Deformation of orthotropic porous elastic bars under lateral loading

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THE PRESENT PAPER is concerned with the linear theory of inhomogeneous and orthotropic, porous elastic cylinders. The work is motivated by the recent interest in using of orthotropic porous elastic solid as a model for bones and for various engineering materials. A generalization of Saint–Venant's problem to the case when the cylinder is subjected to body forces and to surface forces on the lateral boundary. The three-dimensional problem is reduced to the study of plane problems. The method is applied to investigate the deformation of a uniformly loaded circular cylinder.

Key words: porous elastic solids, orthotropic bodies, loaded cylinders, inhomogeneous bodies.

1. Introduction

THE MECHANICS OF POROUS MATERIALS is of fundamental interest for the fields of soil mechanics, powder technology and mechanics of bone. In recent years the behaviour of porous elastic media has been studied in various papers (see, e.g., [1–3], and references therein). The mechanical behaviour of solids with voids (porous solids) or of solids containing microscopic components, cannot be described by means of the classical theory of elasticity. In [4, 5] NUNZIATO and COWIN established a theory of elastic materials with voids for the treatment of porous solids. This theory introduces an additional degree of kinematical freedom. Very much has been written in the last years on the theories of elastic solids with inner structure, in which the deformation is described not only by the usual vector displacement field, but by other vector or tensor fields as well. The origin of the theories of continua with microstructure goes back to the papers of ERICKSEN and TRUESDELL [6], MINDLIN [7], ERINGEN and SUHUBI [8] and GREEN and RIVLIN [9]. MINDLIN [7] formulated a theory of an elastic solid which has some properties of a crystal lattice as a result of inclusion of the idea of a unit cell. Mindlin begins with the general concept of an elastic continuum,

each material point of which is a deformable medium. Independently, a theory of microelastic continuum was published by Eringen and Suhubi [8], and a theory of multipolar continuum mechanics by Green and Rivlin [9]. Green [10] has established the relation of the theory of multipolar mechanics to the other theories. Much of the theoretical progress is discussed in the books of Kunin [11], Eringen [12] and Mariano [13]. In the theory developed by Mindlin [7], each material point is constrained to deform homogeneously. In this theory, the degrees of freedom for each material point are twelve: three translations, u_i, and nine microdeformations, ψ_{ij}. The theory of bodies with microstructure introduces generalized body forces, surface forces and generalized stresses (or dipolar body forces, dipolar surface forces and stresses [9]). The physical significance of the new forces and stresses has been discussed in the papers [7, 9, 14]. The domain of applicability of the theory of continua with microstructure has been investigated in [11].

A special class of bodies with microstructure [12] is characterized by a microdeformation tensor of the form $\psi_{ij} = \nu\delta_{ij}$, where ν is the microstretch function (or microdilatation function) and δ_{ij} is the Kronecker delta. In this case the material points undergo a uniform microdilatation (a breathing motion). The linear equations which describe the behavior of an elastic body with this kind of microstructure, coincide with the equations of the linear theory of elastic materials with voids, established by Cowin and Nunziato [5] (cf. Eringen [12, 15]). In what follows we shall refer to this model as a porous elastic continuum. The linear theory of elastic materials with voids is the simplest theory of elastic bodies that takes into account the inner structure of the material.

The linear theory of porous elastic bodies has received considerable attention (see, e.g., [16, 17] and references therein). In particular, the deformation of elastic cylinders has been a subject of intensive study. The solution of pure bending of a cylinder made of an isotropic and homogeneous porous elastic material has been investigated by Cowin and Nunziato [5]. The result has been extended to inhomogeneous porous elastic bodies in [16]. Dell'Isola and Batra [18] have investigated the Saint–Venant problem for isotropic and homogeneous porous elastic bars. In the present paper we study the deformation of orthotropic porous elastic bars subjected to resultant forces and moments on the ends, to body forces, and to surface forces on the lateral boundary. The solution is new even for homogeneous and isotropic porous elastic solids. The work is motivated by the recent interest in using of the orthotropic elastic solid as a model for bones (see, e.g., [19–22] and references therein), and for various engineering materials (see., e.g., [23–26]). In biomechanics, the bone tissue is typically described as an orthotropic elastic material. Earlier studies reported that bone tissue could be assumed to be transversely isotropic. Subsequent investigations have confirmed that in reality, orthotropy is the case that most closely describes mechanical

anisotropy of a bone [20, 22]. We note that the cancellous bone is considered as a porous material.

In Sec. 2 we present the equations of the linear theory of orthotropic porous elastic bodies and formulate the problem of loaded cylinders. Section 3 is devoted to the plane strain problem for inhomogeneous and orthotropic, porous elastic solids. We introduce three special plane strain problems characterized by external data, which depend only on the constitutive coefficients and the domain under consideration. These plane strain problems shall be used in the study of the deformation of loaded bars. In Sec. 4 we present a solution of the problem of loaded cylinders, when the body forces and surface forces on the lateral surface are independent of the axial coordinate. In the classical elasticity, this problem was initiated by ALMANSI [27] and MICHELL [28] and it is known as Almansi–Michell problem (see, e.g., [29, 30] and references therein). The method presented in this paper is used in Sec. 5 to study the deformation of a loaded circular cylinder.

2. Preliminaries. Statement of the problem

Throughout this paper we denote by B the interior of a right cylinder of length ℓ with open cross-section Σ and lateral boundary Π. Let L be the boundary of Σ. We call ∂B the boundary of B, and denote by n_i the components of the outward unit normal of ∂B. A rectangular cartesian coordinate system Ox_k ($k = 1, 2, 3$) is used. The rectangular cartesian coordinate frame is chosen such that the x_3-axis is parallel to the generators of B and x_1Ox_2 plane contains one of terminal cross-sections. We denote by $\Sigma^{(0)}$ and $\Sigma^{(\ell)}$, respectively, the cross-section located at $x_3 = 0$ and $x_3 = \ell$. We shall employ the usual summation and differentiation conventions: Greek subscripts are understood to range over integers $(1, 2)$ whereas Latin subscripts (unless otherwise specified) to the range $(1, 2, 3)$; summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding cartesian coordinate.

In this paper we consider the equilibrium theory of porous elastic materials. Let u_i be the components of the displacement vector field over B. The linear strain measure e_{ij} is given by

$$(2.1) \qquad e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}).$$

We denote by t_{ij} the stress tensor, and by h_i the equilibrated stress vector. The surface force and the equilibrated surface force at a regular point of ∂B, are given by

$$t_i = t_{ji}n_j, \qquad h = h_j n_j,$$

respectively. The equilibrium equations of porous continua are (cf. [5])

$$(2.2) \qquad t_{ji,j} + f_i = 0, \qquad h_{j,j} + g + q = 0,$$

where f_i is the body force, q is the extrinsic equilibrated body force, and g is the intrinsic equilibrated body force. The constitutive equations of orthotropic porous elastic solids are

$$t_{11} = c_{11}e_{11} + c_{12}e_{22} + c_{13}e_{33} + \beta_1\varphi,$$

$$t_{22} = c_{12}e_{11} + c_{22}e_{22} + c_{23}e_{33} + \beta_2\varphi,$$

$$t_{33} = c_{13}e_{11} + c_{23}e_{22} + c_{33}e_{33} + \beta_3\varphi,$$

$$(2.3)$$

$$t_{23} = 2c_{44}e_{23}, \qquad t_{31} = 2c_{55}e_{13}, \qquad t_{12} = 2c_{66}e_{12},$$

$$h_1 = \alpha_1\varphi_{,1}, \qquad h_2 = \alpha_2\varphi_{,2}, \qquad h_3 = \alpha_3\varphi_{,3},$$

$$g = -\beta_1 e_{11} - \beta_2 e_{22} - \beta_3 e_{33} - \xi\varphi,$$

where φ is the porosity function and c_{rs} $(r,s = 1,2,\ldots,6)$, α_k, β_k and ξ are constitutive coefficients.

The cylinder is supposed to be subjected to surface forces on the lateral surface and to appropriate global conditions over its ends. On the lateral surface of the cylinder we have the conditions

$$(2.4) \qquad t_{\alpha i}n_\alpha = \widetilde{t}_i, \qquad h_\alpha n_\alpha = \widetilde{h} \qquad \text{on } \Pi,$$

where $\widetilde{t}_i$ and $\widetilde{h}$ are the prescribed, piecewise regular functions. We assume that the body loads and the forces applied on the lateral surface are independent of the axial coordinate x_3. Let $\mathbf{R}$ and $\mathbf{M}$ be the prescribed vectors representing the resultant force and the resultant moment about O of the tractions acting on $\Sigma^{(0)}$. On $\Sigma^{(\ell)}$ there are tractions applied so as to satisfy the equilibrium conditions of the body. Thus, for $x_3 = 0$ we have the following conditions:

$$(2.5) \qquad \int_{\Sigma^{(0)}} t_{\alpha 3}\,da = -R_\alpha,$$

$$(2.6) \qquad \int_{\Sigma^{(0)}} t_{33}\,da = -R_3,$$

$$(2.7) \qquad \int_{\Sigma^{(0)}} x_\alpha t_{33}\,da = \varepsilon_{\alpha\beta 3}M_\beta,$$

$$(2.8) \qquad \int_{\Sigma^{(0)}} \varepsilon_{\alpha\beta 3}x_\alpha t_{\beta 3}\,da = -M_3,$$

where ε_{ijk} is the alternating symbol. We note that there is no contribution of the equilibrated surface force in the resultant force and resultant moment (cf. [31]).

In what follows we assume that the constitutive coefficients are functions independent of the axial coordinate,

$$
(2.9) \qquad \begin{aligned} &c_{mn} = \widehat{c}_{mn}(x_1, x_2), \qquad \alpha_k = \widehat{\alpha}_k(x_1, x_2), \qquad \beta_k = \widehat{\beta}_k(x_1, x_2), \\ &\xi = \widehat{\xi}(x_1, x_2), \qquad (x_1, x_2) \in \Sigma^{(0)}. \end{aligned}
$$

Throughout this paper we suppose that the cross-section is C^∞-smooth. Moreover, we assume that the constitutive coefficients belong to $C^\infty(\overline{\Sigma}^{(0)})$. We have chosen these hypotheses in order to emphasize the method of solution of the considered problem.

The problem of Almansi–Michell consists in finding of the functions u_i and φ which satisfy the equations (2.1)–(2.3) on B, the conditions (2.4) on Π, and the conditions (2.5)–(2.8) on $\Sigma^{(0)}$, when the body loads and surface forces are independent of the axial coordinate.

The elastic potential W is defined by

$$
2W = c_{11}e_{11}^2 + c_{22}e_{22}^2 + c_{33}e_{33}^2 + 4c_{44}e_{23}^2 + 4c_{55}e_{31}^2 + 4c_{66}e_{12}^2
$$
$$
+ 2c_{12}e_{11}e_{22} + 2c_{13}e_{11}e_{33} + 2c_{23}e_{22}e_{33} + \xi\varphi^2 + 2\beta_1 e_{11}\varphi
$$
$$
+ 2\beta_2 e_{22}\varphi + 2\beta_3 e_{33}\varphi + \alpha_1(\varphi_{,1})^2 + \alpha_2(\varphi_{,2})^2 + \alpha_3(\varphi_{,3})^2.
$$

Throughout this paper we assume that the elastic potential is a positive definite quadratic form in the variables e_{ij}, φ and $\varphi_{,j}$.

3. Plane strain problems

We recall that in the classical elasticity, solving of Saint–Venant's problem is reduced to the study of two-dimensional problems. In this paper we shall use three special problems of plane strain of the cylinder B. A state of plane strain, parallel to the x_1, x_2-plane, is characterized by

$$
(3.1) \qquad u_\alpha = u_\alpha(x_1, x_2), \qquad u_3 = 0, \qquad \varphi = \varphi(x_1, x_2), \qquad (x_1, x_2) \in \Sigma^{(0)}.
$$

These restrictions, in conjunction with the strain-displacement relations (2.1) and the constitutive equations (2.3), imply that e_{ij}, t_{ij}, h_i and g are all independent of x_3. The non-zero components of the strain tensor are given by

$$
(3.2) \qquad e_{\alpha\beta} = \frac{1}{2}(u_{\alpha,\beta} + u_{\beta,\alpha}).
$$

The constitutive equations imply that $t_{\alpha 3} = 0, h_3 = 0$ and

$$t_{11} = c_{11}e_{11} + c_{12}e_{22} + \beta_1\varphi,$$
$$t_{22} = c_{12}e_{11} + c_{22}e_{22} + \beta_2\varphi,$$
$$(3.3) \qquad t_{12} = 2c_{66}e_{12},$$
$$h_1 = \alpha_1\varphi_{,1}, \qquad h_2 = \alpha_2\varphi_{,2},$$
$$g = -\beta_1 e_{11} - \beta_2 e_{22} - \xi\varphi.$$

The equilibrium equations reduce to

$$(3.4) \qquad t_{\beta\alpha,\beta} + f_\alpha = 0, \qquad h_{\alpha,\alpha} + g + q = 0,$$

on $\Sigma^{(0)}$. The conditions on the lateral surface become

$$(3.5) \qquad t_{\beta\alpha}n_\beta = \widetilde{t}_\alpha, \qquad h_\alpha n_\alpha = \widetilde{h} \qquad \text{on } L.$$

We assume that $f_\alpha, q, \widetilde{t}_\alpha$ and $\widetilde{h}$ are functions of class C^∞.

The plane strain problem consists in finding the displacement u_α and the porosity φ which satisfy the Eqs. (3.2)–(3.4) on $\Sigma^{(0)}$ and the boundary conditions (3.5) on L. The necessary and sufficient conditions for the existence of solutions belonging to $C^\infty(\Sigma^{(0)})$ are (cf. [32, 33])

$$(3.6) \qquad \int_{\Sigma^{(0)}} f_\alpha da + \int_L \widetilde{t}_\alpha ds = 0, \qquad \int_{\Sigma^{(0)}} \varepsilon_{\alpha\beta 3}x_\alpha f_\beta da + \int_L \varepsilon_{\alpha\beta 3}x_\alpha \widetilde{t}_\beta ds = 0.$$

In what follows we introduce three plane strain problems, in which the body loads and the surface tractions on the lateral boundary depend on the constitutive coefficients of the material. We denote these problems by $P^{(k)}$ ($k = 1, 2, 3$). The problem $P^{(k)}$ is characterized by the equations of equilibrium

$$(3.7) \qquad t^{(k)}_{\beta\alpha,\beta} + f^{(k)}_\alpha = 0, \qquad h^{(k)}_{\alpha,\alpha} + g^{(k)} + q^{(k)} = 0,$$

the constitutive equations

$$t^{(k)}_{11} = c_{11}e^{(k)}_{11} + c_{12}e^{(k)}_{22} + \beta_1\varphi^{(k)},$$
$$t^{(k)}_{22} = c_{12}e^{(k)}_{11} + c_{22}e^{(k)}_{22} + \beta_2\varphi^{(k)},$$
$$(3.8) \qquad t^{(k)}_{12} = 2c_{66}e^{(k)}_{12},$$
$$h^{(k)}_1 = \alpha_1\varphi^{(k)}_{,1}, \qquad h^{(k)}_2 = \alpha_2\varphi^{(k)}_{,2},$$
$$g^{(k)} = -\beta_1 e^{(k)}_{11} - \beta_2 e^{(k)}_{22} - \xi\varphi^{(k)},$$

and the geometrical equations

$$(3.9) \qquad 2e^{(k)}_{\alpha\beta} = u^{(k)}_{\alpha,\beta} + u^{(k)}_{\beta,\alpha},$$

on $\Sigma^{(0)}$, and the boundary conditions

$$(3.10) \qquad t^{(k)}_{\beta\alpha}n_\beta = \widetilde{t}^{(k)}_\alpha, \qquad h^{(k)}_\alpha n_\alpha = \widetilde{h}^{(k)} \qquad \text{on } L,$$

where the body loads and the surface forces are given by

$$
\begin{aligned}
& f_1^{(1)} = (c_{13}x_1)_{,1}, && f_2^{(1)} = (c_{23}x_1)_{,2}, && q^{(1)} = -\beta_3 x_1, \\
& f_1^{(2)} = (c_{13}x_2)_{,1}, && f_2^{(2)} = (c_{23}x_2)_{,2}, && q^{(2)} = -\beta_3 x_2, \\
(3.11) \quad & f_1^{(3)} = c_{13,1}, && f_2^{(3)} = c_{23,2}, && q^{(3)} = -\beta_3, \\
& \widetilde{t}_1^{(1)} = -c_{13}x_1 n_1, && \widetilde{t}_2^{(1)} = -c_{23}x_1 n_2, && \widetilde{h}^{(1)} = 0, \\
& \widetilde{t}_1^{(2)} = -c_{13}x_2 n_1, && \widetilde{t}_2^{(2)} = -c_{23}x_2 n_2, && \widetilde{h}^{(2)} = 0, \\
& \widetilde{t}_1^{(3)} = -c_{13}n_1, && \widetilde{t}_2^{(3)} = -c_{23}n_2, && \widetilde{h}^{(3)} = 0 \qquad (k=1,2,3).
\end{aligned}
$$

We can see that the necessary and sufficient conditions (3.6) for the existence of the solutions are satisfied for each problem $P^{(k)}$ $(k = 1, 2, 3)$.

4. Almansi–Michell problem

In this section we present the solution of the Almansi–Michell problem formulated in Sec. 2. We seek the solution of the problem of loaded cylinders in the form

$$
u_\alpha = -\frac{1}{2}a_\alpha x_3^2 - \frac{1}{6}b_\alpha x_3^3 - \frac{1}{24}c_\alpha x_3^4 + \varepsilon_{3\beta\alpha}\left(\tau_1 x_3 + \frac{1}{2}\tau_2 x_3^2\right)x_\beta
$$

$$
+ \sum_{s=1}^{3}\left(a_s + b_s x_3 + \frac{1}{2}c_s x_3^2\right)u_\alpha^{(s)} + v_\alpha(x_1, x_2),
$$

$$(4.1) \quad u_3 = (a_1 x_1 + a_2 x_2 + a_3)x_3 + \frac{1}{2}(b_1 x_1 + b_2 x_2 + b_3)x_3^2$$

$$
+ \frac{1}{6}(c_1 x_1 + c_2 x_2 + c_3)x_3^3 + (\tau_1 + x_3 \tau_2)\varPhi + \varPsi(x_1, x_2) + x_3\chi(x_1, x_2),
$$

$$
\varphi = \sum_{s=1}^{3}\left(a_s + b_s x_3 + \frac{1}{2}c_s x_3^2\right)\varphi^{(s)} + w(x_1, x_2),
$$

where $(u_\alpha^{(k)}, \varphi^{(k)})$ is the solution of the problem $P^{(k)}$ $(k = 1, 2, 3)$, introduced in Sec. 3, $v_\alpha, \varPhi, \varPsi, \chi$ and w are unknown functions independent of x_3, and a_k, b_k, c_k, τ_1 and τ_2 are unknown constants. Let us prove that we can determine the functions $v_\alpha, \varPhi, \varPsi, \chi$ and w, and the constants a_j, b_j, c_j, τ_1 and τ_2, such that the functions (4.1) will be a solution of the Eqs. (2.1)–(2.3) with conditions (2.4)–(2.8).

In view of the geometrical equations (2.1) and (4.1) we find that

$$e_{\alpha\beta} = \sum_{s=1}^{3}\left(a_s + b_s x_3 + \frac{1}{2}c_s x_3^2\right)e_{\alpha\beta}^{(s)} + \gamma_{\alpha\beta},$$

(4.2) $\quad 2e_{\alpha 3} = (\tau_1 + \tau_2 x_3)(\Phi_{,\alpha} + \varepsilon_{3\beta\alpha}x_\beta) + \Psi_{,\alpha} + x_3\chi_{,\alpha} + \sum_{s=1}^{3}(b_s + c_s x_3)u_\alpha^{(s)},$

$$e_{33} = a_1 x_1 + a_2 x_2 + a_3 + (b_1 x_1 + b_2 x_2 + b_3)x_3$$
$$+ \frac{1}{2}(c_1 x_1 + c_2 x_2 + c_3)x_3^2 + \tau_2\Phi + \chi,$$

where $e_{\alpha\beta}^{(s)}$, $(s = 1, 2, 3)$, are given by (3.8) and $\gamma_{\alpha\beta}$ are defined by

(4.3) $$\gamma_{\alpha\beta} = \frac{1}{2}(v_{\alpha,\beta} + v_{\beta,\alpha}).$$

From (4.2) and the constitutive equations (2.3) we obtain

$$t_{11} = c_{13}\left[a_1 x_1 + a_2 x_2 + a_3 + (b_1 x_1 + b_2 x_2 + b_3)x_3 + \frac{1}{2}(c_1 x_1 + c_2 x_2 + c_3)x_3^2\right]$$
$$+ \sum_{s=1}^{3}\left(a_s + b_s x_3 + \frac{1}{2}c_s x_3^2\right)t_{11}^{(s)} + \sigma_{11} + c_{13}(\tau_2\Phi + \chi),$$

$$t_{22} = c_{23}\left[a_1 x_1 + a_2 x_2 + a_3 + (b_1 x_1 + b_2 x_2 + b_3)x_3 + \frac{1}{2}(c_1 x_1 + c_2 x_2 + c_3)x_3^2\right]$$
$$+ \sum_{s=1}^{3}\left(a_s + b_s x_3 + \frac{1}{2}c_s x_3^2\right)t_{22}^{(s)} + \sigma_{22} + c_{23}(\tau_2\Phi + \chi),$$

$$t_{33} = c_{33}\left[a_1 x_1 + a_2 x_2 + a_3 + (b_1 x_1 + b_2 x_2 + b_3)x_3 + \frac{1}{2}(c_1 x_1 + c_2 x_2 + c_3)x_3^2\right]$$

(4.4)
$$+ \sum_{s=1}^{3}\left(a_s + b_s x_3 + \frac{1}{2}c_s x_3^2\right)(c_{13}e_{11}^{(s)} + c_{23}e_{22}^{(s)} + \beta_3\varphi^{(s)})$$
$$+ c_{13}\gamma_{11} + c_{23}\gamma_{22} + \beta_3 w + c_{33}(\tau_2\Phi + \chi),$$

$$t_{12} = \sum_{s=1}^{3}\left(a_s + b_s x_3 + \frac{1}{2}c_s x_3^2\right)t_{12}^{(s)} + \sigma_{12},$$

$$t_{23} = c_{44}\left[(\tau_1 + \tau_2 x_3)(\Phi_{,2} + x_1) + \Psi_{,2} + x_3\chi_{,2} + \sum_{s=1}^{3}(b_s + c_s x_3)u_2^{(s)}\right],$$

$$t_{13} = c_{55}\left[(\tau_1 + \tau_2 x_3)(\Phi_{,1} - x_2) + \Psi_{,1} + x_3\chi_{,1} + \sum_{s=1}^{3}(b_s + c_s x_3)u_1^{(s)}\right],$$

$$h_\alpha = \sum_{s=1}^{3} \left(a_s + b_s x_3 + \frac{1}{2} c_s x_3^2 \right) h_\alpha^{(s)} + H_\alpha,$$

$$(4.4) \quad h_3 = \alpha_3 \sum_{s=1}^{3} (b_s + c_s x_3) \varphi^{(s)},$$
[cont.]

$$g = \sum_{s=1}^{3} \left(a_s + b_s x_3 + \frac{1}{2} c_s x_3^2 \right) g^{(s)} - \beta_3 \Big[a_1 x_1 + a_2 x_2 + a_3$$

$$+ (b_1 x_1 + b_2 x_2 + b_3) x_3 + \frac{1}{2} (c_1 x_1 + c_2 x_2 + c_3) x_3^2 + \tau_2 \Phi + \chi \Big] + G,$$

where $t_{\alpha\beta}^{(s)}$, $h_\alpha^{(s)}$ and $g^{(s)}$ are given by (3.8), and we have used the notations

$$\sigma_{11} = c_{11} \gamma_{11} + c_{12} \gamma_{22} + \beta_1 w,$$

$$\sigma_{22} = c_{12} \gamma_{11} + c_{22} \gamma_{22} + \beta_2 w,$$

$$(4.5) \quad \sigma_{12} = 2 c_{66} \gamma_{12},$$

$$H_1 = \alpha_1 w_{,1}, \qquad H_2 = \alpha_2 w_{,2},$$

$$G = -\beta_1 \gamma_{11} - \beta_2 \gamma_{22} - \xi w.$$

It follows from Eqs. (3.7), (3.11) and (4.4) that the equilibrium equations (2.2) are satisfied if $v_\alpha, \Phi, \Psi, \chi$ and w satisfy the equations

$$(4.6) \qquad \sigma_{\beta\alpha,\beta} + F_\alpha = 0, \qquad H_{\alpha,\alpha} + G + Q = 0,$$

$$(4.7) \qquad (c_{55} \Phi_{,1})_{,1} + (c_{44} \Phi_{,2})_{,2} = \Lambda,$$

$$(4.8) \qquad (c_{55} \Psi_{,1})_{,1} + (c_{44} \Psi_{,2})_{,2} = -f_3 - c_{33}(b_1 x_1 + b_2 x_2 + b_3)$$

$$- \sum_{s=1}^{3} b_s [(c_{55} u_1^{(s)})_{,1} + (c_{44} u_2^{(s)})_{,2} + c_{13} e_{11}^{(s)} + c_{23} e_{22}^{(s)} + \beta_3 \varphi^{(s)}],$$

$$(4.9) \qquad (c_{55} \chi_{,1})_{,1} + (c_{44} \chi_{,2})_{,2} = -c_{33}(c_1 x_1 + c_2 x_2 + c_3)$$

$$- \sum_{s=1}^{3} c_s [(c_{55} u_1^{(s)})_{,1} + (c_{44} u_2^{(s)})_{,2} + c_{13} e_{11}^{(s)} + c_{23} e_{22}^{(s)} + \beta_3 \varphi^{(s)}],$$

on $\Sigma^{(0)}$. In (4.6) we have used the notations

$$F_1 = f_1 + [c_{13}(\tau_2\Phi + \chi)]_{,1} + c_{55}\left[\tau_2(\Phi_{,1} - x_2) + \chi_{,1} + \sum_{s=1}^{3} c_s u_1^{(s)}\right],$$

(4.10)
$$F_2 = f_2 + [c_{23}(\tau_2\Phi + \chi)]_{,2} + c_{44}\left[\tau_2(\Phi_{,2} + x_1) + \chi_{,2} + \sum_{s=1}^{3} c_s u_2^{(s)}\right],$$

$$Q = q + \alpha_3 \sum_{s=1}^{3} c_s \varphi^{(s)} - \beta_3(\tau_2\Phi + \chi),$$

$$\Lambda = (c_{55}x_2)_{,1} - (c_{44}x_1)_{,2}.$$

The boundary conditions (2.4) are satisfied if the following conditions hold on L,

(4.11)
$$\sigma_{\beta\alpha}n_\beta = p_\alpha, \qquad H_\alpha n_\alpha = \widetilde{h},$$

(4.12)
$$c_{55}\Phi_{,1}n_1 + c_{44}\Phi_{,2}n_2 = m,$$

(4.13)
$$c_{55}\Psi_{,1}n_1 + c_{44}\Psi_{,2}n_2 = \widetilde{t}_3 - \sum_{s=1}^{3} b_s(c_{55}u_1^{(s)}n_1 + c_{44}u_2^{(s)}n_2),$$

(4.14)
$$c_{55}\chi_{,1}n_1 + c_{44}\chi_{,2}n_2 = -\sum_{s=1}^{3} c_s(c_{55}u_1^{(s)}n_1 + c_{44}u_2^{(s)}n_2),$$

where

(4.15)
$$p_1 = \widetilde{t}_1 - c_{13}(\tau_2\Phi + \chi)n_1, \qquad p_2 = \widetilde{t}_2 - c_{23}(\tau_2\Phi + \chi)n_2,$$
$$m = c_{55}x_2n_1 - c_{44}x_1n_2.$$

The function Φ satisfies the boundary-value problem (4.7), (4.12). The necessary and sufficient condition to solve this problem is [32]

(4.16)
$$\int_{\Sigma^{(0)}} \Lambda da = \int_L m ds.$$

With the help of the divergence theorem, Eqs. (4.10) and (4.12), we see that the condition (4.16) is satisfied. Function Φ is the torsion function in context of the classical elasticity (see, e.g., [30, 34]).

From (4.3), (4.5), (4.6) and (4.11) we conclude that v_α and w satisfy the equations and the boundary conditions of a plane strain problem. The necessary and sufficient conditions to solve this problem are

(4.17)
$$\int_{\Sigma^{(0)}} F_\alpha da + \int_L p_\alpha ds = 0, \qquad \int_{\Sigma^{(0)}} \varepsilon_{\alpha\beta3}x_\alpha F_\beta da + \int_L \varepsilon_{\alpha\beta3}x_\alpha p_\beta ds = 0.$$

By the divergence theorem, Eqs. (4.4), (4.10) and (4.15), we find that

$$(4.18) \qquad \int_{\Sigma^{(0)}} F_\alpha da + \int_L p_\alpha ds = \int_{\Sigma^{(0)}} f_\alpha da + \int_L \widetilde{t}_\alpha ds + \int_{\Sigma^{(0)}} t_{\alpha 3,3} da,$$

$$\int_{\Sigma^{(0)}} \varepsilon_{\alpha\beta 3} x_\alpha F_\beta da + \int_L \varepsilon_{\alpha\beta 3} x_\alpha p_\beta ds = \int_{\Sigma^{(0)}} \varepsilon_{\alpha\beta 3} x_\alpha f_\beta da$$

$$+ \int_L \varepsilon_{\alpha\beta 3} x_\alpha \widetilde{t}_\beta ds + \int_{\Sigma^{(0)}} \varepsilon_{\alpha\beta 3} x_\alpha t_{\beta 3,3} da.$$

In view of the equilibrium equations, and taking into account that the body forces are independent of the axial coordinate, we obtain

$$t_{\alpha 3,3} = t_{\alpha 3,3} + x_\alpha(t_{j3,j} + f_3)_{,3} = (t_{\alpha 3} + x_\alpha t_{j3,j})_{,3}$$

$$= [(x_\alpha t_{\beta 3})_{,\beta} + x_\alpha t_{33,3}]_{,3} = (x_\alpha t_{\beta 3,3})_{,\beta} + x_\alpha t_{33,33}.$$

By using the conditions on the lateral surface we get

$$(4.19) \qquad \int_{\Sigma^{(0)}} t_{\alpha 3,3} da = \int_{\Sigma^{(0)}} x_\alpha t_{33,33} da.$$

It follows from (4.18), (4.19) and (4.4) that

$$(4.20) \qquad \int_{\Sigma^{(0)}} F_\alpha da + \int_L p_\alpha ds = \int_{\Sigma^{(0)}} f_\alpha da + \int_L \widetilde{t}_\alpha ds + D_{\alpha j} c_j,$$

where

$$(4.21) \qquad \begin{aligned} D_{\alpha\beta} &= \int_{\Sigma^{(0)}} x_\alpha [c_{33} x_\beta + c_{13} e_{11}^{(\beta)} + c_{23} e_{22}^{(\beta)} + \beta_3 \varphi^{(\beta)}] da, \\ D_{\alpha 3} &= \int_{\Sigma^{(0)}} x_\alpha [c_{33} + c_{13} e_{11}^{(3)} + c_{23} e_{22}^{(3)} + \beta_3 \varphi^{(3)}] da. \end{aligned}$$

In view of Eq. (4.20), the first two conditions from (4.17) reduce to

$$(4.22) \qquad D_{\alpha j} c_j = - \int_{\Sigma^{(0)}} f_\alpha da - \int_L \widetilde{t}_\alpha ds.$$

Next, we consider the boundary value problem (4.9), (4.14) for the unknown function χ. The necessary and sufficient condition to solve this problem can be expressed as

(4.23) $$D_{3j}c_j = 0,$$

where we have used the notations

(4.24)
$$D_{3\alpha} = \int_{\Sigma^{(0)}} \left(c_{33}x_\alpha + c_{13}e_{11}^{(\alpha)} + c_{23}e_{22}^{(\alpha)} + \beta_3\varphi^{(\alpha)} \right) da,$$
$$D_{33} = \int_{\Sigma^{(0)}} \left(c_{33} + c_{13}e_{11}^{(3)} + c_{23}e_{22}^{(3)} + \beta_3\varphi^{(3)} \right) da.$$

We note that the constants D_{ij} can be determined after solving of the problems $P^{(k)}$, $(k = 1, 2, 3)$. Following [33], the constants D_{ij} have the properties

(4.25) $$D_{ij} = D_{ji}, \qquad \det(D_{ij}) \neq 0.$$

We conclude that we can determine the constants c_1, c_2 and c_3 so that the conditions (4.22) and (4.23) will be satisfied.

Let us consider now the boundary value problem (4.8), (4.13) for the function Ψ. The necessary and sufficient condition for the existence of the solution of this problem can be expressed in the form

(4.26) $$D_{3j}b_j = -\int_{\Sigma^{(0)}} f_3 da - \int_L \widetilde{t}_3 ds.$$

Next, we consider the end conditions (2.5). First, we note that

(4.27)
$$\int_{\Sigma^{(0)}} t_{\alpha 3} da = \int_{\Sigma^{(0)}} [t_{\alpha 3} + x_\alpha(t_{j3,j} + f_3)] da$$
$$= \int_{\Sigma^{(0)}} x_\alpha t_{33,3} da + \int_{\Sigma^{(0)}} x_\alpha f_3 da + \int_L x_\alpha \widetilde{t}_3 ds.$$

If we take into account Eqs. (4.27), (4.4) and (4.21), then we see that the conditions (2.5) can be written as

(4.28) $$D_{\alpha j}b_j = -R_\alpha - \int_{\Sigma^{(0)}} x_\alpha f_3 da - \int_L x_\alpha \widetilde{t}_3 ds.$$

The Eqs. (4.26) and (4.27) determine the constants b_1, b_2 and b_3. After finding of the functions v_α, w, Φ, Ψ and χ, and the constants b_k and c_k, we can pass to study the other boundary conditions.

If we take into account (4.4) and (4.18), then the third of the conditions (4.17) becomes

$$(4.29) \qquad D\tau_2 = -\int_{\Sigma^{(0)}} \varepsilon_{\alpha\beta 3} x_\alpha f_\beta da - \int_L \varepsilon_{\alpha\beta 3} x_\alpha \widetilde{t}_\beta ds$$

$$- \int_{\Sigma^{(0)}} \left[x_1 c_{44}\left(\chi_{,2} + \sum_{s=1}^{3} c_s u_2^{(s)} \right) - x_2 c_{55}\left(\chi_{,1} + \sum_{s=1}^{3} c_s u_1^{(s)} \right) \right] da,$$

where D is the torsional rigidity

$$(4.30) \qquad D = \int_{\Sigma^{(0)}} [c_{44} x_1 (\Phi_{,2} + x_1) - c_{55} x_2 (\Phi_{,1} - x_2)] da.$$

It is known (see, e.g., [34]) that $D \neq 0$. Thus, the constant τ_2 is determined by (4.29).

In view of (4.4), the conditions (2.6) and (2.7) reduce to the equations

$$(4.31) \qquad D_{kj} a_j = C_k,$$

where

$$(4.32)$$
$$C_\alpha = \varepsilon_{3\alpha\beta} M_\beta - \int_{\Sigma^{(0)}} x_\alpha [c_{13}\gamma_{11} + c_{23}\gamma_{22} + \beta_3 w + c_{33}(\tau_2 \Phi + \chi)] da,$$

$$C_3 = -R_3 - \int_{\Sigma^{(0)}} [c_{13}\gamma_{11} + c_{23}\gamma_{22} + \beta_3 w + c_{33}(\tau_2 \Phi + \chi)] da.$$

From (4.31) we can determine the constants a_1, a_2 and a_3.

With the help of Eq. (4.4), the condition (2.8) becomes

$$(4.33) \quad D\tau_1 = -M_3 - \int_{\Sigma^{(0)}} \left[x_1 c_{44}\left(\Psi_{,2} + \sum_{s=1}^{3} b_s u_2^{(s)} \right) - x_2 c_{55}\left(\Psi_{,1} + \sum_{s=1}^{3} b_s u_1^{(s)} \right) \right] da.$$

This relation determines the constant τ_1. We conclude that the considered problem is solved. First, we have to determine the solutions of the problems $P^{(k)}$ ($k = 1, 2, 3$). Next, we determine the function Φ and calculate the torsional rigidity by (4.30). Then, we determine the constants b_j by (4.26) and (4.28). Since b_j and $(v_\alpha^{(s)}, \varphi^{(s)})$ are known, from (4.8) and (4.13) we can find the func-

tion Ψ. From (4.33) we can calculate the constant τ_1. Next, from (4.21) and (4.24) we determine the constants D_{ij}, and consequently the unknown constants c_k by (4.22) and (4.23). Now, we can find the function χ from (4.9) and (4.14). From (4.29) we determine the constant τ_2. Since the functions F_α, Q and p_α are known, we can determine the solution (v_1, v_2, w) of the plane strain problem (4.3), (4.5), (4.6), (4.11). Next, we determine C_k by (4.32) and the constants a_j from the system (4.31). Thus, the solution of the Almansi–Michell problem is given by (4.1).

We note that in absence of the classical loads $(f_i = 0, \widetilde{t}_i = 0, R_i = 0, M_i = 0)$, we obtain $b_i = c_i = 0$, $\tau_1 = \tau_2 = 0$. It follows from (4.1) that in this case, the generalized forces produce an extension, a bending and a plane strain parallel to the x_1, x_2-plane.

5. Application

In this section we use the solution (4.1) to study the deformation of a uniformly loaded circular cylinder. We consider the cylinder $B = \{x : x_1^2 + x_2^2 < a^2, 0 < x_3 < \ell\}$, $(a > 0)$, and assume that it is occupied by an orthotropic and homogeneous material.

We suppose that the bar is subjected to a constant pressure on the lateral surface, and to extension and torsion. In this case we have

$$
(5.1) \quad
\begin{aligned}
&f_i = 0, \quad q = 0, \quad \widetilde{t}_\alpha = Pn_\alpha, \quad \widetilde{t}_3 = 0, \quad \widetilde{h} = 0, \quad R_\alpha = 0, \quad R_3 = T, \\
&M_\alpha = 0, \quad M_3 = M,
\end{aligned}
$$

where P, T and M are the given constants. From (4.22), (4.23), (5.1) and (4.25) we conclude that $c_j = 0$. By (4.26), (4.28) and (5.1) we find that $b_j = 0$. In this case, from (4.9) and (4.14) we obtain $\chi = 0$. By using this result, from (4.29) we find that $\tau_2 = 0$. It follows from (4.8), (4.13) and (5.1) that $\Psi = 0$. From (4.10), (4.15) and (5.1) we conclude that

$$
(5.2) \quad F_1 = 0, \quad F_2 = 0, \quad Q = 0, \quad p_1 = Pn_1, \quad p_2 = Pn_2, \quad \widetilde{h} = 0.
$$

Let us prove that the solution of the problem (4.3), (4.5), (4.6), (4.11), with the data (5.2), is

$$
(5.3) \quad v_1 = d_1 x_1, \qquad v_2 = d_2 x_2, \qquad w = d_3,
$$

where the constants d_1, d_2 and d_3 are given by

$$
(5.4) \quad
\begin{aligned}
d_1 &= \xi P[\xi(c_{22} - c_{12}) + \beta_1\beta_2 - \beta_2^2]\Omega, \\
d_2 &= \xi P[\xi(c_{11} - c_{12}) + \beta_1\beta_2 - \beta_1^2]\Omega, \\
d_3 &= -(\beta_1 d_1 + \beta_2 d_2)\xi^{-1}, \\
\Omega^{-1} &= (\xi c_{11} - \beta_1^2)(\xi c_{22} - \beta_2^2) - (\xi c_{12} - \beta_1\beta_2)(\xi c_{22} - \beta_2^2).
\end{aligned}
$$

From (4.3), (5.4) and (4.5) we find that

$$\sigma_{11} = c_{11}d_1 + c_{12}d_2 + \beta_1 d_3,$$
$$\sigma_{22} = c_{12}d_1 + c_{22}d_2 + \beta_2 d_3,$$
$$\sigma_{12} = 0,$$
$$H_1 = 0, \qquad H_2 = 0,$$
$$G = -\beta_1 d_1 - \beta_2 d_2 - \xi d_3.$$

It is easy to see that the Eqs. (4.6) and the boundary conditions (4.11) are satisfied on the basis of relations (5.4).

Similarly, we can show that the solution of the problem $P^{(3)}$ is

$$(5.5) \qquad u_1^{(3)} = k_1 x_1, \qquad u_2^{(3)} = k_2 x_2, \qquad \varphi^{(3)} = k_3,$$

where the constants k_j are determined by the following system:

$$c_{11}k_1 + c_{12}k_2 + \beta_1 k_3 = -c_{13},$$
$$(5.6) \qquad c_{12}k_1 + c_{22}k_2 + \beta_2 k_3 = -c_{23},$$
$$\beta_1 k_1 + \beta_2 k_2 + \xi k_3 = -\beta_3.$$

It is a simple matter to see that positive definiteness of the elastic potential implies that the determinant of the system (5.6) is different from zero.

It follows from (4.21), (4.24) and (5.5) that

$$(5.7) \qquad D_{\alpha 3} = 0, \qquad D_{33} = \pi a^2 (c_{33} + c_{13}k_1 + c_{23}k_2 + \beta_3 k_3).$$

In view of (4.32) and (5.3), we find

$$(5.8) \qquad C_\alpha = 0, \qquad C_3 = -R_3 - R_3^0, \qquad R_3^0 = \pi a^2 (c_{13}d_1 + c_{23}d_2 + \beta_3 d_3).$$

Since $D_{ij} = D_{ji}$, we conclude that $D_{3\alpha} = 0$. By (5.7) and (5.8) we see that the system (4.31) reduces to

$$D_{\alpha\beta}a_\beta = 0, \qquad D_{33}a_3 = -R_3 - R_3^0.$$

Thus, we obtain

$$(5.9) \qquad a_1 = a_2 = 0, \qquad a_3 = -\frac{1}{D_{33}}(R_3 + R_3^0).$$

The solution of the boundary value problem (4.7), (4.12) is given by

$$(5.10) \qquad \Phi = \frac{c_{55} - c_{44}}{c_{55} + c_{44}} x_1 x_2, \qquad (x_1 x_2) \in \Sigma^{(0)}.$$

From (4.30) and (5.10) we get

$$D = \frac{\pi a^4 c_{44} c_{55}}{2(c_{44} + c_5)}.$$

The constant τ_1 is given by

(5.11) $\tau_1 = -M_3/D.$

It follows from Eq. (4.1) that the solution of the considered problem is

$$u_1 = a_3 k_1 x_1 + d_1 x_1 - \tau_1 x_2 x_3,$$
$$u_2 = a_3 k_2 x_2 + d_2 x_2 + \tau_1 x_1 x_3,$$
$$u_3 = a_3 x_3 + \tau_1 (c_{55} - c_{44})(c_{55} + c_{44})^{-1} x_1 x_2,$$
$$\varphi = a_3 k_3 + d_3,$$

where a_3, k_j, d_j and τ_1 are defined by Eqs. (5.4), (5.6), (5.9) and (5.11). We note that the loading (5.1) produces a uniform variation of porosity. The porosity is not influenced by the torsion of the cylinder.

Acknowledgments

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