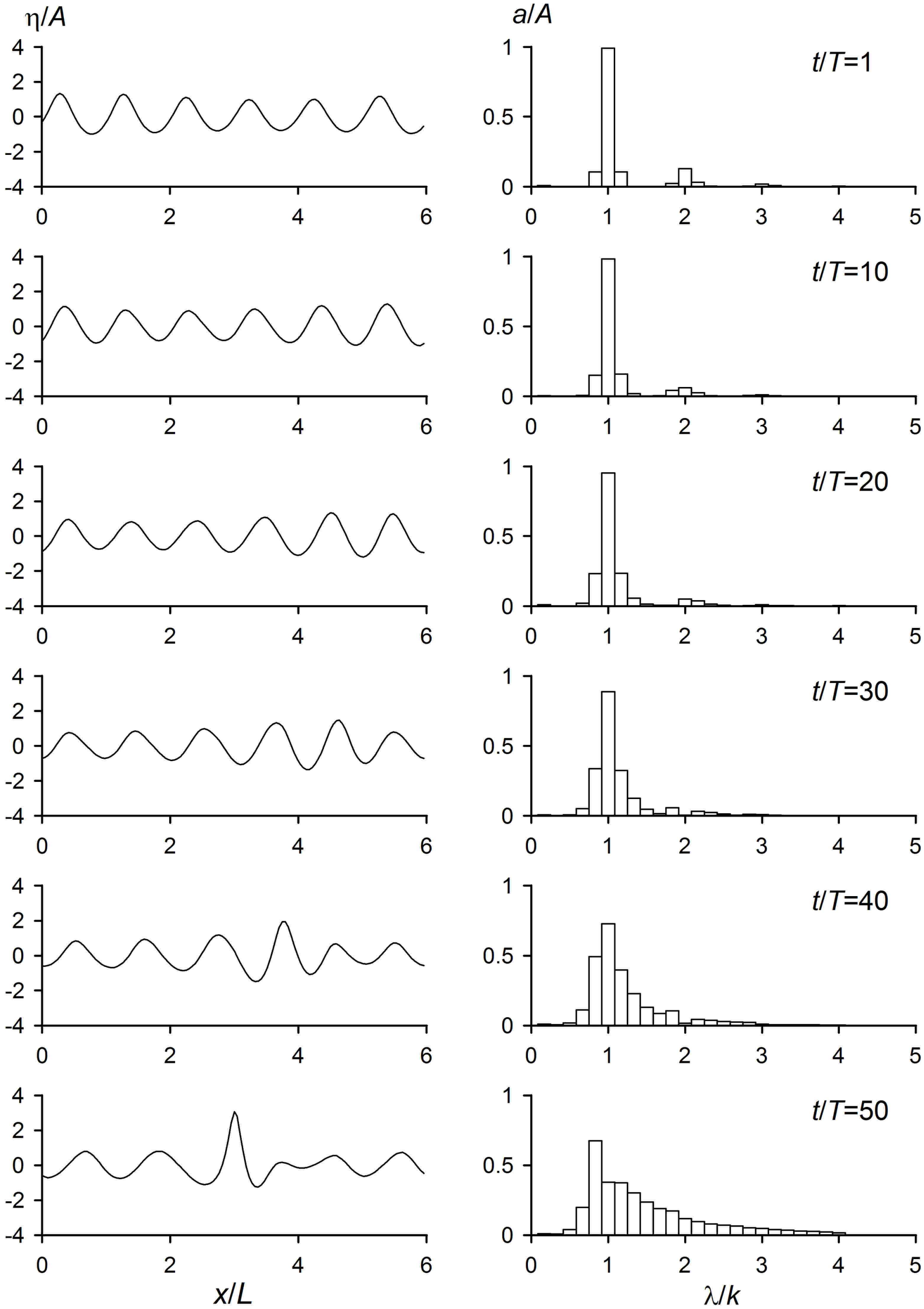

Fig. 4c. Evolution of a wave train for $Ak = 0.14$ and $N = 6$.

4. Formation of freak waves

The nonlinear interaction of wave components in a wave train and a transfer of wave energy to selected wave frequencies may trigger the formation of high waves in a wave train. In fact, this mechanism is believed to be one of the potential sources of the formation of freak waves. The outcome of calculations conducted for four, five and six waves in the modulated wave train segments are presented in Figs. 5–7. The results in Figs. 5–7 show the profiles of free-surface elevation with high waves.

The results in Figs. 5–7 show that for waves of very low steepness for which the nonlinear effects and the changes of wave profile and wave spectrum are of secondary importance, high waves are not formed in a wave train. For waves of higher steepness for which the nonlinear interaction between waves in a wave train are becoming more and more important, the evolution of waves may trigger the formation of high waves in a wave train. The analysis shows that the interaction between waves in a wave train and the significant evolution of a wave profile, often in a fairly short period of time, leads to the formation of freak-type waves. The results indicate that some waves formed in the evolution process may be several times higher than waves in the original wave train.

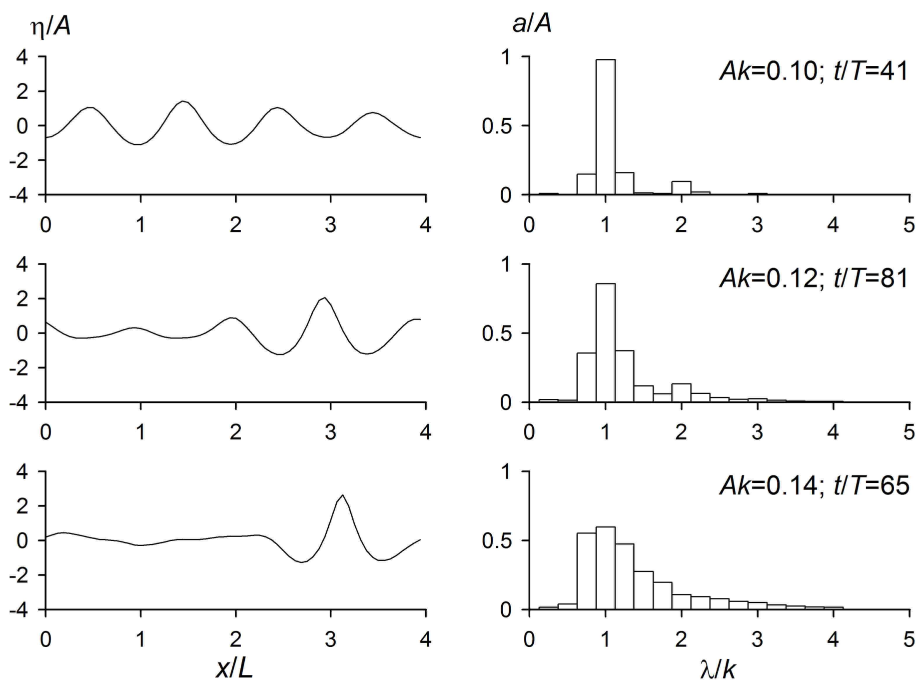

Fig. 5. Maximum free-surface elevation for $N = 4$

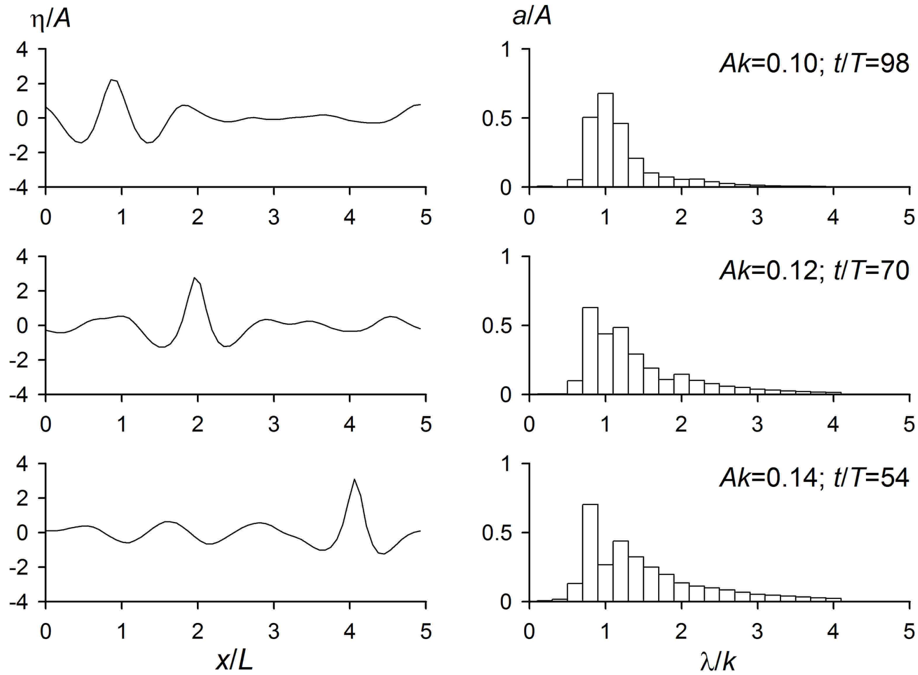

Fig. 6. Maximum free-surface elevation for $N = 5$.

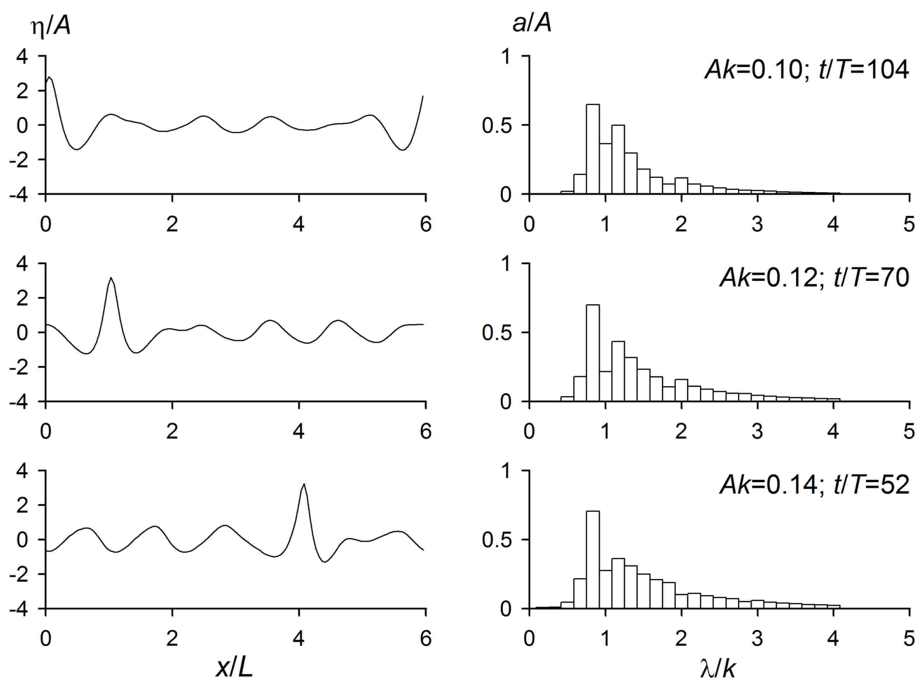

Fig. 7. Maximum free-surface elevation for $N = 6$.

5. Comparison with experiments

Laboratory experiments

Laboratory experiments were conducted in the wave flume at the Institute of Hydroengineering, Polish Academy of Sciences, Gdańsk. The wave flume at the Institute of Hydroengineering is 64 m long, 0.6 m wide and 1.4 m deep. It is equipped with a programmable piston wave generator. A porous wave absorber is supplied at the end of the wave flume as shown in Fig. 8.

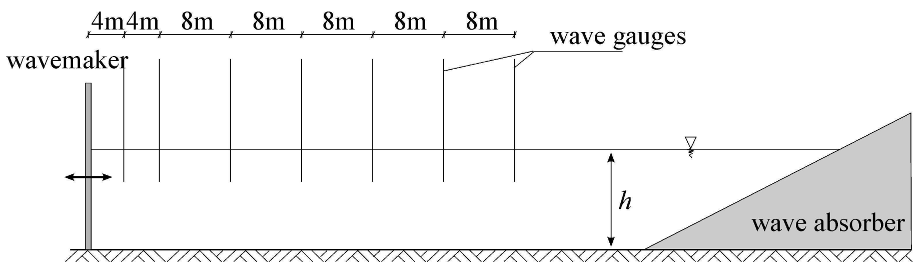

Fig. 8. Wave flume with a system of wave gauges.

The wavemaker generated modulated wave trains of different frequencies to verify the model derived to describe the propagation and evolution of nonlinear water waves. The measurements of the free-surface elevation were conducted by a group of resistance-type wave gauges installed in the wave flume. The free-surface elevation was measured for each wave train for about 100 s and were sampled at the rate of 200 Hz. The analysis of the free-surface elevation was conducted by applying a Fourier method and Kalman filter.

Laboratory experiments in the wave flume were conducted at the water depth $h = 0.6$ m. The measured time series of free-surface elevations, the evolution of wave components in the generated wave trains and the measured wave amplitude spectra, were used to conduct verification of the theoretical approach.

6. Comparisons with experimental data

A comparison between theoretical results and experimental data is shown in Fig. 9. The plots show the predicted and measured time series of the free-surface elevation. The results obtained by application of the derived model were compared with experimental data at eight locations along the wave flume. The comparisons are presented for the first, the fourth, and the last wave gauge. Because water waves in the wave flume are exposed to damping, a standard laminar damping proposed by Larsen and Dancy [17] was incorporated in the theoretical model to calculate theoretical results for comparisons with experimental data.

The plots in Fig. 9 show that the theoretical results obtained by the application of the model derived to describe the evolution of water waves in a wave train are in reasonable agreement with experimental data. A fairly good agreement between the theoretical results and experimental data is observed. A satisfactory agreement between theoretical results and experimental data is observed even for complex changes of the wave train arising from wave instabilities. Some discrepancies observed between the predicted and measured time series of the free-surface elevation are likely related with higher-order nonlinear effects that are not included in the theoretical approach.

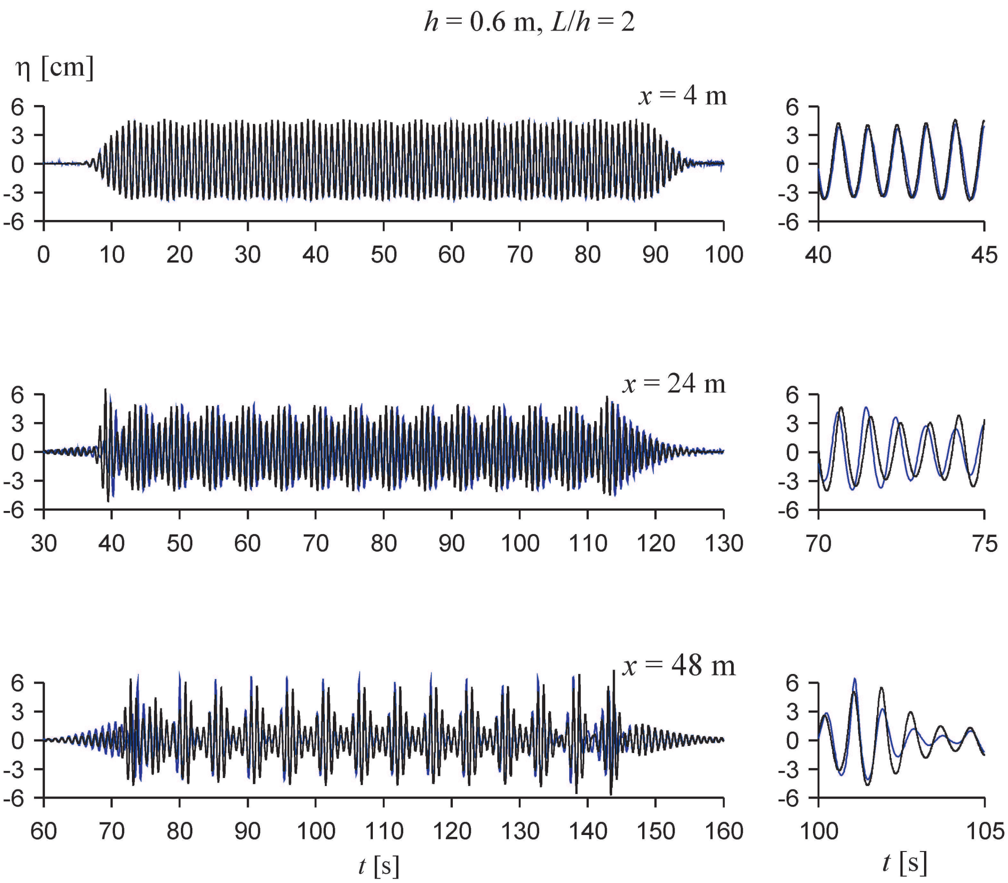

Fig. 9. Predicted and measured free-surface elevation for $L/h = 2$; black line – theoretical results, blue line – experimental data.

A complementary verification of the model derived to describe the evolution of water waves in a wave train was conducted by applying a Fourier analysis and comparing the wave amplitude spectra obtained from theoretical results and experimental data. The outcome of the Fourier analysis is presented in Fig. 10. The plots in Fig. 10 show the amplitudes of wave components for the time series presented in Fig. 9.

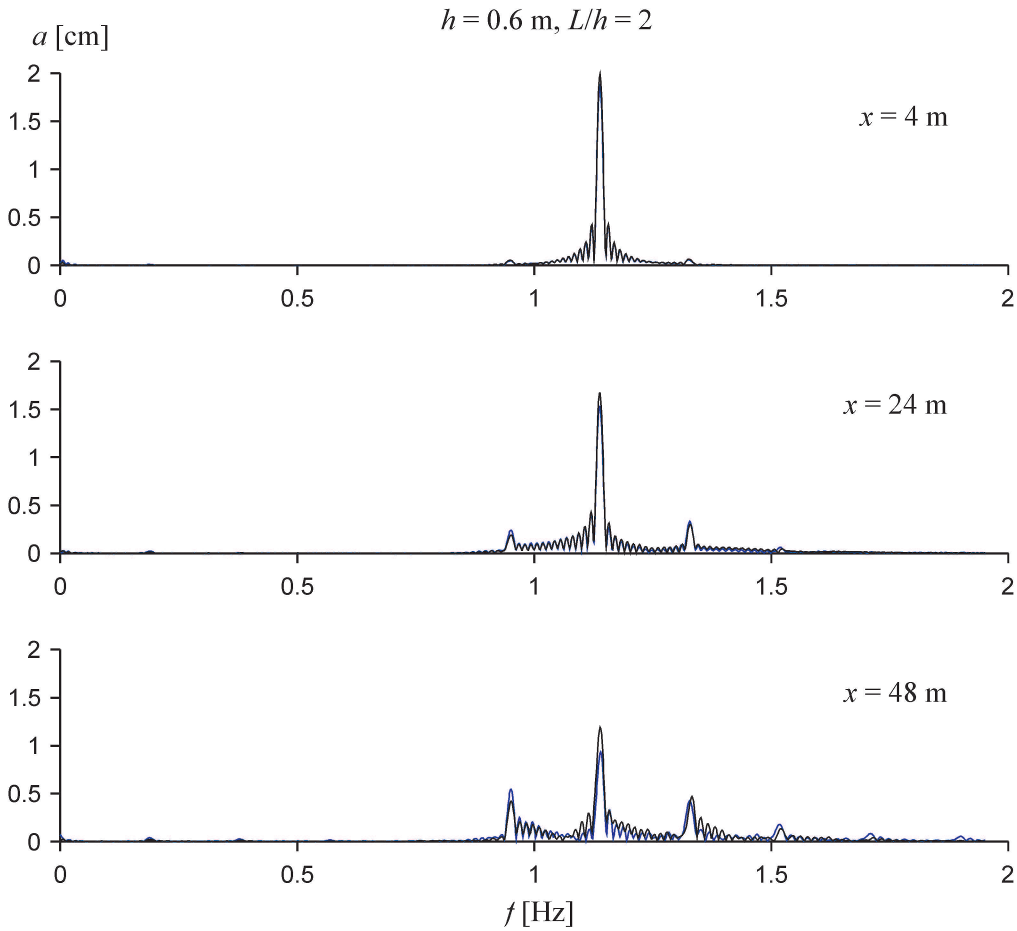

Fig. 10. Outcome of Fourier analysis; black line – theoretical results, blue line – experimental data.

The plots in Fig. 10 show that the theoretical results obtained by the application of the model derived to describe the evolution of water waves in a wave train are in reasonable agreement with experimental data. A fairly good agreement between the predicted wave amplitude distribution and the corresponding experimental data is observed for a wide range of wave frequencies. The plots show that the model predicts fairly well the multi-peak spectra including wave spectra with significant nonlinear wave components.

7. Summary

A theoretical approach is applied to predict the propagation and evolution of nonlinear water waves in a wave train. A semi-analytical solution was derived by applying an eigenfunction expansion method. The solution is applied to study the instability and evolution of nonlinear waves in a wave train and the formation of freak waves. The main attention is paid to the wave profile, the wave spectrum, and the changes of wave profile and wave spectrum due to the interaction of wave components in a wave train.

The results indicate that for waves of very low steepness, the nonlinear effects and the changes of wave profile and wave spectrum are of secondary importance, and weakly nonlinear wave theories can be applied to describe wave propagation in a wave train. For steeper waves the interaction between waves in a wave train are becoming more and more important and weakly nonlinear wave theories cannot be applied to describe substantial changes in the wave profile. The results show that a train of basically sinusoidal waves may drastically change its form within a relatively short distance from its original position and often freak waves are formed. The interaction between waves in a wave train and significant evolution of wave profile has substantial effects on the wave spectrum. A train of initially very narrow-banded spectrum changes its simple one-peak spectrum to a broad-banded or multi-peak spectrum in a fairly short period of time, which often leads to the formation of large freak-type waves. The analysis indicates that these phenomena cannot be properly described by the nonlinear Schrödinger equation or its modifications. The solution of the nonlinear Schrödinger equation provides insight into the instability of weakly nonlinear waves; however, its practical applicability range is very limited because wave evolution is a very sensitive process and the solution often provides confusing results.

Laboratory experiments were conducted in a wave flume to verify the theoretical approach. The free-surface elevation was recorded by a system of wave gauges to compare experimental data with theoretical results provided by the semi-analytical solution. The comparisons have been conducted for wave profiles and wave amplitude spectra. The analysis shows that theoretical results are in a fairly good agreement with experimental data. A reasonable agreement between theoretical results and experimental data is observed, even for complex changes of wave trains arising from wave instabilities.

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