

Electromagnetic solids with irreversible process of local mass displacement

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THE NON-LOCAL MODEL of electromagnetothermomechanics for polarized non-ferromagnetic solids is proposed. It takes into account the process of local mass displacement due to structural changes of a physically small element of a body. An approach which takes into account possible irreversibility of the local mass displacement is also proposed. On this basis, we have obtained the rheological constitutive relations for the vectors of the mass displacement and for the polarization. The proposed model allows to study the surface charge kinetics and the formation of near-surface inhomogeneities of the stress-strained state as well as the electric polarization, surface tension and disjoining pressure.

Key words: local mass displacements, irreversible thermodynamic processes, dielectric materials, non-local theory, interfacial phenomena.

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1. Introduction

NON-LOCAL THEORIES OF DIELECTRICS, which take into account the dependence of the body state on the strain gradients [1], the polarization gradient [2], the electric field gradients or higher electric moments (quadrupoles, octupoles and so on) [3, 4], as well as the theories which predict the constitutive relations of integral type [5], are well known in literature [5–8]. Recently, a new non-local theory of nonferromagnetic dielectric bodies has been proposed. This theory takes into account the process of local displacement of mass due to structural reordering of a physically small element of the body. The aforementioned reordering can be observed, in particular, in close vicinity of newly created surfaces or due to the body polarization, etc. Equations of such a theory with accounting for reversible

processes of local displacement of mass have been obtained in [9, 10]. The irreversibility of the local mass displacement process has been taken into account in [11]. In this paper we summarize the theory of coupled electromechanical processes in nonferromagnetic thermoelastic dielectrics with local displacement of mass. The model equations are used to study the near-surface inhomogeneity of the electromechanical fields in dielectric solids as well as the dynamics of creation of such inhomogeneities due to the surfaces formation. The effect of the coupled mechanical and electric fields in the near-surface regions as well as the size effects are analyzed.

2. Theory

We consider an isotropic thermoelastic polarized nonferromagnetic solid that occupies a region (V) of the Euclidean space and is bounded by a closed smooth surface (Σ). The solid is under the influence of an external load, which induces the mechanical, thermal and electromagnetic processes, and causes ordering of the body structure and electric charge, that manifests itself in the appearance of the efficient mass flux \mathbf{J}_{ms} and electric flux \mathbf{J}_{es} (the polarization current) [9, 10]. To obtain the basic set of equations we used the electrodynamics equations, the equations of mass conservation and also the balance equations of entropy and energy.

The Maxwell's equations in the local form may be represented as [12]

$$(2.1) \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{D} = \rho_e, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{H} = \mathbf{J}_e + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{P}}{\partial t}.$$

Here \mathbf{E} and \mathbf{H} are the electric and magnetic fields, \mathbf{B} and \mathbf{D} are the vectors of electric and magnetic inductions. For nonferromagnetic medium $\mathbf{B} = \mu_0 \mathbf{H}$, $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$, where \mathbf{P} denotes the local displacement of electric charge (polarization), ε_0 and μ_0 are the electric and magnetic constants, ρ_e is the density of free electric charge, \mathbf{J}_e is the density of electric current, $\partial \mathbf{P} / \partial t$ is the density of current caused by ordering of a charged system (polarization current), t is the time and ∇ is the Hamilton operator.

The balance equation of electromagnetic energy can be deduced from Maxwell's equations [9, 12]

$$(2.2) \quad \frac{\partial U_e}{\partial t} + \nabla \cdot \mathbf{S}_e + \left(\mathbf{J}_e + \frac{\partial \mathbf{P}}{\partial t} \right) \cdot \mathbf{E} = 0.$$

In Eq. (2.2), $U_e = (\varepsilon_0 \mathbf{E}^2 + \mu_0 \mathbf{H}^2) / 2$ is the density of electromagnetic energy and $\mathbf{S}_e = \mathbf{E} \times \mathbf{H}$ is the flow of electromagnetic energy (Poynting vector).

With account of the process of local mass displacement, the conservation of mass can be expressed as [10]

$$(2.3) \quad \frac{d}{dt} \int_{(V)} \rho dV = - \int_{(\Sigma)} (\rho \mathbf{v}_* + \mathbf{J}_{ms}) \cdot \mathbf{n} d\Sigma,$$

where ρ is the mass density, \mathbf{v}_* is a velocity of the convective displacement of the fixed body element, and \mathbf{n} is the outward unit normal to the surface (Σ) .

We introduce the vector of local mass displacement by the formula

$$\mathbf{\Pi}_m(\mathbf{r}, t) = \int_0^t \mathbf{J}_{ms}(\mathbf{r}, t') dt'.$$

Here \mathbf{r} is the position vector. For vector \mathbf{J}_{ms} one obtains

$$(2.4) \quad \mathbf{J}_{ms} = \frac{\partial \mathbf{\Pi}_m}{\partial t}.$$

The velocity vector \mathbf{v} of the centre of mass we define by relation

$$\mathbf{v} = \frac{1}{\rho} \left(\rho \mathbf{v}_* + \frac{\partial \mathbf{\Pi}_m}{\partial t} \right).$$

Then the mass balance equation in the local form acquires a standard form

$$(2.5) \quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0.$$

By analogy with the induced electric charge [12], we introduce the density of induced mass $\rho_{m\pi}$, which has the dimension of mass density. We require that for an arbitrary solid of finite size (domain (V)), the vector $\mathbf{\Pi}_m$ of the local mass displacement and the density of induced mass $\rho_{m\pi}$ satisfy [10]

$$\int_{(V)} \mathbf{\Pi}_m dV = \int_{(V)} \rho_{m\pi} \mathbf{r} dV.$$

From this relation we deduce that $\rho_{m\pi} = -\nabla \cdot \mathbf{\Pi}_m$ [10]. It is easy to show that equation

$$(2.6) \quad \frac{\partial \rho_{m\pi}}{\partial t} + \nabla \cdot \mathbf{J}_{ms} = 0$$

is satisfied. This equation has the form of the conservation law of induced mass.

In the local form, the entropy balance equation is [13]

$$(2.7) \quad \rho T \frac{ds}{dt} = -\nabla \cdot \mathbf{J}_q + \frac{1}{T} \mathbf{J}_q \cdot \nabla T + T \sigma_s + \rho \mathfrak{R}.$$

Here s is the specific entropy, T is the absolute temperature, σ_s is the entropy production, \mathfrak{R} denotes the distributed thermal sources, \mathbf{J}_q is the density of heat flux, and $d\dots/dt = \partial\dots/\partial t + \mathbf{v} \cdot \nabla\dots$

The balance equation of energy of the system “solid-electromagnetic field” in the integral form is given by BURAK *et al.* [10]:

$$(2.8) \quad \frac{d}{dt} \int_{(V)} \left(\rho u + U_e + \frac{1}{2} \rho \mathbf{v}^2 \right) dV \\ = - \oint_{(\Sigma)} \left[\rho \left(u + \frac{1}{2} \mathbf{v}^2 \right) \mathbf{v} - \hat{\boldsymbol{\sigma}} \cdot \mathbf{v} + \mathbf{S}_e + \mathbf{J}_q + \mu \mathbf{J}_m + \mu_\pi \frac{\partial \boldsymbol{\Pi}_m}{\partial t} \right] \cdot \mathbf{n} d\Sigma \\ + \int_{(V)} (\rho \mathbf{F} \cdot \mathbf{v} + \rho \mathfrak{R}) dV.$$

Here u and $\rho \mathbf{v}^2/2$ are the specific internal and kinetic energies, $\hat{\boldsymbol{\sigma}}$ is the Cauchy’s stress tensor, $\mu \mathbf{J}_m$ is the flux of energy connected with the mass transport relative to the center of mass, $\mathbf{J}_m = \rho(\mathbf{v}_* - \mathbf{v})$, $\mu_\pi \partial \boldsymbol{\Pi}_m / \partial t$ is the flux of energy related with structure ordering (local mass displacement), μ is the chemical potential, μ_π is the energy measure of the influence of the mass displacement on the internal energy, and \mathbf{F} is the mass force.

Taking into account the mass conservation law (2.5), the entropy balance (2.7) and the electromagnetic energy equation (2.2) and using the Ostrogradsky–Gauss theorem, from Eq. (2.8) we obtain the following local form of balance equation of internal energy:

$$(2.9) \quad \rho \frac{du}{dt} = \rho T \frac{ds}{dt} + \hat{\boldsymbol{\sigma}}_* : \frac{d\hat{\mathbf{e}}}{dt} + \rho \mathbf{E}_* \cdot \frac{d\mathbf{p}}{dt} + \rho \mu'_\pi \frac{d\rho_m}{dt} - \rho \nabla \mu'_\pi \cdot \frac{d\boldsymbol{\pi}_m}{dt} \\ + \mathbf{J}_{e*} \cdot \mathbf{E}_* - \mathbf{J}_q \cdot \frac{\nabla T}{T} - T \sigma_s + \mathbf{v} \cdot \left(-\rho \frac{d\mathbf{v}}{dt} + \nabla \cdot \hat{\boldsymbol{\sigma}}_* + \mathbf{F}_e + \rho \mathbf{F}_* \right).$$

Here

$$(2.10) \quad \hat{\mathbf{e}} = \frac{1}{2} \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right], \\ \hat{\boldsymbol{\sigma}}_* = \hat{\boldsymbol{\sigma}} - \rho \left(\mathbf{E}_* \cdot \mathbf{p} - \rho_m \mu'_\pi - \boldsymbol{\pi}_m \cdot \nabla \mu'_\pi \right) \hat{\mathbf{I}}, \\ \mathbf{F}_* = \mathbf{F} + \rho_m \nabla \mu'_\pi - \boldsymbol{\pi}_m \cdot \nabla \nabla \mu'_\pi, \\ \mathbf{F}_e = \rho_e \mathbf{E}_* + \left[\mathbf{J}_{e*} + \frac{\partial(\rho \mathbf{p})}{\partial t} \right] \times \mathbf{B} + \rho (\nabla \mathbf{E}_*) \cdot \mathbf{p}, \\ \mathbf{E}_* = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \quad \mathbf{J}_{e*} = \mathbf{J}_e - \rho_e \mathbf{v}, \quad \mu'_\pi = \mu_\pi - \mu,$$

$\hat{\mathbf{e}}$ is the strain tensor, \mathbf{u} is the displacement vector, the superscript T denotes the transpose, $\mathbf{p} = \mathbf{P}/\rho$, $\boldsymbol{\pi}_m = \boldsymbol{\Pi}_m/\rho$, $\rho_m = \rho_{m\pi}/\rho$, and $\hat{\mathbf{I}}$ is the unit tensor.

We represent the vectors \mathbf{E}_* and $\nabla\mu'_\pi$ as a sum of the reversible \mathbf{E}_*^r , $(\nabla\mu'_\pi)^r$ and irreversible \mathbf{E}_*^i , $(\nabla\mu'_\pi)^i$ components, namely

$$(2.11) \quad \mathbf{E}_* = \mathbf{E}_*^r + \mathbf{E}_*^i, \quad \nabla\mu'_\pi = (\nabla\mu'_\pi)^r + (\nabla\mu'_\pi)^i.$$

Then Eq. (2.9) becomes

$$(2.12) \quad \rho \frac{du}{dt} = \rho T \frac{ds}{dt} - \hat{\boldsymbol{\sigma}}_* : \frac{d\hat{\mathbf{e}}}{dt} + \rho \mathbf{E}_*^r \cdot \frac{d\mathbf{p}}{dt} + \rho \mu'_\pi \frac{d\rho_m}{dt} - \rho (\nabla\mu'_\pi)^r \cdot \frac{d\boldsymbol{\pi}_m}{dt} \\ + \rho \mathbf{E}_*^i \cdot \frac{d\mathbf{p}}{dt} - \rho (\nabla\mu'_\pi)^i \cdot \frac{d\boldsymbol{\pi}_m}{dt} + \mathbf{J}_{e*} \cdot \mathbf{E}_* - \mathbf{J}_q \cdot \frac{\nabla T}{T} - T\sigma_s - \mathbf{v} \cdot \left(\rho \frac{d\mathbf{v}}{dt} - \nabla \cdot \hat{\boldsymbol{\sigma}}_* - \mathbf{F}_e - \rho \mathbf{F}_* \right).$$

Introducing the generalized Helmholtz free energy $f = u - Ts - \mathbf{E}_*^r \cdot \mathbf{p} + (\nabla\mu'_\pi)^r \cdot \boldsymbol{\pi}_m$, and from the requirement that the balance equation of free energy is invariant with respect to translations, we obtain the following Gibbs equation, relation for the entropy production, and the momentum equation [11, 14]:

$$(2.13) \quad df = -sdT + \frac{1}{\rho} \hat{\boldsymbol{\sigma}}_* : d\hat{\mathbf{e}} - \mathbf{p} \cdot d\mathbf{E}_*^r + \mu'_\pi d\rho_m + \boldsymbol{\pi}_m \cdot d(\nabla\mu'_\pi)^r,$$

$$(2.14) \quad \sigma_s = \mathbf{J}_{e*} \cdot \frac{\mathbf{E}_*}{T} - \mathbf{J}_q \cdot \frac{\nabla T}{T^2} + \rho \frac{d\mathbf{p}}{dt} \cdot \frac{\mathbf{E}_*^i}{T} - \rho \frac{d\boldsymbol{\pi}_m}{dt} \cdot \frac{(\nabla\mu'_\pi)^i}{T},$$

$$(2.15) \quad \rho \frac{d\mathbf{v}}{dt} = \nabla \cdot \hat{\boldsymbol{\sigma}}_* + \mathbf{F}_e + \rho \mathbf{F}_*.$$

Note also that Eq. (2.13) is the generalization of the Gibbs relation for polarized thermoelastic nonferromagnetic medium with regard to irreversibility of both the local mass and electric charges displacements. This formula contains two additional terms, namely $\mu'_\pi d\rho_m$ and $\boldsymbol{\pi}_m \cdot d(\nabla\mu'_\pi)^r$, which describe the process of local mass displacement with respect to its irreversibility. Since $f = f(T, \hat{\mathbf{e}}, \rho_m, \mathbf{E}_*^r, (\nabla\mu'_\pi)^r)$ and the parameters T , ρ_m , \mathbf{E}_*^r , $(\nabla\mu'_\pi)^r$ and $\hat{\mathbf{e}}$ are independent, we obtain the following constitutive equations from the Gibbs relation

$$(2.16) \quad s = -\frac{\partial f}{\partial T}, \quad \hat{\boldsymbol{\sigma}}_* = \rho \frac{\partial f}{\partial \hat{\mathbf{e}}}, \quad \mu'_\pi = \frac{\partial f}{\partial \rho_m}, \quad \mathbf{p} = -\frac{\partial f}{\partial \mathbf{E}_*^r}, \quad \boldsymbol{\pi}_m = \frac{\partial f}{\partial (\nabla\mu'_\pi)^r}.$$

Let $\hat{\mathbf{e}} = 0$, $T = T_0$, $\rho_m = 0$, $\mathbf{E}_*^r = 0$, $(\nabla\mu'_\pi)^r = 0$, $\hat{\boldsymbol{\sigma}}_* = 0$, $s = s_0$, $\mu'_\pi = \mu'_{\pi 0}$, $\mathbf{p} = 0$, $\boldsymbol{\pi}_m = 0$ in the reference state. Then in the linear approximation for isotropic elastic medium Eq. (2.16) may be written in the form

$$(2.17) \quad s = s_0 + \frac{C_V}{T_0}(T - T_0) + \frac{1}{\rho_0} K \alpha_T e + \beta_{Tm} \rho_m,$$

$$(2.18) \quad \hat{\boldsymbol{\sigma}}_* = 2G\hat{\mathbf{e}} + \left[\left(K - \frac{2}{3}G \right) e - K\alpha_T (T - T_0) - K\alpha_m \rho_m \right] \hat{\mathbf{1}},$$

$$(2.19) \quad \mu'_\pi = \mu'_{\pi 0} + d_m \rho_m - \frac{1}{\rho_0} K \alpha_m e - \beta_{Tm} (T - T_0),$$

$$(2.20) \quad \mathbf{p} = \chi_E \mathbf{E}^r - \chi_{Em} (\nabla \mu'_\pi)^r, \quad \boldsymbol{\pi}_m = -\chi_m (\nabla \mu'_\pi)^r + \chi_{Em} \mathbf{E}^r,$$

where $e \equiv \hat{\mathbf{e}} : \hat{\mathbf{I}}$ is the first invariant of the strain tensor, K , G , d_m , C_V , α_T , α_m , β_{Tm} , χ_E , χ_m and χ_{Em} are the characteristics of material, s_0 , T_0 and $\mu'_{\pi 0}$ are the entropy, the temperature and the reduced potential μ'_π in the reference state, respectively.

Using the Onsager principle and Eq. (2.14) for the entropy production, one finds in the linear approximation [13], that

$$(2.21) \quad \mathbf{j}_l = \sum_{k=1}^4 L'_{lk} \mathbf{X}_k, \quad l = \overline{1, 4}.$$

Here \mathbf{j}_k , \mathbf{X}_k are the fluxes and thermodynamic forces,

$$\begin{aligned} \mathbf{j}_1 &= \mathbf{J}_q, & \mathbf{j}_2 &= \mathbf{J}_{e^*}, & \mathbf{j}_3 &= \rho \frac{d\mathbf{p}}{dt}, & \mathbf{j}_4 &= \rho \frac{d\boldsymbol{\pi}_m}{dt}, \\ \mathbf{X}_1 &= -T^{-2} \nabla T, & \mathbf{X}_2 &= -T^{-1} \mathbf{E}_*, & \mathbf{X}_3 &= T^{-1} \mathbf{E}_*^i, & \mathbf{X}_4 &= -T^{-1} (\nabla \mu'_\pi)^i \end{aligned}$$

and L'_{lk} ($k, l = \overline{1, 4}$) are kinetic coefficients. Taking into account the state equations (2.20) and formulas (2.11), we rewrite the kinetic equations (2.21) in the form

$$(2.22) \quad \begin{aligned} \mathbf{J}_q &= -L_{11} \nabla T + L_{12} \mathbf{E} + L_{13} \mathbf{p} - L_{14} \nabla \mu'_\pi - L_{15} \boldsymbol{\pi}_m, \\ \mathbf{J}_{e^*} &= -L_{21} \nabla T + L_{22} \mathbf{E} + L_{23} \mathbf{p} - L_{24} \nabla \mu'_\pi - L_{25} \boldsymbol{\pi}_m, \\ \rho \frac{d\mathbf{p}}{dt} &= -L_{31} \nabla T + L_{32} \mathbf{E} + L_{33} \mathbf{p} - L_{34} \nabla \mu'_\pi - L_{35} \boldsymbol{\pi}_m, \\ \rho \frac{d\boldsymbol{\pi}_m}{dt} &= -L_{41} \nabla T + L_{42} \mathbf{E} + L_{43} \mathbf{p} - L_{44} \nabla \mu'_\pi - L_{45} \boldsymbol{\pi}_m. \end{aligned}$$

Here

$$\begin{aligned} L_{k1} &= \frac{1}{T^2} L'_{k1}, & L_{k2} &= \frac{1}{T} (L'_{k2} - L'_{k3}), \\ L_{k3} &= \frac{1}{T} \frac{L'_{k3} \chi_m + L'_{k4} \chi_{Em}}{\chi_E \chi_m - \chi_{Em}^2}, \\ L_{k4} &= \frac{1}{T} L'_{k4}, & L_{k5} &= \frac{1}{T} \frac{L'_{k3} \chi_{Em} + L'_{k4} \chi_E}{\chi_E \chi_m - \chi_{Em}^2}, & k &= \overline{1, 4}. \end{aligned}$$

Note that two last equations of the set (2.22) are rheological relations for determination of vectors \mathbf{p} and $\boldsymbol{\pi}_m$. Unlike the processes of reversible displacements

of mass and electric charges for which the vectors of local displacement of mass $\boldsymbol{\pi}_m$ and polarization \mathbf{p} depend only on the $\nabla\mu'_\pi$ and \mathbf{E} , in this case, vectors $\boldsymbol{\pi}_m$ and \mathbf{p} depend not only on the vectors $\nabla\mu'_\pi$ and \mathbf{E} , but also on the temperature gradient.

The Maxwell's equations (2.1), the conservation laws of momentum (2.15), mass (2.5), induced mass (2.6), and entropy (2.7), the constitutive equations (2.17)–(2.19) and (2.22) with relations (2.4) and (2.10)₁, form a complete set of equations which describes the electro-thermo-mechanical processes in a polarized solid with regard to irreversibility of the local mass and charge displacements.

In the case where the local mass displacement and polarization are reversible processes, the following equation for the specific density of the induced mass ρ_m obtains in the linear approximation [15] the form

$$\Delta\rho_m - \lambda^2\rho_m = \frac{1}{d_m} \left[K \frac{\alpha_m}{\rho_0} \Delta(\nabla \cdot \mathbf{u}) + \beta_{Tm} \Delta T + \frac{\chi_{Em}}{\chi_m} \nabla \cdot \mathbf{E} \right],$$

where $\lambda^2 = (d_m\chi_m)^{-1} > 0$, because $d_m > 0$ and $\chi_m > 0$ [15]. The quantity λ^{-1} has the dimension of the length and describes the characteristic distances of the investigated problem.

By means of integration of this equation we can define ρ_m as a function of \mathbf{u} , T , \mathbf{E} . If we exclude the parameters which describe the process of local mass displacement from the basic set of Eqs. (2.1), (2.4)–(2.7), (2.10), (2.15), (2.17)–(2.19), and (2.22), we receive a set of spatially non-local integro-differential equations. The constitutive equations become functional (of spatial type). For example, for the entropy this relation becomes

$$s(\mathbf{r}) = s_0 + \frac{1}{T_0} \left(C_V - \frac{\beta_{Tm}^2}{T_0 d_m} \right) [T(\mathbf{r}) - T_0] + \frac{K}{\rho_0} \left(\alpha_t - \alpha_m \frac{\beta_{Tm}}{d_m} \right) e(\mathbf{r}) - \frac{\beta_{Tm}\lambda^2}{4\pi d_m} \int f(\mathbf{r} - \mathbf{r}') \left\{ K \frac{\alpha_m}{\rho_0} e(\mathbf{r}') + \beta_{Tm} [T(\mathbf{r}') - T_0] - d_m \chi_{Em} \nabla' \cdot \mathbf{E}(\mathbf{r}') \right\} d\mathbf{r}'.$$

Here $f(\mathbf{r}) = e^{-\lambda r}/r$, $r \equiv |\mathbf{r}|$.

3. Applications

Equations, obtained earlier by BURAK *et al.* [10], allow us to study the non-homogeneity of stationary distributions of mechanical stresses, electric potential, polarization etc., caused by the presence of surfaces in a dielectric body. The results for an infinite elastic polarized layer with traction-free surfaces are presented in [9, 10, 14, 16]. The effect of layer thickness on distributions of these

quantities and on the value of coupled surface charge density (the size effect) has been investigated in these papers too. Such a model adequately describes the high-frequency dispersion of elastic waves [17] as well as the anomalous dependence of capacity of a thin dielectric film on its thickness [16] observed experimentally by MEAD [18]. We note that similar results have also been obtained by MINDLIN [19], KAFADAR [3], YANG [20], YANG and YANG [4], YANG *et al.* [21] who used other known non-local theories of piezoelectricity.

The set of equations (2.1), (2.4)–(2.7), (2.10), (2.15), (2.17)–(2.19) and (2.22), which takes into account the irreversibility of processes of local mass and charge displacements, allows us to investigate the dynamics of the electromechanical fields, caused by formation of a body surface.

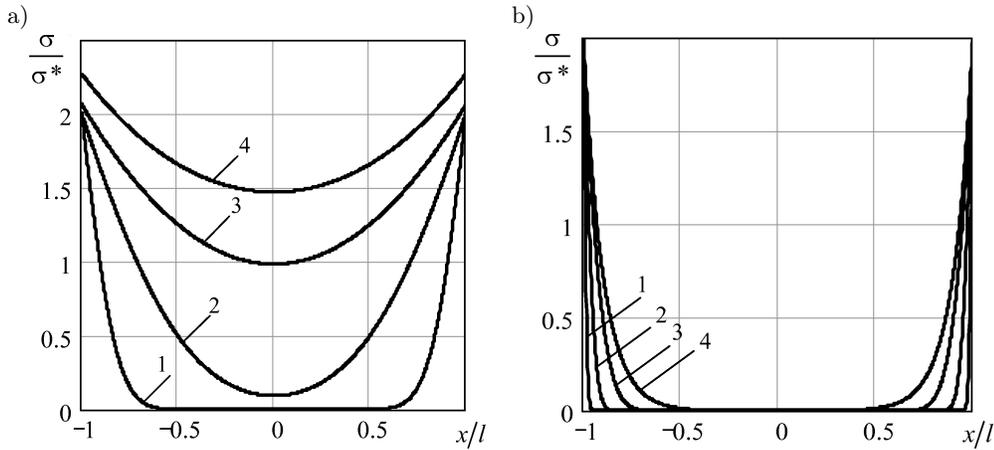


FIG. 1. The distribution of the stress σ/σ^* in layer for (a) $\xi = 1$ and (b) $\xi = 10$; $\tau = 10^{-2}$, 10^{-1} , $5 \cdot 10^{-1}$, 5 (curves 1–4 respectively).

Figure 1 illustrates the dynamics of dimensionless stresses σ/σ_* in thin ($\xi = 1$) and thick ($\xi = 10$) traction-free elastic layers (the region $|x| \leq l$) [14]. Here strains e_{yy} and e_{zz} are absent,

$$\sigma_{yy} = \sigma_{zz} \equiv \sigma, \quad \sigma^* = \mu'_{\pi 0} K G \frac{\alpha_m}{d_m} \left(K - \frac{2}{3} G \right)^{-1}, \quad \xi = l \lambda_*$$

is dimensionless thickness of the layer and

$$\lambda_* = \lambda \left[1 + \frac{K^2 \alpha_m^2}{\rho_0 d_m [K + 4G/3 - K^2 \alpha_m^2 (\rho_0 d_m)^{-1}]} \right]^{-1/2}.$$

For simplicity we consider an isotropic approximation and neglect the polarization process. As one can see, the stresses distributions can be considered as

stationary for $\tau \geq 5$, where $\tau = t/\tau_\pi$ is dimensionless time, $\tau_\pi = \rho\chi_m/L_{44}$ is the time of relaxation of the process of mass displacement. Layers of small thickness (thin films) are characterized by the overlay of the near-surface inhomogeneities (Fig. 1a) for $\tau \geq 5$, while there is a well-defined bulk region characterized by the uniform profile for thicker layers (Fig. 1b). Such an overlay of distributions of near-surface stresses causes the appearance of so-called disjoining pressure, which has been investigated earlier for liquid films [22, 23]. Disjoining pressure is defined as the difference between the pressures

$$p = - \int_{-1}^l \sigma_{xx}(\zeta) d\zeta$$

in a layer (with the clamped boundary) with and without the interface inhomogeneities [10]. For non-overlapping inhomogeneities, the surface tension is practically independent of the thickness (of a body), and the disjoining pressure vanishes. The sign of the surface stresses as well as of the disjoining pressure (see Figs. 1 and 2) is determined by the sign of σ^* . As $|\mu| > |\mu_\pi|$, and μ, K, G, d_m are positive, the characteristic stress σ^* is positive as well, provided $\alpha_m > 0$. In this case the surface stress causes stretching, thus worsening the layer's stretching resistance. Figure 2 illustrates the dynamics of the disjoining pressure in elastic layers of different thickness.

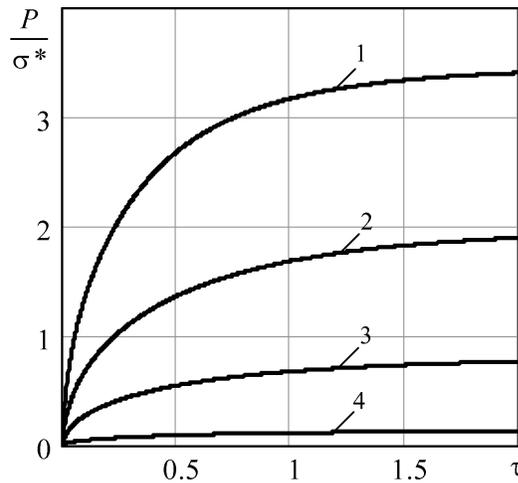


FIG. 2. Dynamics of disjoining pressure p/σ^* in layers with $\xi = 1, 2, 5, 30$ (curves 1–4, respectively).

In a layer of variable thickness, the lateral force $F_L^y = \partial\sigma_{yy}/\partial y$ (or $F_L^z = \partial\sigma_{zz}/\partial z$) acts along the layer. This force results from the difference between the stresses σ_{yy} (or σ_{zz}) taken in arbitrary cross-sections $y = \text{const}$ (or $z = \text{const}$)

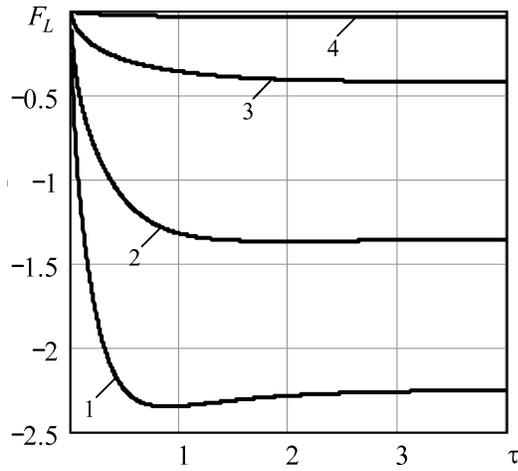


FIG. 3. Dynamics of lateral force F_L in layers with $\xi = 1, 1.5, 3, 10$ (curves 1–4, respectively).

of the layer. The value of the lateral force can be estimated by using the solution for a layer of constant (non-variable) thickness. Figure 3 illustrates the dynamics of the lateral force in thin (curves 1–3) and thick (curve 4) elastic layers. For $\alpha_m > 0$ the lateral force acts in the direction from wider to narrower parts of the layer, which means that it tends to smooth out the layer thickness, thus increasing its resistance. Note that the lateral force and the disjoining pressure are thickness-dependent (size effects). Although for thick films the lateral forces as well as the disjoining pressure are negligible, their effect is significant for thin films/layers. We stress therefore that it is very important to take them into account in case of thin films, as they can have a significant influence on the strength and resistance of such films (as in liquid films [22, 23]).

4. Conclusions

The complete set of equations of non-local (gradient) theory for description of electro-magneto-thermo-mechanical processes in polarized nonferromagnetic solids, which takes into account the irreversibility of processes of local mass and electric charge displacements, has been obtained. It is shown that the theory allows us to describe correctly such phenomena as the Meed anomaly, high-frequency dispersion of elastic waves, as well as the near-surface inhomogeneities of the stress-strained state, polarization and electric potential. This theory allows us to study the surface charge, disjoining pressure and lateral forces in layers of variable thickness. The dynamics of electromechanical fields, caused by formation of a new surface, as well as the size effect can also be studied.

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