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Description of anisotropic pore space structure of permeable materials based on Minkowski metric space

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This paper contains an extended presentation of a new approach to the macroscopic description of anisotropic pore space structure of rigid permeable material, utilizing the concept of Minkowski metric space. The metrics of the Minkowski and Euclidean spaces are used to determine double measures of any line, surface and volume elements, and to define geometrical macro-parameters characterizing anisotropic pore space structure: the volume and surface porosities, and the tortuosity of the pores. It is shown that the metric tensor of the Minkowski space can be interpreted as a tensor of the pore tortuosity, and its inverse as a tensor of the surface porosity. This means that the pore tortuosity has a pure geometrical character and is of fundamental importance for description of anisotropic pore space structure and for all physical processes taking place in the pore space. The approach presented in this paper allows a description of fluid dynamics in the anisotropic pore space in the framework of rational mechanics.

Key words: Minkowski metric space, pore space structure, permeable materials.

1. Introduction

Description of the pore space structure of a permeable porous material is one of the fundamental problems in modeling transport phenomena in saturated porous materials. It plays an important role in many physical and chemical processes: transport of mass, momentum and energy, wave propagation, and chemical reactions [38, 24, 7, 1]. The presence of pore structure is the reflection of two basic features of such materials: separation of both physical constituents (fluid and skeleton) at the microscopic level, and relative stiffness of the skeleton. Their interface defines boundary limits of the pore space which is the real space where the fluid motion takes place. Its structure imposes constraints on this motion, strongly influencing the character and intensity of interactions between fluid and skeleton. At the macroscopic level it appears as viscous, dynamical and dilatational couplings of both constituents.

There were many approaches to the description of permeable porous materials: microscopic modeling [38, 1, 3], methods of homogenization [4, 21, 39] and averaging [6, 33, 25, 28], theory of mixtures [12, 43, 7], variational methods

[9, 10, 22], and mixed methods [23, 29, 30, 32]. In each of these, the parameters of pore space structure, except the volume porosity, were defined and introduced in different ways.

In models formulated at the microscopic level, the structure of the pore space is determined by the form of the elementary cell. In the averaging procedures, the pore structure parameters are often introduced as coefficients relating fluctuations of kinetic energy of the fluid about its value represented by macroscopic velocity. In the macroscopic models, the influence of pore space structure is simply described by constitutive relations for interaction forces and stresses, then the pore structure parameters become coefficients in these relations. Similarly, in descriptions based on variational principles, they appear in the equations as a consequence of the assumed forms of dissipation function and kinetic energy of the medium.

Another direction of studies on porous media structure is presented in papers based on stochastic methods [26, 27, 11, 42, 41]. They apply such concepts as local porosity distribution [26, 27, 11] and the correlation functions [42, 41] for quantitative characterization of pore structure from two- and three-dimensional images of microscopic pore geometry and for determination of their relations with physical properties of the medium.

The existing macro-continuum models of the dynamics of fluid-saturated porous materials do not explicitly recognize the kinematical domination of the skeleton on the fluid motion. The kinematics is formulated in the context of mixture theory, which does not incorporate explicitly the pore space structure. This results in considerable difficulties in description of the related problems of the dynamical, dilatational and even viscous couplings between the fluid and the skeleton. This applies also to an isotropic structure and is unsolved for media with anisotropic structure.

The aim of this paper is an extended presentation of a new approach to macroscopic description of anisotropic pore space structure of rigid permeable porous materials and modeling of fluid motion in such media, based on the concept of anisotropic metric space as a model of anisotropic pore space.

The fluid flow through a rigid skeleton of an anisotropic pore structure is considered in the paper as motion of the material continuum (fluid) in an anisotropic plane metric space. In this model the pore space of the permeable skeleton forms at a macroscopic level the anisotropic metric space in which fluid motion takes place. Therefore, the constraints imposed by the pore space structure on fluid motion correspond to those imposed by the metric of the anisotropic space and quantities describing the anisotropic properties of the metric space can be interpreted as parameters characterizing the pore space structure. Such approach enables a description of fluid dynamics in the anisotropic pore space, in the framework of rational mechanics.

The concept of a plane anisotropic space in modern geometry and differential geometry is termed the Minkowski space [36, 40]. This plane space and its generalization – the Finsler space (curved) [31, 36] – are useful in modelling various physical, biological and mechanical phenomena [2, 8, 37].

Results of this paper in the condensed form were partially presented in the papers [17] and [18] published in the proceedings of international symposia, and their extended form have been published till now only in Polish [19]. Due to the mathematical complexity of the metric-based approach and applied original notation of vector and tensor algebra, the fundamentals of the proposed description and implementation of the Minkowski metric space to the needs of anisotropic pore space modeling are presented in extended form. The explicit form of metrics for surface and volume elements in the anisotropic space are derived and examples of Minkowski metrics are given. Additionally, the principal directions in anisotropic pore space and corresponding structure parameters (tortuosity of pores and surface porosity) have been determined. To illustrate the power of the metric-based approach to description of the pore space structure of permeable porous materials, an example of the metric for tetragonal pore space has been presented.

It was shown that metric tensor of Minkowski space plays a fundamental role in the description of the geometrical structure of the pore space. It determines measures of any line, surface and volume elements in the pore space, which in turn define geometrical parameters characterizing the pore space structure, i.e. the volume and surface porosities and the tortuosity of the pores. This metric tensor can be interpreted as a tensor of the pore tortuosity, and its inverse as a tensor of the surface porosity.

2. Basic definitions and notations

Vectors and tensors. Denote by $\mathcal{V}$ the three-dimensional real vector space, and by $\mathcal{V}^\star$ the dual vector space of a space $\mathcal{V}$. If $\mathbf{u} \in \mathcal{V}$ and $\mathbf{v}^\star \in \mathcal{V}^\star$, the scalar $\mathbf{u} \cdot \mathbf{v}^\star = \mathbf{v}^\star \cdot \mathbf{u} \in R$ is the dual product of vector $\mathbf{u}$ and covector $\mathbf{v}^\star$, and the "dot" $((\ ) \cdot (\ ))$ denotes the bilinear operation (the dual multiplication) defined on $\mathcal{V}$ and $\mathcal{V}^\star$.

The multilinear transformations of vector spaces are called tensors. They are elements of linear spaces formed by the tensor products of vector spaces. For example, a tensor $\mathbf{A} \in \mathcal{V} \otimes \mathcal{V}^\star$ is an endomorphism of $\mathcal{V}$ and $\mathcal{V}^\star$. For $\mathbf{u} \in \mathcal{V}$ and $\mathbf{v}^\star \in \mathcal{V}^\star$,

$$\mathbf{A} \cdot \mathbf{u} \in \mathcal{V}, \qquad \mathbf{v}^\star \cdot \mathbf{A} \in \mathcal{V}^\star.$$

The tensor product $\mathbf{u} \otimes \mathbf{v}^\star$ of a vector $\mathbf{u}$ and covector $\mathbf{v}^\star$ is the simplest form of a tensor in the space $\mathcal{V} \otimes \mathcal{V}^\star$.

Since tensors of order three and four arise later, operations on them are simplified by introducing new, alternative, notations for tensor products. For $\mathbf{t}, \mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathcal{V}$,

$$\text{(2.1)} \qquad \mathbf{t} \otimes \mathbf{u} \equiv \begin{pmatrix} \mathbf{t} \\ \mathbf{u} \end{pmatrix} \in \mathcal{V} \otimes \mathcal{V} = \otimes \mathcal{V}^2, \qquad \mathbf{t} \otimes \mathbf{u} \otimes \mathbf{v} \equiv \begin{pmatrix} \mathbf{t} \\ \mathbf{u} \\ \mathbf{v} \end{pmatrix} \in \otimes \mathcal{V}^3,$$

$$\text{(2.2)} \qquad \mathbf{t} \otimes \mathbf{u} \otimes \mathbf{v} \otimes \mathbf{w} \equiv \begin{pmatrix} \mathbf{t}\, \mathbf{v} \\ \mathbf{u}\, \mathbf{w} \end{pmatrix} \equiv \begin{pmatrix} \mathbf{t} \otimes \mathbf{v} \\ \mathbf{u} \otimes \mathbf{w} \end{pmatrix} \equiv \begin{pmatrix} \mathbf{t} \\ \mathbf{u} \end{pmatrix} \otimes \begin{pmatrix} \mathbf{v} \\ \mathbf{w} \end{pmatrix} \in \otimes \mathcal{V}^4.$$

Such representations of the tensor products render their factors accessible for various operations, e.g. the dual multiplication or transposition. Since each tensor can be represented as a linear combination of the tensor products of the basis vectors of $\mathcal{V}$ (and of $\mathcal{V}^\star$), the notations (2.1) and (2.2) also have consequences for operations performed on general tensors. For example, taking into account $(2.1)_2$, three different dual products of a tensor $\mathbf{B} \in \otimes \mathcal{V}^{\star 3}$ and a vector $\mathbf{u} \in \mathcal{V}$ can be denoted as follows:

$$\text{(2.3)} \qquad \mathbf{B}^{\,\cdot \mathbf{u}}, \qquad \mathbf{B} \cdot\! \mathbf{u}, \qquad \mathbf{B}_{\,\cdot \mathbf{u}}.$$

In this notation, the position of the vector $\mathbf{u}$ indicates the type of dual product of both quantities.

Representations of surface and volume elements. Similarly to a vector which is an algebraic model of a directed segment, the skew-symmetric tensors of order two and three are natural models of surface and volume elements [13], respectively. These elements, spanned by two $(\mathbf{u}, \mathbf{v})$ and three $(\mathbf{u}, \mathbf{v}, \mathbf{w})$ linearly independent vectors, can be represented by their exterior products $\mathbf{u} \wedge \mathbf{v}$ and $\mathbf{u} \wedge \mathbf{v} \wedge \mathbf{w}$ defined by

$$\text{(2.4)} \qquad \boldsymbol{\sigma} = \mathbf{u} \wedge \mathbf{v} \equiv \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} \equiv \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix} - \begin{pmatrix} \mathbf{v} \\ \mathbf{u} \end{pmatrix},$$

$$\text{(2.5)} \qquad \boldsymbol{\vartheta} = \mathbf{u} \wedge \mathbf{v} \wedge \mathbf{w} \equiv \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{w} \end{bmatrix}$$

$$\equiv \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{w} \end{pmatrix} + \begin{pmatrix} \mathbf{v} \\ \mathbf{w} \\ \mathbf{u} \end{pmatrix} + \begin{pmatrix} \mathbf{w} \\ \mathbf{u} \\ \mathbf{v} \end{pmatrix} - \begin{pmatrix} \mathbf{v} \\ \mathbf{u} \\ \mathbf{w} \end{pmatrix} - \begin{pmatrix} \mathbf{u} \\ \mathbf{w} \\ \mathbf{v} \end{pmatrix} - \begin{pmatrix} \mathbf{w} \\ \mathbf{v} \\ \mathbf{u} \end{pmatrix}.$$

The surface elements $\boldsymbol{\sigma}$ form a three-dimensional space $\wedge \mathcal{V}^2$ of the skew-symmetric tensors of order two,

$$\boldsymbol{\sigma} \in \wedge \mathcal{V}^2 \subset \otimes \mathcal{V}^2,$$

and the volume elements $\boldsymbol{\vartheta}$ form a one-dimensional space $\wedge\mathcal{V}^3$ of the skew-symmetric tensors of order three,

$$\boldsymbol{\vartheta} \in \wedge\mathcal{V}^3 \subset \otimes\mathcal{V}^3.$$

Any two elements of the space $\wedge\mathcal{V}^3$ are related to each other by a scalar multiplier. For example, for two skew-symmetric tensors of the form

$$\begin{bmatrix}\mathbf{u}\\ \mathbf{v}\\ \mathbf{w}\end{bmatrix}, \begin{bmatrix}\mathbf{A}\cdot\mathbf{u}\\ \mathbf{A}\cdot\mathbf{v}\\ \mathbf{A}\cdot\mathbf{w}\end{bmatrix} \in \wedge\mathcal{V}^3,$$

the scalar is the determinant of automorphism $\mathbf{A} \in \mathcal{V}\otimes\mathcal{V}^\star$; that is,

$$\begin{bmatrix}\mathbf{A}\cdot\mathbf{u}\\ \mathbf{A}\cdot\mathbf{v}\\ \mathbf{A}\cdot\mathbf{w}\end{bmatrix} = \begin{pmatrix}\mathbf{A}\\ \mathbf{A}\\ \mathbf{A}\end{pmatrix} \vdots \begin{bmatrix}\mathbf{u}\\ \mathbf{v}\\ \mathbf{w}\end{bmatrix} = \det(\mathbf{A}) \begin{bmatrix}\mathbf{u}\\ \mathbf{v}\\ \mathbf{w}\end{bmatrix}. \tag{2.6}$$

This identity can be used as a definition of the determinant of tensor $\mathbf{A}$.

Normed vector spaces. A vector space $\mathcal{V}$ is called a normed space if there is a real-valued function $L_A(\mathbf{u})$ defined on $\mathcal{V}$, that each vector $\mathbf{u}$ ascribes a real value $u \in R$, called the length of vector $\mathbf{u}$,

$$u = L_A(\mathbf{u}),$$

and satisfies the following axioms [5]:

$$L_A(\mathbf{u}) > 0 \qquad \text{for } \mathbf{u} \neq \mathbf{0}, \text{ and } L_A(\mathbf{0}) = 0; \tag{2.7}$$

$$L_A(k\,\mathbf{u}) = k\,L_A(\mathbf{u}) \qquad \text{for } k > 0; \tag{2.8}$$

$$L_A(\mathbf{u}+\mathbf{v}) < L_A(\mathbf{u}) + L_A(\mathbf{v}); \tag{2.9}$$

for all linearly independent vectors $\mathbf{u}, \mathbf{v} \in \mathcal{V}$.

The function $L_A(\mathbf{u})$ is called the norm of space $\mathcal{V}$. It is positive definite, positively homogeneous of order one, and strictly convex. Due to the positive homogeneity (2.8), the length of a vector $\mathbf{u}$ can depend on its sense,

$$L_A(-\mathbf{u}) \neq L_A(\mathbf{u}),$$

which means that the norm does not have to be symmetric. The norm $L_A(\mathbf{u})$ generates a metric in the vector space $\mathcal{V}$ that defines the distance $d_A(\mathbf{u},\mathbf{v})$ between any of its two elements $\mathbf{u}$ and $\mathbf{v}$,

$$d_A(\mathbf{u},\mathbf{v}) \equiv L_A(\mathbf{v}-\mathbf{u}). \tag{2.10}$$

Below, we present examples of a symmetric and an asymmetric metric

$$L_A^4(\mathbf{u}) = \begin{pmatrix}\mathbf{u}\\ \mathbf{u}\end{pmatrix} : \mathbf{C} : \begin{pmatrix}\mathbf{u}\\ \mathbf{u}\end{pmatrix}, \tag{2.11}$$

$$L_A(\mathbf{u}) = \frac{a\,\mathbf{u}\cdot\mathbf{M}\cdot\mathbf{u}}{\sqrt{\mathbf{u}\cdot\mathbf{M}\cdot\mathbf{u}} + b\,\mathbf{u}\cdot\mathbf{M}\cdot\mathbf{n}}, \tag{2.12}$$

where

$$\mathbf{C}\in\otimes\mathcal{V}^{\star 4}, \qquad \mathbf{M}\in\otimes\mathcal{V}^{\star 2}, \qquad \mathbf{n}\in\mathcal{V}, \qquad a,\ b\in R,$$

and the tensors $\mathbf{C}$, $\mathbf{M}$, and scalar parameters a, b must be chosen to satisfy the axioms (2.7) and (2.9).

Affine spaces. The pair $(P,\ \mathcal{V})$ composed of a point P and a vector space $\mathcal{V}$ defines an affine point space, because the algebraic structures of both objects are isomorphic. Point P is called the reference point and $\mathcal{V}$ is the space of the position vectors of points. The affine space $(P,\ \mathcal{V})$ is normed if the space $\mathcal{V}$ of the position vectors is normed.

3. Geometry of Euclidean point space

The Euclidean point space is the simplest special case of a normed affine point space. The affine space is Euclidean when the norm $L(\mathbf{u})$ generating its metric is given by

$$L^2(\mathbf{u}) \equiv \mathbf{u}\cdot\mathbf{M}\cdot\mathbf{u} = \mathbf{M} : \begin{pmatrix}\mathbf{u}\\ \mathbf{u}\end{pmatrix}, \tag{3.1}$$

where the metric tensor $\mathbf{M}\in\otimes\mathcal{V}^{\star 2}$ of the Euclidean space is independent of $\mathbf{u}$. Tensor $\mathbf{M}$ is non-singular, symmetric and positive definite, and therefore $L(\mathbf{u})$ satisfies the axioms of a symmetric norm.

The pair $(\mathcal{V},\ \mathbf{M})$ form the Euclidean vector space, and the triple $(P,\ (\mathcal{V},\ \mathbf{M}))$ form the Euclidean point space. Due to its affine structure, this space is plane. Because metric tensor $\mathbf{M}$ is independent of the position vector of points and of the vector $\mathbf{u}$, it is also homogeneous and isotropic. Euclidean point space is used in the classical mechanics as a model of the physical space.

The existence of a distance metric in the Euclidean space $(P,\ (\mathcal{V},\ \mathbf{M}))$ allows definitions of metrics for surface elements $\boldsymbol{\sigma}\in\wedge\mathcal{V}^2$, and for volume elements $\boldsymbol{\vartheta}\in\wedge\mathcal{V}^3$. They are given by

$$S^2(\boldsymbol{\sigma}) = \boldsymbol{\sigma} : \mathbf{S} : \boldsymbol{\sigma}, \qquad V^2(\boldsymbol{\vartheta}) = \boldsymbol{\vartheta} \,\vdots\, \mathbf{V} \,\vdots\, \boldsymbol{\vartheta}, \tag{3.2}$$

where

$$\mathbf{S} = \frac{1}{2}\begin{pmatrix}\mathbf{M}\\ \mathbf{M}\end{pmatrix}, \qquad \mathbf{V} = \frac{1}{6}\begin{pmatrix}\mathbf{M}\\ \mathbf{M}\\ \mathbf{M}\end{pmatrix}, \tag{3.3}$$

are the metric tensors in the spaces of surface elements $\wedge \mathcal{V}^2$ and volume elements $\wedge \mathcal{V}^3$, respectively. These tensors are of order four and six.

The metrics defined by expressions (3.2) satisfy all the axioms of a norm. They are positive definite, homogeneous, symmetric and strictly convex. Their application to the elements represented by the exterior products (2.4) and (2.5) of two and three linearly independent vectors, gives the Gram's determinants that define the area and volume of the parallelogram and parallelepiped, respectively.

4. Geometry of Minkowski point space

The normed affine point space, the norm of which has the general properties (2.7)–(2.9), is called Minkowski space [36]. Like the Euclidean space, it is plane but has anisotropic properties, and may not be symmetric. This determines the peculiar features of its internal, geometrical structure.

The metric tensor of distance in Minkowski space is

$$\mathbf{M}_A(\mathbf{u}) = \frac{1}{2}\frac{\partial^2 L_A^2(\mathbf{u})}{\partial \mathbf{u}\,\partial \mathbf{u}} \in \mathcal{V}^\star \otimes \mathcal{V}^\star, \tag{4.1}$$

and due to the homogeneity of $L_A(\mathbf{u})$ given by (2.8), has the properties:

$$\mathbf{u}\cdot\mathbf{M}_A(\mathbf{u})\cdot\mathbf{u} = L_A^2(\mathbf{u}), \qquad \mathbf{M}_A(k\,\mathbf{u}) = \mathbf{M}_A(\mathbf{u}) \quad \text{for } k>0. \tag{4.2}$$

In expressions (4.1) $\partial L_A^2(\mathbf{u})/\partial\mathbf{u}$ denotes the gradient of the function $L_A^2(\mathbf{u})$, defined by the identity

$$\mathbf{v}\cdot\frac{\partial}{\partial\mathbf{u}}L_A^2(\mathbf{u}) \equiv \frac{\partial}{\partial h}L_A^2(\mathbf{u}+h\,\mathbf{v})\Big|_{h=0}, \tag{4.3}$$

which must be satisfied for all $\mathbf{v}\in\mathcal{V}$.

From $(4.2)_2$, the metric tensor $\mathbf{M}_A(\mathbf{u})$, in general, depends on the direction and sense of the vector $\mathbf{u}$ and is independent of its length. This defines the anisotropic properties of Minkowski space. The metric tensor $\mathbf{M}_A(\mathbf{u})$ determines a set of anisotropic spheres (Minkowski spheres), i.e. closed and convex surfaces, the points of which are equally distant from the reference point P with respect to the Minkowski metric. The sphere of unit radius is defined by equation

$$L_A^2(\mathbf{N}) = \mathbf{N}\cdot\mathbf{M}_A(\mathbf{N})\cdot\mathbf{N} = 1,$$

and is called the indicatrix of the metric space. The indicatrix may also be used for characterizing the geometry of Minkowski space. The restrictions imposed on the norm $L_A(\mathbf{u})$ by the axioms (2.7)–(2.9) imply that the indicatrix can be formed by any strictly convex surface surrounding the reference point.

For the metric of Minkowski space given by (2.12), the indicatrix is asymmetric and forms a surface of revolution, the cross-section of which is the limaçon curve [5], shown in Fig. 1. In turn, the form of indicatrix generated by the metric (2.11) depends on the properties of tensor $\mathbf{C}$, which can be interpreted as the structure tensor of Minkowski space. It can define the indicatrix for each three-dimensional anisotropic pore space characterized by any group of its point symmetries. The example of such a metric for the tetragonal pore space structure is presented in the Sec. 8. The indicatrix of Euclidean space is a special case of the Minkowski indicatrix which can be represented by the usual sphere (Fig. 1).

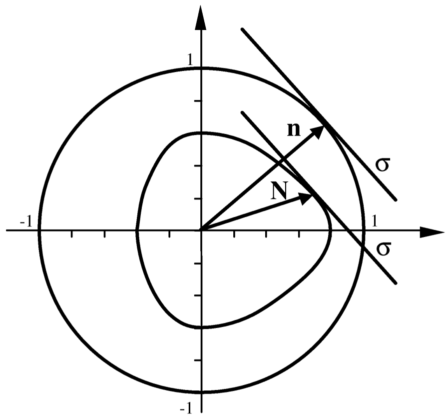

FIG. 1. Cross-section of the Euclidean indicatrix (circle) and the Minkowski indicatrix (the limaçon given by (2.12)) and their unit vectors $\mathbf{n}$ and $\mathbf{N}$, normal to the surface element $\boldsymbol{\sigma}$.

The metric tensor $\mathbf{M}_A(\mathbf{u})$ determining the distance of points in the Minkowski space is used in the paper to define the metrics of surface and volume elements in this space.

Similar to the Euclidean space, these metrics have the form

$$S_A^2(\boldsymbol{\sigma}) = \boldsymbol{\sigma} : \mathbf{S}_A(\mathbf{r}) : \boldsymbol{\sigma}, \qquad V_A^2(\boldsymbol{\vartheta}) = \boldsymbol{\vartheta} \,\vdots\, \mathbf{V}_A \,\vdots\, \boldsymbol{\vartheta}, \tag{4.4}$$

where

$$\mathbf{S}_A(\mathbf{r}) = \alpha(\mathbf{r})\,\frac{1}{2}\begin{pmatrix}\mathbf{M}_A(\mathbf{r})\\ \mathbf{M}_A(\mathbf{r})\end{pmatrix}, \qquad \mathbf{V}_A = \alpha(\mathbf{r})\frac{1}{3!}\begin{pmatrix}\mathbf{M}_A(\mathbf{r})\\ \mathbf{M}_A(\mathbf{r})\\ \mathbf{M}_A(\mathbf{r})\end{pmatrix}, \tag{4.5}$$

are the metric tensors in the spaces of surface elements $\wedge\mathcal{V}^2$ and volume elements $\wedge\mathcal{V}^3$, respectively. These tensors are of order four and six.

The vector $\mathbf{r}$ in $(4.5)_1$ is orthogonal to the surface element $\boldsymbol{\sigma}$ with respect to metric $\mathbf{M}_A(\mathbf{r})$, and so it satisfies the condition

$$\boldsymbol{\sigma} \cdot \mathbf{M}_A(\mathbf{r}) \cdot \mathbf{r} = 0.$$

Moreover, in expressions (4.5), the scalar-valued vector function $\alpha(\mathbf{r})$ has been introduced to ensure that the measure of volume elements $(4.4)_2$ is independent of the vector $\mathbf{r}$. Applying definition (2.6) of the determinant, the following relation can be written:

$$\boldsymbol{\vartheta} \vdots \begin{pmatrix} \mathbf{M}_A \\ \mathbf{M}_A \\ \mathbf{M}_A \end{pmatrix} = \boldsymbol{\vartheta} \vdots \begin{pmatrix} \mathbf{M} \\ \mathbf{M} \\ \mathbf{M} \end{pmatrix} \vdots \begin{pmatrix} \mathbf{M}^{-1} \cdot \mathbf{M}_A \\ \mathbf{M}^{-1} \cdot \mathbf{M}_A \\ \mathbf{M}^{-1} \cdot \mathbf{M}_A \end{pmatrix} = \det(\mathbf{M}^{-1} \cdot \mathbf{M}_A(\mathbf{r}))\, \boldsymbol{\vartheta} \vdots \begin{pmatrix} \mathbf{M} \\ \mathbf{M} \\ \mathbf{M} \end{pmatrix}.$$

Then, the measure $(4.4)_2$ is independent of $\mathbf{r}$ if the coefficient $\alpha(\mathbf{r})$ has the form

$$\alpha(\mathbf{r}) = \frac{\beta}{\det(\mathbf{M}^{-1} \cdot \mathbf{M}_A(\mathbf{r}))}, \tag{4.6}$$

where β is a positive constant parameter. In that case the Minkowski measure $(4.4)_2$ of a volume element becomes proportional to the Euclidean measure $(3.2)_2$

$$V_A(\boldsymbol{\vartheta}) = \sqrt{\beta}\, V(\boldsymbol{\vartheta}), \tag{4.7}$$

and the parameter β is independent of the Minkowski metric $\mathbf{M}_A(\mathbf{r})$. This means that the metric of the surface and volume elements in Minkowski space are not fully determined by the metric of distance.

From equality (4.7), the Minkowski measure of volume $V_A(\boldsymbol{\vartheta})$ satisfies all the axioms of the norm, satisfied by the Euclidean measure $V(\boldsymbol{\vartheta})$. It can be shown that the metric of surface elements $(4.4)_1$ also satisfies all the axioms of the norm. It is positive definite, homogeneous and strictly convex.

5. Modelling the anisotropic pore space

The concepts of Euclidean and Minkowski spaces introduced in the previous sections are now applied to describe the pore space of a permeable porous material, and to define the parameters characterizing its structure.

The interconnected pores in permeable porous materials form a very complicated network of channels, which are filled with a fluid, determine the real space for the fluid motion. Therefore, at the microscopic level, the fluid particles moving in pores between any two points of the medium have a much longer distance to travel than the distance between these points in the physical space. Generally, that length depends on the direction of the macroscopic (mean) flow of the fluid. It may also happen that fluid particles flowing in the opposite direction would travel a different distance. From the macroscopic point of view, such situations may be modelled by an unsymmetric pore space metric. Such complex geometry of a microscopic fluid flow through porous materials strongly influences the mechanical behavior of the fluid at the macroscopic level, and motivates the use of the Minkowski metric space as a macroscopic description of the pore space.

The porous skeleton is embedded in the physical space described in classical mechanics by an Euclidean point space. Therefore, the anisotropic Minkowski space describing the pore space, is considered here to be embedded in the Euclidean space and points of both spaces are identified. They may be represented by

$$(P,(\mathcal{V},\mathbf{M})), \qquad (P,(\mathcal{V},\mathbf{M}_A)).$$

Such embedding imposes restrictions on the relations of the measures of any line, surface and volume elements in the two spaces, following from the physical requirement that embedding should not disturb the geometrical structure of the basic (Euclidean) space. This means that the length of a line segment measured with respect to the Minkowski metric, cannot be smaller than its length with respect to the Euclidean metric. The measures of volume elements in both spaces should satisfy the inverse relation.

For any vector $\mathbf{u} \in \mathcal{V}$ and any volume element $\boldsymbol{\vartheta} \in \wedge\mathcal{V}^3$, the following inequalities hold:

$$L_A(\mathbf{u}) \geq L(\mathbf{u}), \qquad V_A(\boldsymbol{\vartheta}) \leq V(\boldsymbol{\vartheta}). \tag{5.1}$$

It can be shown that the inequalities (5.1) also induce a relation between the measures of surface elements in Minkowski and Euclidean spaces. For any surface element $\boldsymbol{\sigma} \in \wedge\mathcal{V}^2$,

$$S_A(\boldsymbol{\sigma}) \leq S(\boldsymbol{\sigma}). \tag{5.2}$$

In turn, from the inequality $(5.1)_1$, it follows that the Minkowski indicatrix is always contained in the Euclidean indicatrix.

Note that the above relations between the Euclidean and Minkowski measures of line, surface and volume elements do not satisfy the commonly used Kolmogoroff's monotonicity requirement [40], described by [14]: if a space has two metrics, one of which dominates the other, then the smaller metric should induce smaller areas.

6. Macroscopic parameters of pore space structure

The line, surface and volume elements measured with respect to the Minkowski and the Euclidean metrics allow defitions of the tortuosity of pores, and the volume and surface porosities. These parameters characterize the macroscopic pore space structure of an anisotropic porous medium and play very important role in the mechanics of fluid-saturated porous materials.

Tortuosity of pores. The concept of tortuosity was first introduced into the theory of porous materials by CARMAN [15]. The tortuosity of the pores was defined as the square of ratio of the averaged length of fluid particles paths, measured in the pore space in the direction of macroscopic fluid flow, to the distance in the physical space. Later [16], the tortuosity parameter was defined as the ratio of these quantities.

Here, the tortuosity parameter for the pores in the direction determined by a vector $\mathbf{u} \in \mathcal{V}$ in the porous medium, is defined as the ratio of the length of the vector $\mathbf{u}$ measured with respect to the Minkowski metric, to the length of this vector with respect to the Euclidean metric. Denoting the tortuosity parameter by $\delta(\mathbf{u})$,

$$\delta^2(\mathbf{u}) \equiv \left(\frac{u_A}{u}\right)^2 = \frac{\mathbf{u} \cdot \mathbf{M}_A(\mathbf{u}) \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{M} \cdot \mathbf{u}}, \tag{6.1}$$

or

$$\delta^2(\mathbf{n}) = \mathbf{n} \cdot \mathbf{M}_A(\mathbf{n}) \cdot \mathbf{n}, \tag{6.2}$$

where $\mathbf{n} = \mathbf{u}/u$ is the Euclidean unit vector $(\mathbf{n} \cdot \mathbf{M} \cdot \mathbf{n} = 1)$ indicating direction in the physical space. The parameter $\delta(\mathbf{u})$ is a homogeneous scalar-valued function of order zero:

$$\delta(k\,\mathbf{u}) = \delta(\mathbf{u}), \qquad k > 0,$$

which for an unsymmetric metric of anisotropic pore space will take different values for the opposite sense of the vector $\mathbf{u}$,

$$\delta(-\mathbf{u}) \neq \delta(\mathbf{u}).$$

It follows from the restriction $(5.1)_1$ that the tortuosity parameter $\delta(\mathbf{n})$ takes values not less than unity; that is,

$$\delta(\mathbf{n}) \geq 1. \tag{6.3}$$

From (6.2), the metric tensor $\mathbf{M}_A(\mathbf{n})$ fully determines the value of the tortuosity parameter $\delta(\mathbf{n})$, and can therefore be interpreted as a tortuosity tensor of the pore space. This tensor characterizes the anisotropic properties of the pore space structure. Even if the tensor $\mathbf{M}_A(\mathbf{n})$ does not depend on the unit vector $\mathbf{n}$, the Minkowski and Euclidean spaces may be relatively anisotropic, i.e. if one is isotropic, the metric of the other space may be anisotropic with respect to the first one.

The tensor $\mathbf{M}_A(\mathbf{n})$ describes an isotropic pore structure if it is proportional to the Euclidean metric tensor $\mathbf{M}$. By (6.2),

$$\mathbf{M}_A(\mathbf{n}) = \delta^2\,\mathbf{M}, \tag{6.4}$$

where δ denotes the tortuosity of pores in the isotropic pore space.

Volume porosity. Similar to tortuosity, the volume porosity f_v can be defined by the ratio of the measures of volume element $\boldsymbol{\vartheta} \in \wedge \mathcal{V}^3$ with respect to the Minkowski and Euclidean metrics. Then, with the definitions $(3.2)_2$ and $(4.4)_2$,

$$f_v^2 \equiv \left(\frac{V_A(\boldsymbol{\vartheta})}{V(\boldsymbol{\vartheta})}\right)^2 = \frac{\boldsymbol{\vartheta} \,\vdots\, \mathbf{V}_A \,\vdots\, \boldsymbol{\vartheta}}{\boldsymbol{\vartheta} \,\vdots\, \mathbf{V} \,\vdots\, \boldsymbol{\vartheta}} = \mathbf{E}_3 \,\vdots\, \mathbf{V}_A \,\vdots\, \mathbf{E}_3, \tag{6.5}$$

where $\mathbf{E}_3 = \boldsymbol{\vartheta}/V$ is the Euclidean unit volume element ($\mathbf{E}_3 \,\vdots\, \mathbf{V} \,\vdots\, \mathbf{E}_3 = 1$). The volume porosity parameter f_v, due to the restriction $(5.1)_2$, must satisfy the condition

$$f_v \leq 1. \tag{6.6}$$

The definition (6.5) and relation (4.7) determine the constant parameter β in the coefficient $\alpha(\mathbf{r})$ of the metric tensors (4.5); thus

$$\beta = f_v^2. \tag{6.7}$$

Surface porosity. The third parameter characterizing the pore space structure is the surface porosity denoted by $\lambda(\boldsymbol{\sigma})$. This parameter is defined as the ratio of measures of a surface element $\boldsymbol{\sigma} \in \wedge \mathcal{V}^2$ with respect to the Minkowski and Euclidean metrics. With the definitions $(3.2)_1$ and $(4.4)_1$,

$$\lambda^2(\boldsymbol{\sigma}) \equiv \left(\frac{S_A(\boldsymbol{\sigma})}{S(\boldsymbol{\sigma})}\right)^2 = \frac{\boldsymbol{\sigma} : \mathbf{S}_A(\mathbf{N}) : \boldsymbol{\sigma}}{\boldsymbol{\sigma} : \mathbf{S} : \boldsymbol{\sigma}}, \tag{6.8}$$

or

$$\lambda^2(\mathbf{E}_2) = \mathbf{E}_2 : \mathbf{S}_A(\mathbf{N}) : \mathbf{E}_2, \tag{6.9}$$

where $\mathbf{E}_2 = \boldsymbol{\sigma}/S$ is the Euclidean unit surface element ($\mathbf{E}_2 : \mathbf{S} : \mathbf{E}_2 = 1$), and $\mathbf{N}$ is the Minkowski unit vector orthogonal to the surface element $\boldsymbol{\sigma}$ with respect to the metric $\mathbf{M}_A(\mathbf{N})$ (Fig. 1). The parameter $\lambda(\boldsymbol{\sigma})$, due to the restriction (5.2), must satisfy the condition

$$\lambda \leq 1. \tag{6.10}$$

From the definition (6.9), the metric tensor of a surface element $\boldsymbol{\sigma}$ can be interpreted as the tensor of surface porosity.

The definition (6.9) can be reduced to a simpler form if the unit surface element $\mathbf{E}_2$ is replaced by the Euclidean unit vector $\mathbf{n}$ orthogonal to that element. Since

$$\mathbf{E}_2 = \mathbf{E}_3^{\cdot \mathbf{M} \cdot \mathbf{n}}, \tag{6.11}$$

(6.9) gives

$$\lambda^2(\mathbf{E}_2) = \mathbf{n} \cdot \mathbf{M}_S(\mathbf{N}) \cdot \mathbf{n} \tag{6.12}$$

where, due to the definition (2.6) and identity

$$\mathbf{E}_3 : \binom{\mathbf{M}}{\mathbf{M}} : \mathbf{E}_3^{\cdot\mathbf{M}} = 2\,\mathbf{I}, \tag{6.13}$$

the tensor $\mathbf{M}_S(\mathbf{N})$ is given by

$$\mathbf{M}_S(\mathbf{N}) = f_v^2\,\mathbf{M}\cdot\mathbf{M}_A^{-1}(\mathbf{N})\cdot\mathbf{M}, \tag{6.14}$$

whereas $\mathbf{I} \in \mathcal{V}\otimes\mathcal{V}^\star$ is the identity automorphism of spaces $\mathcal{V}$ and $\mathcal{V}^\star$. In expressions (6.11) and (6.13) the notation (2.3) has been used.

Relations (6.12) and (6.14) show that the inverse of the Minkowski metric tensor $\mathbf{M}_A(\mathbf{N})$, together with the volume porosity f_v, defines the value of the surface porosity $\lambda(\boldsymbol{\sigma})$. Therefore the tensor $\mathbf{M}_A^{-1}(\mathbf{N})$ may be interpreted as the surface porosity tensor. This means that the directional properties of the surface porosity are closely related to the directional properties of the pore tortuosity, characterized by the metric tensor of an anisotropic pore space.

In the case of the isotropic pore space structure, due to relation (6.4), the tensor (6.14) reduces to the form

$$\mathbf{M}_S(\mathbf{N}) = \left(\frac{f_v}{\delta}\right)^2\mathbf{M}, \tag{6.15}$$

then from definition (6.12),

$$f_v = \lambda\delta. \tag{6.16}$$

Since the tortuosity of the pores is not less than unity, from (6.16) it follows that, in general, the volume porosity cannot be equal to the surface porosity. This means that the surface porosity λ defined at the macroscopic level, does not have a simple microscopic interpretation. It can be shown that, for a pore space composed of cylindrical tubes (pores), the surface porosity defined by (6.8) represents the total area of normal cross-sections of tubes on the unit surface of the porous medium. Representations of the surface porosity and the tortuosity of pores in isotropic and anisotropic pore space, in terms of microscopic geometrical parameters, will be considered in a separate paper.

7. Principal directions of the pore space

The existence of the two metrics: $\mathbf{M}_A(\mathbf{N})$ and $\mathbf{M}$, defined on a vector space $\mathcal{V}$, which describe the geometry of the anisotropic pore space embedded in a physical space, allows the formulation of an automorphism of the space $\mathcal{V}$:

$$\mathbf{M}^{-1}\cdot\mathbf{M}_A(\mathbf{u}) \in \mathcal{V}\otimes\mathcal{V}^\star.$$

The properties of this automorphism are closely connected with the geometrical properties of the pore space. Specifically, its eigenvalues $\mu \in R$, and eigenvectors $\mathbf{k} \in \mathcal{V}$, defined by

$$\mathbf{M}^{-1} \cdot \mathbf{M}_A(\mathbf{k}) \cdot \mathbf{k} = \mu\,\mathbf{k}, \tag{7.1}$$

determine the principal directions, in the pore space, of particular properties. These are now determined from relations between the Euclidean unit vector $\mathbf{n}$, and the Minkowski unit vector $\mathbf{N}$ orthogonal to the surface element $\boldsymbol{\sigma}$, with respect to a suitable metric (Fig. 1).

Similar to (6.11), the unit surface element $\mathbf{E}_2^A$ is related to its Minkowski unit normal vector $\mathbf{N}$ by

$$\mathbf{E}_2^A = \mathbf{E}_3^{A\,\cdot \mathbf{M}_A(\mathbf{N})\cdot \mathbf{N}}, \tag{7.2}$$

where $\mathbf{E}_3^A$ is the Minkowski unit volume element ($\mathbf{E}_3^A \,\vdots\, \mathbf{V}_A \,\vdots\, \mathbf{E}_3^A = 1$). In expression (7.2) the notation (2.3) has been used. For a surface element $\boldsymbol{\sigma}$ co-planar with $\mathbf{E}_2$ and $\mathbf{E}_2^A$, and with the same orientation,

$$\boldsymbol{\sigma} = S\,\mathbf{E}_2 = S_A\,\mathbf{E}_2^A.$$

Since

$$S_A = \lambda(\boldsymbol{\sigma})\,S, \qquad \mathbf{E}_3 = f_v\,\mathbf{E}_3^A, \tag{7.3}$$

from (6.11) and (7.2) follows a relation between the unit vectors $\mathbf{n}$ and $\mathbf{N}$ orthogonal to surface element $\boldsymbol{\sigma}$, with respect to Euclidean and Minkowski metrics, respectively:

$$f_v\,\mathbf{M} \cdot \mathbf{n} = \lambda(\boldsymbol{\sigma})\,\mathbf{M}_A(\mathbf{N}) \cdot \mathbf{N}. \tag{7.4}$$

Relation (7.4) is fundamental for the geometry of the anisotropic pore space, since it contains the basic information about its structure. For example, the definition of surface porosity (6.12) can be obtained directly from (7.4).

Comparing relation (7.4) with condition (7.1), the eigenvectors $\mathbf{k}$ of the automorphism $\mathbf{M}^{-1} \cdot \mathbf{M}_A(\mathbf{k})$ determine directions in the pore space which are orthogonal to the surface element $\boldsymbol{\sigma}$, with respect to both the Euclidean and Minkowski metric. That is, the principal directions in the pore space are those for which the unit vectors $\mathbf{n}$ and $\mathbf{N}$ are co-linear. Then,

$$\mathbf{n} = \delta(\mathbf{N})\,\mathbf{N}, \tag{7.5}$$

and setting $\mathbf{n} = \mathbf{k}$ in (7.4) yields

$$\delta(\mathbf{k})\,\lambda(\boldsymbol{\sigma}) = f_v, \tag{7.6}$$

which must be satisfied for each principal direction of the pore space. Therefore, the eigenvalue of the automorphism $\mathbf{M}^{-1} \cdot \mathbf{M}_A(\mathbf{k})$ for the eigenvector $\mathbf{k}$ is given by

$$\mu = \delta^2(\mathbf{k}). \tag{7.7}$$

It can be proved that the principal directions of the pore space determine the directions for which the parameters of pore tortuosity take their extreme values.

8. Description of tetragonal pore space

This section illustrates the power of the metric-based approach to description of the pore space structure of permeable porous materials. It presents the main results of the paper [20] devoted to macroscopic description of the pore space with tetragonal symmetry. The anisotropic properties of such media are predicted by the theory given in this paper. Such prediction is not consistent with the description presented e.g. in papers [35] and [34], in which problems of mass, linear momentum and energy transport are considered in porous media that have the tetragonal symmetry of microscopic structure formed by a bundle of parallel cylindrical fibers arranged in a square network. It is proved that the macroscopic pore space structure of such media is isotropic in the plane perpendicular to the axes of fibres.

The solution of the apparent contradiction between these two descriptions is extension of the class of porous materials, the pore space of which is modeled as the Minkowski metric space. An example of microscopic structure of such a medium is shown in Fig. 2. In that case, the material consists of a bundle of parallel fibres with rhomboidal cross-sections which are able to rotate free around the axis formed by one of their edges. These axes are ordered in a square net.

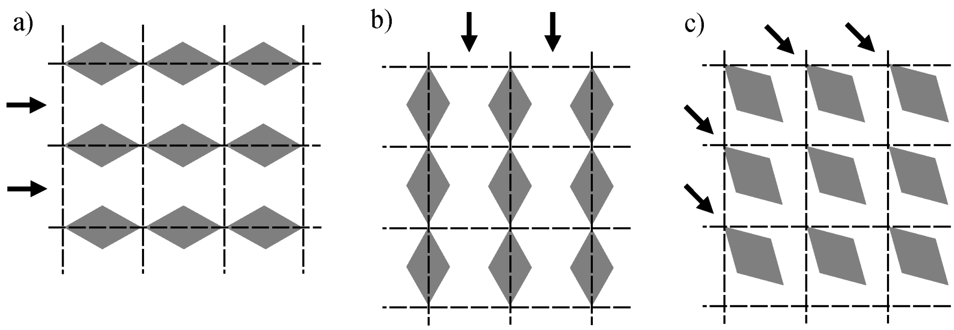

Fig. 2. Tetragonal architecture of porous medium with rotary rhomboidal fibres. Configuration of fibres cross-section for transport processes in the main (a), (b) and diagonal (c) directions of the pore space [20].

It was assumed that during the process (e.g. a fluid flow), the fibres take position of balance minimizing the resistance to flow. For a transport process in the main directions of the square net (Fig. 2a, b), the configurations of fibres placement are the same. Therefore, the properties of such a medium in both directions are identical. In turn, for a transport process in the diagonal direction of the square net (Fig. 2c), the configuration of fibres placement is essentially different from those for a transport process in the main direction.

To describe the anisotropic properties of the tetragonal space, the norm of the form (2.11) has been applied. It was assumed that the fourth-order tensor has internal properties of the compliance tensor used in the linear theory of elasticity of anisotropic materials. After spectral decomposition of this tensor and application of the automorphisms group describing point symmetries of the square net, the general form of the norm (2.11), for tetragonal pore space was obtained,

$$L_A^4(\mathbf{u}) = \delta_o^4 \left[(\mathbf{u} \circ \mathbf{u})^2 + 4(\alpha - 1)(\mathbf{u} \circ \mathbf{e}_1)^2 (\mathbf{u} \circ \mathbf{e}_2)^2 \right] \tag{8.1}$$

where

$$\alpha = (\delta_p/\delta_o)^4 \,, \tag{8.2}$$

and $\mathbf{u} \circ \mathbf{v} \equiv \mathbf{u} \cdot \mathbf{M} \cdot \mathbf{v}$ is the Euclidean scalar product of vectors in the space $\mathcal{V}$, whereas $\mathbf{e}_1$ and $\mathbf{e}_2$ ($\mathbf{e}_1 \circ \mathbf{e}_2 = 0$) are the Euclidean unit vectors indicating main directions of the tetragonal pore space.

Then, the pore tortuosity $\delta(\mathbf{n})$ in direction $\mathbf{n}$ of the pore space, defined by the formula (6.2) can be represented by expression

$$\delta^4(\mathbf{n}) = \delta_o^4 \left[1 + 4(\alpha - 1)(\mathbf{n} \circ \mathbf{e}_1)^2 \; (\mathbf{n} \circ \mathbf{e}_2)^2 \right] , \tag{8.3}$$

and in the polar coordinates it reduces to the form

$$\delta^4 = \delta_o^4 \left[1 + (\alpha - 1) \sin^2(2\varphi) \right] , \tag{8.4}$$

where φ is the angle between versors $\mathbf{e}_1$ and $\mathbf{n}$.

Quantities δ_o and δ_p in expressions (8.1) and (8.2) stand for parameters of tortuosity in the main directions of the pore space, defined by vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ and in the diagonal directions defined by $(\mathbf{e}_2 + \mathbf{e}_1)/\sqrt{2}$ and $(\mathbf{e}_2 - \mathbf{e}_1)/\sqrt{2}$, respectively. The norm (8.1) is positively definite for any values of parameters δ_o and δ_p and their ratio α is restricted by the convexity condition (2.9) of function $L_A(\mathbf{u})$,

$$1/2 < \alpha < 2.$$

9. Concluding remarks

This paper is the first one of a series devoted to a description of the dynamics of fluid-saturated porous materials, utilizing the concept of a Minkowski metric space to model the anisotropic pore space. It introduces the basic concepts for the proposed approach. Special attention was paid to the mathematical description of anisotropic metric spaces, and their adaptation to a model of anisotropic pore space. The metric of Minkowski space was used to define macroscopic measures of distance, surface and volume elements, in the pore space, which in turn allow the definitions of macroscopic parameters characterizing the anisotropic pore space structure: surface and volume porosities and pore tortuosity, together with their associated tensors directly related to the metric tensor of the pore space. Examples of metrics for spaces with symmetric and asymmetric anisotropy were proposed.

From considerations performed in the paper it results that the pore tortuosity parameter and its tensor characteristics are essential for proper macroscopic description of the pore space structure and transport processes in anisotropic porous materials. Its importance was not estimated enough in the literature, mainly because of big troubles with its definition and introduction into description of physical processes. The approach proposed in the paper radically changes this situation. The definition of the tortuosity results directly from the applied model and the fact that it is defined by the metric tensor of the pore space indicate that this parameter, similarly to the volume porosity and the specific pore surface, has a purely geometrical character. It means also that tortuosity is of fundamental importance for all physical processes taking place in the pore space. For example, it strongly influences the velocity of wave propagation in a fluid filling a rigid skeleton and also the resistivity of an electrolyte filling a non-conductive skeleton. Therefore, this parameter can be determined from the microscopic geometry of the pore space, e.g. by simulation of current passage. At the pore level it is described by the Laplace equation, and macroscopic relative resistivity (formation factor) of such material depends only on the pore structure parameters: volume porosity and pore tortuosity. Also, measurements of both quantities: wave velocity and electric resistivity in porous materials are commonly used for identification of the pore tortuosity.

Since the pore tortuosity is defined by the metric tensor of the Minkowski (pore) space (see Eq. (6.2)), both types of measurement can be used to determine components of the metric tensor. This, in turn, allows determination of the surface porosity for any cross-section of the porous material (see Eq. (6.12)) provided that the volume porosity is measured by the other method. Such identification needs, however, a proper description of the transport processes in porous materials which should relate the coefficients characterizing these processes to the pore structure parameters.

The problem of a fluid motion in an anisotropic pore space, and its constitutive description, will be treated in subsequent papers. It should be emphasized that the proposed approach is also applicable to modeling deformable fluid-saturated porous materials with anisotropic pore space, where the concept of a deforming anisotropic space would have to be used.

References

1. P. Adler, *Porous Media. Geometry and Transports*, Butter-Worth-Heinemann, Boston-London-Sydney 1992.
2. P.L. Antonelli, R.S. Ingarden, M. Matsumoto, *Theory of Sprays and Finsler Spaces with Application in Physics and Biology*, Kluwer Academic Publishers, 1993.
3. K. Atteborough, *Acoustical Characteristics of Rigid Fibrous Absorbents and Granular Materials*, J. Acoust. Soc. Am., **73**, 785–799, 1983.
4. J. Auriault, L. Borne, R. Chambon, *Dynamics of Porous Saturated Media*, J. Acoust. Soc. Am., **77**, 427–444, 1985.
5. M. Bao, S. Chern, Z. Shen, *Introduction to Riemann-Finsler Geometry*, Springer Verlag, New York-Berlin 2000.
6. J. Bear, *Dynamics of Fluid in Porous Media*, Elsevier, Amsterdam 1972.
7. A. Bedford, D.S. Drumheller, *Theories of Immiscible and Structured Media*, Int. J. Engng. Sci., **21**, 863–960, 1983.
8. A. Bejancu, *Finsler Geometry and Applications*, Ellis Horwood, New York 1990.
9. M.A. Biot, *Theory of Propagation of Elastic Waves in Fluid Saturated Porous Solid*, J. Acoust. Soc. Am., **28**, 161–191, 1956.
10. M.A. Biot, *Mechanics of Deformation and Acoustic Propagation in Porous Media*, J. Acoust. Soc. Am., **4**, 1482–1498, 1962.
11. B. Biswal, C. Manwart, R. Hilfer, *Three-Dimensional Local Porosity Analysis of Porous Media*, Physica A, **255**, 221–241, 1998.
12. R.M. Bowen, *Theory of Mixtures*, [in:] *Continuum Physics*, A.C. Eringen [Ed.], Academic Press, New York 1960.
13. M. Bowen, C.C. Wang, *Introduction to Vectors and Tensors*, Plenum Press, New York-London 1976.
14. W. Busemann, *Intrinsic Area*, Ann. of Math., **48**, 234–267, 1947.
15. P. Carman, *Fluid Flow through Granular Beds*, Trans. Inst. Chem. Engng., **15**, 150–166, 1937.
16. P. Carman, *Flow of Gases through Porous Media*, Butterworths Scientific Publ., London 1956.

17. M. Cieszko, *Minkowski Space as a Model of Anisotropic Pore Space of Permeable Materials. Modelling of the Skeleton Pore Structure*, [in:] Proceedings of International Symposium on Trends in Continuum Physics, World Scientific, 1998, 87–98.

18. M. Cieszko, *Application of Minkowski Space to Description of Anisotropic Pore Space Structure in Porous Materials*, ZAMM, **80** SI: 129–132, 2000.

19. M. Cieszko, *Description of Anisotropic Pore Space Structure and Fluid Dynamics in Porous Materials. Aplication of Minkowski Metric Space* [in Polish], Kazimierz Wielki Univ. Press, Bydgoszcz 1999.

20. M. Cieszko, W. Kriese, *Description of Tetragonal Pore Space Structure of Porous Materials*, Arch. Mech., **58**, 477–488, 2006.

21. D. Cioranescu, *An Introduction to Homogenization*, Oxford Univ. Press, London 1999.

22. O. Coussy, *Mechanics of Porous Continua*, J. Wiley and Sons, New York-London 1995.

23. W. Derski, S. Kowalski, *Fluid Flow through Granular Beds*, Stud. Geot. et Mech., **2**, 3–12, 1980.

24. F.A.L. Dullien, *Porous Media*, Acad. Press, New York 1979.

25. M. Hassanizadeh, W. Grey, *General Conservation Equation for Multiphase Systems*, Adw. Water Res., **2**, 131–144, 1979.

26. R. Hilfer, *Local Porosity Theory for Flow in Porous Media*, Physical Review B, **45**, 7115–7128, 1992.

27. R. Hilfer, *Transport and Relaxation Phenomena in Porous Media*, Adv. Chem. Phys., **XCII**, 299–324, 1996.

28. M. Kaczmarek, *Extended Internal Geometry Description in Modeling of Dynamics of Fluid-Filled Permeable Media*, J. Transport in Porous Media, **9**, 113–121, 1992.

29. J. Kubik, *A Macroscopic Description of Geometrical Pore Structure of Porous Solids*, Int. J. Engng. Sci., **24**, 971–980, 1986.

30. J. Kubik, *Pore Structure in Dynamic Behavior of Saturated Materials*, J. Transport in Porous Media, **9**, 15–24, 1992.

31. M. Matsumoto, *Foundations of Finsler Geometry and Spatial Finsler Spaces*, Kaiseisha Press, Shigaken 1986.

32. L.W. Morland, H.S. Sellers, *Multiphase Mixtures and Singular Surfaces*, Int. J. Non-Linear Mech., **36**, 131–146, 2000.

33. R. Nigmatulin, *The Fundamentals of Mechanics of Heteogeneous Media* [in Russian], Nauka, Moskva 1978.

34. W.T. Perris, D.R. McKenzie, R.C. McPhedran, *Transport Properties of Regular Arrays of Cylinders*, Proc. R. Soc. Lond., A **369**, 207–225, 1979.

35. Lord Rayleigh, *On the Influence of Obstacles Arranged in Rectangular Order upon the Properties of a Medium*, Phil. Mag., **34**, 481–502, 1892.

36. H. Rund, *The Differential Geometry of Finsler Spaces*, Springer-Verlag, Berlin-Göttingen-Heidelberg 1959.

37. J. Saczuk, *Finslerian Foundations of Solid Mechanics*, Reports of Institute of Fluid-Flow Machinery, Gdańsk, 1996.

38. A. Scheideger, *The Physics of Flow through Porous Media*, Univ. of Toronto, Toronto 1974.

39. J.J. Telega, W. Bielski, *Flow in Random Porous Media: Effective Models*, Comp. Geomechanics, **30**, 271–288, 2003.

40. A.C. Thompson, *Minkowski Geometry*, Cambridge University Press, Cambridge 1996.

41. S. Torquato, *Statistical Description of Microstructures*, Annu. Rev. Mater. **32**, 77–111, 2003.

42. S. Torquato, *Random Heterogeneous Materials: Microstructure and Macroscopic Properties*, Springer-Verlag, New York 2002.

43. C. Truesdell, R.A. Toupin, *The Classical Field Theory*, [in:] Handbuch der Physik. Vol. III/1, Springer-Verlag, Berlin-Göttingen-Heidelberg 1960.

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