Arch. Mech., **61**, 2, pp. 121–136, Warszawa 2009

Influences of magnetic field on wave propagation in generalized thermoelastic solid with diffusion

S. M. ABO-DAHAB[1], B. SINGH[2]

[1] *Mathematics Department, Faculty of Science*
Qena 83523, South Valley University, Egypt
e-mail: sdahb@yahoo.com

[2] *Department of Mathematics, Government College, Sector 11*
Chandigarh 160 011, India
e-mail: baljeet@networkindia.net

This paper is devoted to estimation of the influence of magnetic field in an elastic solid half-space under thermoelastic diffusion. The governing equations in xz-plane are solved taking into consideration the GL model. The reflection of dilatational (P) wave and shear vertical (SV) wave splits into four waves, namely: P wave, thermal wave, mass diffusion wave and SV wave. The reflection phenomena of P and SV waves from the free surface of an elastic solid with thermoelastic diffusion, under the influence of magnetic field is considered. The expressions for the reflection coefficients for the four reflected waves are obtained. These reflection coefficients are found to depend upon the angle of incidence θ of P and SV waves, thermoelastic diffusion, magnetic field and other material parameters. The numerical values for the reflection coefficients are calculated analytically and presented graphically for various values of these parameters. Relevant results of previous investigations are deduced as special cases from this study.

Key words: GL model, thermoelasticity, P and SV waves, magnetic field, mass diffusion, reflection coefficients.

Notations

τ_{ij} components of Maxwell's stress tensor,
T_0 natural state temperature of the medium,
u_i components of the displacement tensor,
$\vec{H}_0$ primary, constant magnetic field vector,
a, b measures of thermoelastic diffusion,
α_c coefficient of linear diffusion expansion,
α_t coefficient of linear thermal expansion,
P chemical potential per unit mass,
$\vec{h}$ perturbed magnetic field over $\vec{H}_0$,
σ_{ij} components of the stress tensor,
τ^0, τ^1 diffusion relaxation times,
τ_0, τ_1 thermal relaxation times,
$\vec{F}$ Lorenz's body forces vector,

D thermoelastic diffusion constant,
C_v specific heat per unit mass,
$\vec{H}$ total magnetic field vector,
μ_e magnetic permeability,
λ, μ Lamé's constants,
S entropy per unit mass,
$\vec{B}$ magnetic induction,
K thermal conductivity,
Θ predicted finite speed,
$\vec{j}$ electric intensity,
C concentration,
k wave number,
v phase speed,
$\beta_1 = \alpha_t(3\lambda + 2\mu)$,
$\beta_2 = \alpha_c(3\lambda + 2\mu)$,
$\Theta = T - T_0$.

1. Introduction

Thermoelasticity theories that predict a finite speed for the propagation of thermal signals have arisen much interest in the last four decades. Danilovskaya [1] was the first author to solve an actual problem in the theory of elasticity with nonuniform heat distribution. The problem was the half-space subjected of thermal shock, in the context of what it became known as the theory of uncoupled thermoelasticity. In this theory, the temperature is governed by a parabolic partial differential equation that does not contain any elastic terms, unlike the conventional thermoelasticity theory [2], based on a parabolic heat equation, which predicts an infinite speed for the propagation of heat, generalized and modified into various thermoelastic models based on hyperbolic thermoelasticity [3]. These theories, referred to as generalized thermoelasticity, were introduced in the literature in an attempt to eliminate the shortcomings of the classical dynamical thermoelasticity. For example, Lord and Shulman [4], by incorporating a flux-rate term into Fourier's law of heat conduction, formulated a generalized theory which involves a hyperbolic heat transport equation admitting finite speed for thermal signals. Green and Lindsay [5], by including temperature rate among the constitutive variables, developed a temperature-rate dependent thermoelasticity that did not violate the classical Fourier law of heat conduction, when the body under consideration has a center of symmetry, and this theory also predicts a finite speed of heat propagation.

During the second half of the twentieth century, non-isothermal problems of the theory of elasticity became increasingly attractive. This is due mainly to their numerous applications in widely diverse fields. First, the high velocities of modern aircraft give rise to aerodynamic heating, which produces high thermal stresses, reducing the strength of the aircraft structure. Secondly, in the nuclear field, the external high temperatures and temperature gradients originating inside the nuclear reactors influence their design and operations (Nowinski [6]). Chandrasekharaiah [7] referred to this wavelike thermal disturbance as a "second sound". The Lord and Shulman theory of generalized thermoelasticity was further extended by Dhaliwal and Sherief [8] to include the anisotropic case. A survey article of representative theories in the range of generalized thermoelasticity is due to Hetnarski and Ignaczak [9]. Chandrasekharaiah [10] introduced a review literature about hyperbolic thermoelasticity. Sinha and Sinha [11] and Sinha and Elsibai [12, 13] studied the reflection of thermoelastic waves from the free surface of a solid half-space and at the interface of two semi-infinite media in welded contact, in the context of generalized thermoelasticity. Abd-Alla and Al-Dawy [14] studied the reflection phenomena of SV waves in a generalized thermoelastic medium. Recently, Sharma *et al.* [15] investigated the problem of thermoelastic wave reflection from the insulated and isothermal stress-free as well as rigidly fixed boundaries of a solid half-space, in the context of different theories of generalized thermoelasticity.

In recent years, the theory of magneto-thermoelasticity which deals with the interactions between strain, temperature and electromagnetic fields, have drawn the attention of many researchers because of their extensive applications in diverse fields, such as geophysics, for understanding of the effect of the Earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiation from nuclear devices, development of a highly sensitive superconducting magnetometers, electrical power engineering, optics, etc. Knopoff [16] and Chadwick [17] studied these types of problems in the beginning, developed later by Kaliski and Petykiewicz [18]. Abd-Alla *et al.* [19] studied the reflection of generalized magneto-thermo-viscoelastic plane waves. Ezzat and Youssef [20] studied the generalized magneto-thermoelasticity in a perfectly conducting medium. Baksi *et al.* [21] illustrate magneto-thermoelastic problems with thermal relaxation and heat sources in a three-dimensional, infinite, rotating elastic medium.

Diffusion may be defined as a random walk of an ensemble of particles, from regions of high concentration to regions of lower concentration. The study of this phenomenon is of great concern due to its many geophysical and industrial applications. In integrated circuit fabrication, diffusion is used to introduce "dopants" in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source/drain regions in Metal Oxide Semiconductor (MOS) transistors and dope poly-silicon gates in MOS transistors. The phenomenon of diffusion is used to improve the conditions of oil extraction (seeking ways of more efficient recovering of oil from oil deposits). These days, oil companies are interested in the process of thermoelastic diffusion for more efficient extraction of oil from oil deposits. Using the coupled thermoelastic model, Nowacki [22–24] developed the theory of thermoelastic diffusion. Using the L-S Model, Sherief *et al.* [25] generalized the theory of thermoelastic diffusion, which allows finite speeds of propagation of waves. The present study is motivated by the importance of thermoelastic diffusion process in the field of oil extraction. Recently, Singh [26, 27] investigated the reflection phenomena of P and SV waves with generalized thermo-elastic diffusion.

In this article, the P and SV waves on an isotropic homogeneous solid half-space under influence of the magnetic field with generalized thermoelastic diffusion, is studied for the GL model. The paper is classified as follows: In Sec. 2, we present the governing equations and solutions. The expressions of reflection coefficients upon magnetic field, thermal relaxation times, diffusion relaxation times, angle of incidence of P and SV waves and other thermal and diffusion parameters at free surface, are derived in Sec. 3. The numerical results are presented and the effect of magnetic field is shown graphically and discussed in Sec. 4. Finally, the conclusions are drawn in Sec. 5.

2. Governing equations and solution

The governing equations for an isotropic, homogeneous, elastic solid with generalized thermoelastic diffusion at reference temperature T_0 with the body forces are:

(i) The constitutive equations:

$$\sigma_{ij} = \left[\lambda e_{kk} - \beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\Theta - \beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C\right]\delta_{ij} + 2\mu e_{ij}, \tag{2.1}$$

$$\rho T_0 S = \rho C_v\left(1+\tau_0\frac{\partial}{\partial t}\right)\Theta + \beta_1 T_0 e_{kk} + aT_0\left(1+\tau^0\frac{\partial}{\partial t}\right)C, \tag{2.2}$$

$$P = -(\beta_2 - b)\left(1+\tau^1\frac{\partial}{\partial t}\right)e_{kk} - a\left(1+\tau_1\frac{\partial}{\partial t}\right)\Theta. \tag{2.3}$$

(ii) Maxwell's stress equation:

$$\tau_{ij} = \mu_e[H_i h_j + H_j h_i - H_k.h_k\delta_{ij}]. \tag{2.4}$$

Let us assume that the medium is a perfect electric conductor, and that the linearized Maxwell equations are governing the electromagnetic field, taking into account absence of the displacement current (SI); then we have

$$\operatorname{curl}\vec{h} = \vec{j},$$
$$\operatorname{curl}\vec{E} = -\mu_e\frac{\partial\vec{h}}{\partial t}, \tag{2.5}$$
$$\operatorname{div}\vec{h} = 0,$$
$$\operatorname{div}\vec{E} = 0,$$

where

$$\vec{h} = \operatorname{curl}(\vec{u}\times\vec{H}_0), \tag{2.6}$$
$$\vec{H} = \vec{H}_0 + \vec{h}, \qquad \vec{H}_0 = (0, H, 0),$$

H denoting constant primary magnetic field acting in direction y.

(iii) Equation of motion:

$$\sigma_{ji,j} + \tau_{ji,j} = \rho\ddot{u}_i, \tag{2.7}$$

which tends to

$$\mu u_{i,jj} + (\lambda+\mu)u_{j,ij} - \beta_1\Big(1+\tau_1\frac{\partial}{\partial t}\Big)\Theta_{,i} - \beta_2\Big(1+\tau^1\frac{\partial}{\partial t}\Big)C_{,i} + F_i = \rho\ddot{u}_i, \tag{2.8}$$

where

$$\vec{F} = \vec{j}\times\vec{B}.$$

(iv) Equation of heat conduction

$$\rho C_v\Big(1+\tau_0\frac{\partial}{\partial t}\Big)\dot{\Theta} + \beta_1 T_0\dot{e}_{kk} + aT_0\Big(1+\tau^0\frac{\partial}{\partial t}\Big)\dot{C} = K\Theta_{,kk}. \tag{2.9}$$

(v) Equation of mass diffusion

$$D\beta_2 e_{,ii} + Da\Big(1+\tau_1\frac{\partial}{\partial t}\Big)\Theta_{,ii} + \dot{C} - Db\Big(1+\tau^1\frac{\partial}{\partial t}\Big)C_{,ii} = 0. \tag{2.10}$$

If the body forces are neglected and the L-S model is applied, the relevant equations are deduced as in [26].

The thermal relaxation times τ_1 and τ_0 satisfy the inequality $\tau_1 \geq \tau_0 > 0$, the diffusion relaxation times τ^1 and τ^0 also satisfy the inequality $\tau^1 \geq \tau^0 > 0$.

For two-dimensional motion in xz plane, the Eqs. (2.6), (2.9) and (2.10) are written as

$$(\lambda+2\mu)u_{1,11} + (\lambda+\mu)u_{3,13} + \mu u_{1,33} - \beta_1\tau_\theta^1\Theta_{,1} - \beta_2\tau_c^1 C_{,1} + F_1 = \rho\ddot{u}_1, \tag{2.11}$$

$$(\lambda+2\mu)u_{3,33} + (\lambda+\mu)u_{1,31} + \mu u_{3,11} - \beta_1\tau_\theta^1\Theta_{,3} - \beta_2\tau_c^1 C_{,3} + F_3 = \rho\ddot{u}_3, \tag{2.12}$$

$$K\nabla^2\Theta = \rho C_v\tau_\theta^0\dot{\Theta} + \beta_1 T_0\dot{e} + aT_0\tau_c^0\dot{C}, \tag{2.13}$$

$$D\beta_2\nabla^2 e + Da\tau_\theta^1\nabla^2\Theta - Db\tau_c^1\nabla^2 C + \dot{C} = 0, \tag{2.14}$$

where

$$\tau_\theta^1 = 1 + \tau_1 \frac{\partial}{\partial t}, \qquad \tau_c^1 = 1 + \tau^1 \frac{\partial}{\partial t}, \qquad \tau_\theta^0 = 1 + \tau_0 \frac{\partial}{\partial t}, \qquad \tau_c^0 = 1 + \tau^0 \frac{\partial}{\partial t},$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}.$$

The displacement components u_1 and u_3 may be written in terms of scalar and vector potential functions ϕ and ψ as

$$\vec{u} = \vec{\nabla}\phi + \vec{\nabla} \times \vec{\psi}, \tag{2.15}$$

and take the form

$$u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \qquad u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}. \tag{2.16}$$

Substituting the displacements from Eqs. (2.16) into Eqs. (2.11)–(2.14), we get

$$\mu \nabla^2 \psi = \rho \frac{\partial^2 \psi}{\partial t^2}, \tag{2.17}$$

$$(\lambda + 2\mu + \mu_e H^2)\nabla^2 \phi - \beta_1 \tau_\theta^1 \Theta - \beta_2 \tau_c^1 C = \rho \frac{\partial^2 \phi}{\partial t^2}, \tag{2.18}$$

$$K\nabla^2 \Theta = \rho C_v \tau_\theta^0 \frac{\partial \Theta}{\partial t} + \beta_1 T_0 \frac{\partial}{\partial t} \nabla^2 \phi + a T_0 \tau_c^0 \frac{\partial C}{\partial t}, \tag{2.19}$$

$$D\beta_2 \nabla^2 \phi + Da\tau_\theta^1 \nabla^2 \Theta - Db\tau_c^1 \nabla^2 C + \frac{\partial C}{\partial t} = 0. \tag{2.20}$$

It follows from Eqs. (2.17)–(2.20) that the SV wave does not influence the thermal, magnetic and diffusion fields but the P-wave does. The solution of Eq. (2.17) corresponds to the propagation of SV waves with velocity $V_s = \sqrt{\mu/\rho}$.

For solving analytically the Eqs. (2.18)–(2.20) in the form of a harmonic travelling wave, we suppose the solution to take the form

$$\{\phi, \Theta, C\} = \{\phi_0, \Theta_0, C_0\} e^{ik(x \sin\theta + z\cos\theta - vt)}, \tag{2.21}$$

where the pair $(\sin\theta, \cos\theta)$ denotes the projection of the wave normal onto the xz-plane.

The homogeneous system of equations in ϕ_0, Θ_0 and C_0 obtained by inserting (2.21) into Eqs. (2.18) to (2.20), admits non-trivial solutions and enables to conclude that it satisfies the cubic equation

$$\xi^3 + L\xi^2 + M\xi + N = 0, \tag{2.22}$$

where

$$\xi = \rho v^2, \qquad \tau_\theta = \tau_0 + i\omega^{-1}, \qquad \tau_c = \tau^0 + i\omega^{-1}, \qquad \tau_\theta^{11} = 1 - i\omega\tau_1,$$

$$\tau_c^{11} = 1 - i\omega\tau^1, \quad d_1 = \frac{K}{c_v \tau_\theta}, \qquad d_2 = -i\omega\rho D b \tau_c^{11},$$

$$\epsilon = \frac{\beta_1^2 T_0 \tau_\theta^{11}}{i\omega\rho c_v \tau_\theta}, \quad \epsilon_1 = -\frac{a}{\beta_1\beta_2}, \qquad \epsilon_2 = -i\omega\rho D \beta_2^2 \tau_c^{11}, \qquad \epsilon_3 = \frac{i\omega a \tau_c}{\beta_1\beta_2\tau_c^{11}},$$

$$M = (\lambda + 2\mu + \mu_e H^2)(d_1 + d_2 + \epsilon\epsilon_1\epsilon_2\epsilon_3) + d_1 d_2 + d_2\epsilon - \epsilon\epsilon_2(\epsilon_1 + \epsilon_3) - \epsilon_2,$$

$$N = -d_1 d_2(\lambda + 2\mu + \mu_e H^2) + \epsilon_2 d_1,$$

$$L = -(\epsilon + \epsilon\epsilon_1\epsilon_2\epsilon_3 + d_1 + d_2 + \lambda + 2\mu + \mu_e H^2).$$

The roots of Eq. (2.22) give three values of ξ; each value of ξ corresponds to a wave if v^2 is real and positive. Hence, three positive values of v will be the velocities of propagation of three possible waves.

Using Cardan's method, Eq. (2.22) is written as:

$$\Lambda^3 + 3Q\Lambda + G = 0, \tag{2.23}$$

where

$$\Lambda = \xi + \frac{L}{3}, \qquad Q = \frac{3M - L^3}{9}, \qquad G = \frac{27N - 9LM + 2L^3}{27}. \tag{2.24}$$

For all the three roots of Eq. (23) to be real, Δ_0 $(= G^2 + 4Q^3)$ should be negative. Assuming Δ_0 to be negative, we obtain the three roots of Eq. (23) as

$$\Lambda_n = 2\sqrt{-Q}\cos\left(\frac{\Phi + 2\pi(n-1)}{3}\right), \qquad n = 1, 2, 3, \tag{2.25}$$

where

$$\Phi = \tan^{-1}\left(\frac{\sqrt{|\Delta_0|}}{-G}\right). \tag{2.26}$$

Hence,

$$v_n = \sqrt{\frac{3\Lambda_n - L}{3\rho}}, \qquad n = 1, 2, 3, \tag{2.27}$$

satisfying the inequality $v_1 > v_2 > v_3$, which refer to the P wave, MD wave and T wave respectively; this fact may be verified, when we solve the Eq. (2.23) using a computer program of Cardan's Method.

3. Boundary conditions and reflection coefficients

In the previous section, it has been shown that there exist three kinds of dilatational waves and one SV wave in an isotropic elastic solid with generalized thermodiffusion. Any wave incident at the interface of two elastic solid bodies, in general, produces dilatational and rotational waves in both media [28, 29]. Let us now consider an incident P or SV wave shown in Fig. 1.

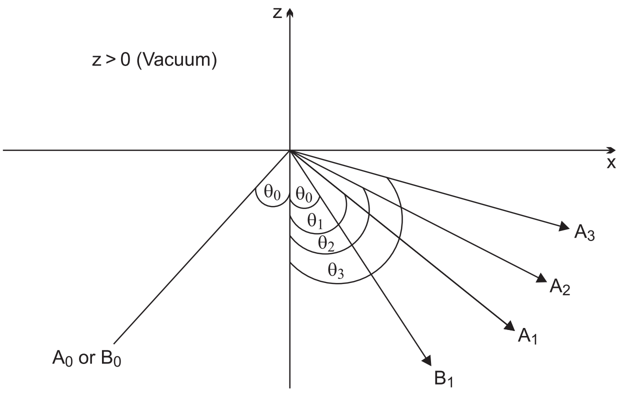

Fig. 1. Incident P and SV waves (for incident P wave, $\theta_0 = \theta_1$, for incident SV wave, $\theta_0 = \theta_4$).

Now, we take into consideration: if the wave normal of the incident wave makes angle θ_0 with the positive direction of the z-axis, and those of reflected P, T and SV waves make angles $\theta_1, \theta_2, \theta_3$ with the z-axis, the displacement potentials ψ and ϕ, the temperature Θ and the concentration C, take the following forms:

$$\begin{aligned} \psi &= B_0 \exp[ik_0(x\sin\theta_0 + z\cos\theta_0) - i\omega t] \\ &\quad + B_1 \exp[ik_3(x\sin\theta_3 - z\cos\theta_3) - i\omega t], \end{aligned} \tag{3.1}$$

$$\begin{aligned} \phi &= A_0 \exp[ik_0(x\sin\theta_0 + z\cos\theta_0) - i\omega t] \\ &\quad + A_1 \exp[ik_1(x\sin\theta_1 - z\cos\theta_1) - i\omega t] \\ &\quad + A_2 \exp[ik_2(x\sin\theta_2 - z\cos\theta_2) - i\omega t] \\ &\quad + A_3 \exp[ik_3(x\sin\theta_3 - z\cos\theta_3) - i\omega t], \end{aligned} \tag{3.2}$$

$$\begin{aligned} \Theta &= \zeta_0 A_0 \exp[ik_0(x\sin\theta_0 + z\cos\theta_0) - i\omega t] \\ &\quad + \zeta_1 A_1 \exp[ik_1(x\sin\theta_1 - z\cos\theta_1) - i\omega t] \\ &\quad + \zeta_2 A_2 \exp[ik_2(x\sin\theta_2 - z\cos\theta_2) - i\omega t] \\ &\quad + \zeta_3 A_3 \exp[ik_3(x\sin\theta_3 - z\cos\theta_3) - i\omega t], \end{aligned} \tag{3.3}$$

$$C = \eta_0 A_0 \exp[ik_0(x\sin\theta_0 + z\cos\theta_0) - i\omega t] + \eta_1 A_1 \exp[ik_1(x\sin\theta_1 - z\cos\theta_1) - i\omega t] + \eta_2 A_2 \exp[ik_2(x\sin\theta_2 - z\cos\theta_2) - i\omega t] + \eta_3 A_3 \exp[ik_3(x\sin\theta_3 - z\cos\theta_3) - i\omega t], \tag{3.4}$$

where

$$\begin{aligned} \eta_j &= k_j^2 G_j[\rho\upsilon_j^2 - \lambda - 2\mu - \mu_e H^2], \\ \zeta_j &= k_j^2 R_j[\rho\upsilon_j^2 - \lambda - 2\mu - \mu_e H^2], \end{aligned} \qquad (j = 1, 2, 3), \tag{3.5}$$

and

$$G_j = \frac{\epsilon\rho\upsilon_j^2(\epsilon_1\epsilon_2 - d_2 + \rho\upsilon_j^2)}{d_1\epsilon_2 + \rho\upsilon_j^2[\epsilon(d_1 - \rho\upsilon_j^2) - \epsilon_2 - 2\epsilon\epsilon_1\epsilon_2]}, \tag{3.6}$$

$$R_j = \frac{\epsilon_2[\rho\upsilon_j^2(\epsilon\epsilon_1 + 1) - d_1]}{d_1\epsilon_2 + \rho\upsilon_j^2[\epsilon(d_1 - \rho\upsilon_j^2) - \epsilon_2 - 2\epsilon\epsilon_1\epsilon_2]}. \tag{3.7}$$

A_0 and B_0 are the amplitudes of the incident P and SV waves, respectively, and A_1, A_2, A_3 and B_1 are the amplitudes of the reflected P, M, T and SV waves, respectively.

The boundary conditions at the free surface, i.e. $z = 0$, take the form

$$\sigma_{zz} + \tau_{zz} = 0, \quad \sigma_{zx} + \tau_{zx} = 0, \quad \frac{\partial\Theta}{\partial z} = 0, \quad \frac{\partial C}{\partial z} = 0 \qquad \text{on } z = 0. \tag{3.8}$$

For the reflected waves, the wave numbers and the reflected angles may be written as

$$k_0\sin\theta_0 = k_1\sin\theta_1 = k_2\sin\theta_2 = k_3\sin\theta_3, \tag{3.9}$$

which take the equivalent form

$$\frac{\sin\theta_0}{v_0} = \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3}. \tag{3.10}$$

Substituting the components from Eqs. (2.1), (2.4) and (3.1)–(3.4) into the boundary conditions of Eq. (3.8), we obtain a system of four algebraic equations which take the forms:

$$\sum A_{ij} Z_j = B_i \qquad (i, j = 1, 2, 3, 4), \tag{3.11}$$

where

$$A_{1j} = -\left(\lambda + 2\mu\cos^2\theta_j + \mu_e H^2 + \beta_1\tau_\theta^{11}\frac{\zeta_j}{k_j^2} + \beta_2\tau_c^{11}\frac{\eta_j}{k_j^2}\right)\left(\frac{k_j}{\ell}\right)^2,$$

$$A_{14} = \mu\sin 2\theta_4\left(\frac{k_4}{\ell}\right)^2,$$

$$A_{2j} = \sin(2\theta_j)\left(\frac{k_j}{\ell}\right)^2, \qquad A_{24} = \cos 2\theta_4\left(\frac{k_4}{\ell}\right)^2,$$

$$A_{3j} = \cos 2\theta_j\frac{\zeta_j}{k_j^2}\left(\frac{k_j}{\ell}\right)^3, \qquad A_{34} = 0,$$

$$A_{4j} = \cos\theta_j\frac{\eta_j}{k_j^2}\left(\frac{k_j}{\ell}\right)^3, \qquad A_{44} = 0.$$

Z_j and B_j for incident P and SV waves may be written in the following forms:

(i) For incident P wave

$$B_1 = -A_{11}, \quad B_2 = A_{21}, \quad B_3 = A_{31}, \quad B_4 = A_{41}, \quad \ell = k_0,$$

$$Z_1 = \frac{A_1}{A_0}, \quad Z_2 = \frac{A_2}{A_0}, \quad Z_3 = \frac{A_3}{A_0}, \quad Z_3 = \frac{B_1}{A_0}.$$

(ii) For incident SV wave

$$B_1 = A_{14}, \quad B_2 = -A_{24}, \quad B_3 = A_{34}, \quad B_4 = A_{44}, \quad \ell = k_4,$$

$$Z_1 = \frac{A_1}{B_0}, \quad Z_2 = \frac{A_2}{B_0}, \quad Z_3 = \frac{A_3}{B_0}, \quad Z_3 = \frac{B_1}{B_0}.$$

4. Numerical results and discussion

For computational work, the following material constants at $T_0 = 27°C$ are used for an elastic body subject to generalized thermoelastic diffusion and magnetic field:

$$\lambda = 5.775\times 10^{11}\ \text{dyne/cm}^2, \qquad \mu = 2.646\times 10^{11}\ \text{dyne/cm}^2,$$
$$\rho = 2.7\ \text{gm/cm}^3, \qquad C_v = 2.361\ \text{cal/gm}\,^\circ\text{C},$$
$$K = 0.492\ \text{cal/cm}\,\text{s}\,^\circ\text{C}, \qquad t = 0.05\ \text{cm}^3\text{/gm},$$
$$c = 0.06\ \text{cm}^3\text{/gm}, \qquad D = 0.5\ \text{g}\,\text{s/cm}^3,$$
$$a = 0.005\ \text{cm}^2\text{/s}^2\,^\circ\text{C}, \qquad b = 0.05\ \text{cm}^5\text{/gm}\,\text{s},$$
$$\alpha_t = 0.005, \qquad \alpha_c = 0.05,$$
$$\omega = 20\ \text{s}^{-1}, \qquad \tau_0 = \tau_1 = 0.05,$$
$$\tau^0 = \tau^1 = 0.04.$$

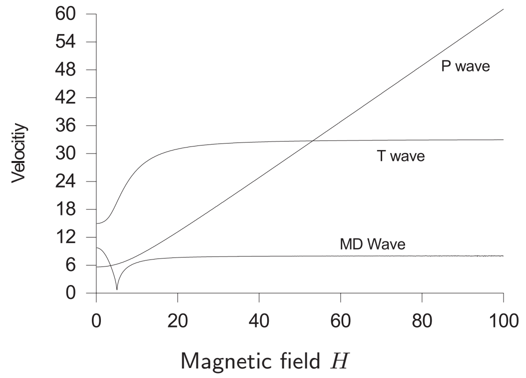

FIG. 2. Variations of (velocities of P wave, MD wave and T wave) $\times 10^2$ as functions of the magnetic field H.

Figure 2 displays the influence of the magnetic field on the velocities of P, MD and T waves for various material parameters. It is shown that with small values of the magnetic field, the P wave is the slowest but the T wave is the fastest. With increasing values of the magnetic field, it is shown that MD wave is the slowest and P wave is the fastest.

The numerical values of reflection coefficients of various reflected waves are computed for angles of incidence P wave varying from 1° to 90° for the GL model, when $0 < H < 100$ Oersted. These numerical values of reflection coefficients are shown graphically in Figs. 3–6. Figure 3 shows the reflection coefficients of SV waves for the GL model. It begins from its minimum at $\theta = 0°$, increases and then decreases to its minimum, to return to its value zero at $\theta = 90°$. Figures 4–6 show the influence of the magnetic field on reflected P, MD and T waves depending on the angle of incidence of P wave, respectively. It is shown

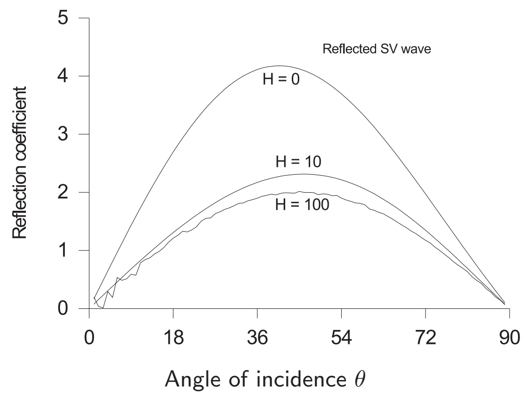

FIG. 3. Effects of the magnetic field on reflection coefficients of SV waves as functions of the angle of incidence θ of the P wave.

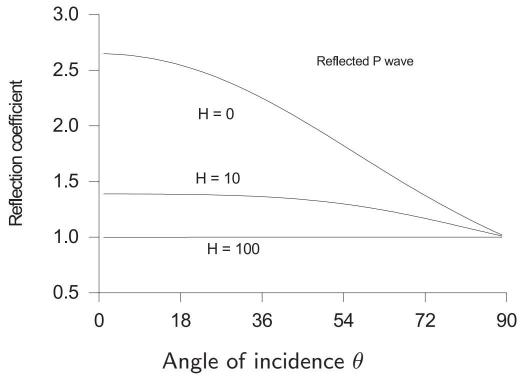

Fig. 4. Effects of the magnetic field on reflection coefficient P wave as functions of the angle of incidence θ of the P wave.

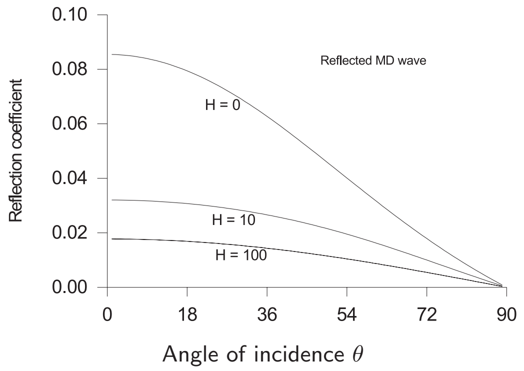

Fig. 5. Effects of the magnetic field $[H = 0, (H = 10) \times 10^3, (H = 100) \times 10^8]$ on reflection coefficient of the Mass Diffusion wave as functions of the angle of incidence θ of the P wave.

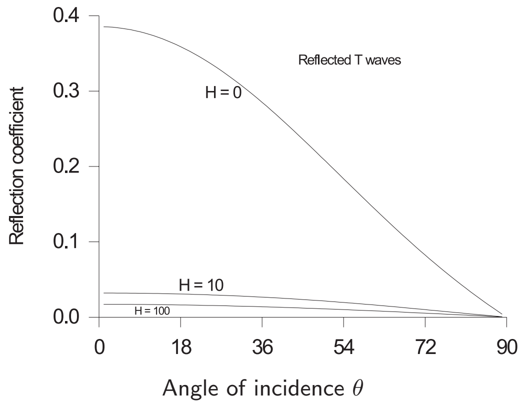

Fig. 6. Effects of the magnetic field $[H = 0, H = 10, (H = 100) \times 10^4]$ on reflection coefficient of T as functions of the angle of incidence θ of the P wave.

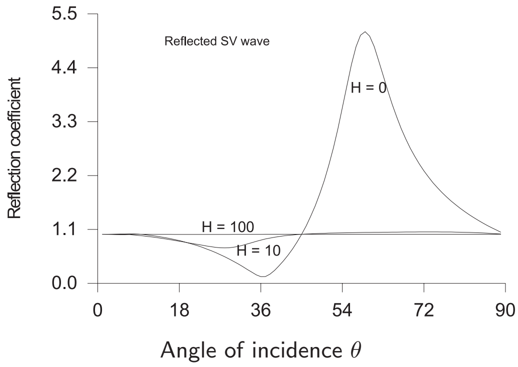

Fig. 7. Effects of the magnetic field on the reflection coefficient of SV wave as functions of the angle of incidence gθ of the SV wave.

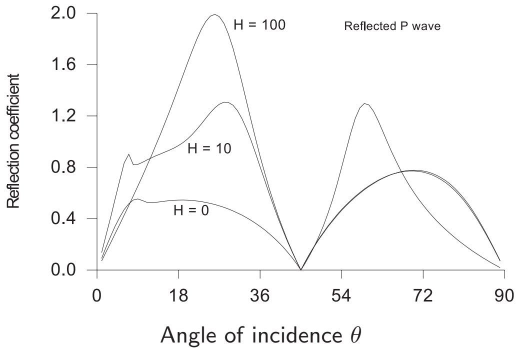

Fig. 8. Effects of the magnetic field on the reflection coefficient of P wave as functions of the angle of incidence θ of the SV wave.

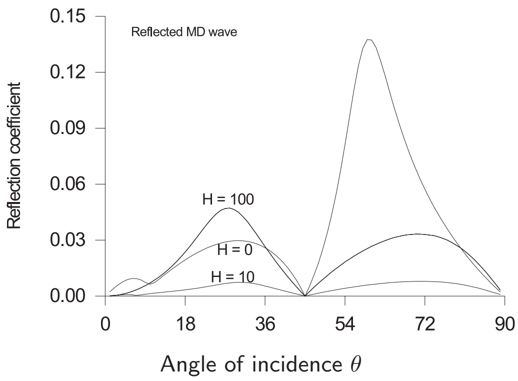

Fig. 9. Effects of the magnetic field $[H = 0$, $(H = 10) \times 10^2$, $(H = 100) \times 10^7]$ on the reflection coefficient of MD as functions of the angle of incidence θ of the SV wave.

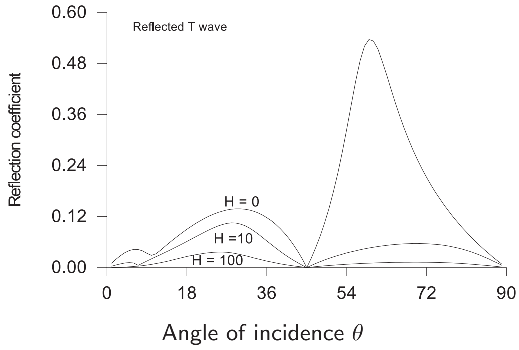

Fig. 10. Effects of the magnetic field [$H = 0$, $H = 10$, $(H = 100) \times 10^3$] on the reflection coefficient of T wave as functions of the angle of incidence θ of the SV wave.

that with increasing angle of incidence of P wave, all reflection coefficients increase from their minima to maxima and then decrease to their minimal value. Also, it can be concluded that increasing of the magnetic field leads to reduction of all reflection coefficients.

Effects of the magnetic field on reflected SV, P, MD and T waves as functions of the angle of incidence of SV wave are displayed in Figs. 7–10 respectively. It is seen that the reflection coefficients for the reflected waves start from their minima at $\theta = 0°$, increase with increasing of the angle of incidence, attain their critical values at $\theta = 45°$ and next decrease and return to their minima at $\theta = 90°$. Also, and with influence of the magnetic field, one can see that the reflection coefficient of SV wave reflected from the incident SV wave, increases with increasing of the magnetic field at the interval $0° < \theta < 45°$, and after the critical value the reflection coefficient decreases with increasing H. Dependences in Fig. 8 are more complex. From Fig. 9 it is seen that the reflection coefficient of MD wave increases with larger values of the magnetic field.

Finally, from Fig. 10 it can be seen that the reflection coefficient of T wave decreases with increasing H.

5. Conclusions

The effects of the magnetic field, thermal relaxation times and diffusion relaxation times on the reflection coefficients of SV, P, MD and T waves reflected from incident P and SV waves have been shown. The conclusions can be summarized as follows:

The angle of incidence θ affects the reflection coefficients in various ways. Analytically, it is seen that the magnetic field and thermal relaxation times have a strong influence on reflection coefficients for incident P and SV waves. For the reflection coefficients of the incident SV wave, the angle of incidence $\theta = 45°$ is a critical value for the reflected waves. Graphically, it is shown that the influence of the magnetic field is very pronounced and either increasing or decreasing the reflection coefficients. Finally, the results obtained may be applicable to seismic waves, earthquakes, geophysics, volcanos, nuclear fields, geology, etc.

References

1. V. Danilovskaya, *Thermal stresses in an elastic half-space due to sudden heating of its boundary*, Prikl. Mat. Mekh., **14**, 316–324, 1950.
2. M.A. Biot, *Thermoelasticity and irreversible thermodynamics*, J. Appl. Phys., **27**, 240–253, 1956.
3. D.E. Carlson, *Linear thermoelasticity*, Handbuch der Physik, Via/2, 97–346, 1972.
4. H.W. Lord, Y. Shulman, *A generalized dynamical theory of thermoelasticity*, J. Mech. Phys. Solids, **7**, 71–75, 1967.
5. A.E. Green, A. Lindsay, *Thermoelasticity*, J. Elasticity, **2**, 1–7, 1972.
6. W. Nowinski, *Theory of thermoelasticity with applications*, Sijthoff and Noordhoof Int., Netherlands, 1978.
7. D.S. Chandrasekharaiah, *Thermoelasticity with second sound: A Review*, Appl. Mech. Rev., **39**, 355–376, 1986.
8. R.S. Dhaliwal, H. H. Sherief, *Generalized thermoelasticity for anisotropic media*, Quart. Appl. Math., **33**, 1–8, 1980.
9. R.B. Hetnarski, J. Ignaczak, *Generalized thermoelasticity*, Journal of Thermal Stresses, **22**, 451–476, 1999.
10. D. S. Chandrasekharaiah, *Hyperbolic thermoelasticity, A review of recent literature*, Applied Mechanics Review, **51**, 705–729, 1988.
11. A.N. Sinha, S.B. Sinha, *Reflection of thermoelastic waves at a solid half-space with thermal relaxation*, J. Phys. Earth, **22**, 237–244, 1974.
12. S.B. Sinha, K.A. Elsibai, *Reflection of thermoelastic waves at a solid half-space with two thermal relaxation times*, J. Thermal Stresses, **19**, 763–777, 1996.
13. S.B. Sinha, K.A. Elsibai, *Reflection and refraction of thermoelastic waves at an interface of two semi-infinite media with two thermal relaxation times*, J. Thermal Stresses, **20**, 129–146, 1997.
14. A.N. Abd-Alla, A.S. Al-Dawy, *The reflection phenomena of SV waves in a generalized thermoelastic medium*, Int. J. Math. Math. Sci., **23**, 529–546, 2000.
15. J.N. Sharma, V. Kumar, D. Chand, *Reflection of generalized thermoelastic waves from the boundary of a half-space*, J. Thermal Stresses, **26**, 925–942, 2003.
16. L. Knopoff, *The interaction between elastic wave motions and a magnetic field in electrical conductors*, J. Geophys. Res., **60**, 441–456, 1955.
17. P. Chadwick, *Elastic waves propagation in a magnetic field*, [in:] Proceedings of the International Congress of Applied Mechanics, Brusseles, Belgium, 143–153, 1957.

18. S. Kaliski, J. Petykiewicz, *Equation of motion coupled with the field of temperature in a magnetic field involving mechanical and electrical relaxation for anisotropic bodies*, Proc. Vibr. Probl., **4**, 1–12, 1959.

19. A.N. Abd-Alla, A.A. Yahia, S.M. Abo-Dahab, *On reflection of the generalized magneto-thermo-viscoelastic plane waves*, Chaos, Solitons & Fractals, **16**, 211–231, 2003.

20. M.A. Ezzat, H.M. Youssef, *Generalized magneto-thermoelasticity in a perfectly conducting medium*, International Journal of Solids and Structures, **42**, 6319–6334, 2005.

21. A. Baksi, R.K. Bera, L. Debnath, *A study of magneto-thermoelastic problems with thermal relaxation and heat sources in a three-dimensional infinite rotating elastic medium*, International Journal of Engineering Science, **43**, 1419–1434, 2005.

22. W. Nowacki, *Dynamical problems of thermoelastic diffusion in solids I*, Bull. Acad. Pol. Sci. Ser. Sci. Tech., **22**, 55–64, 1974.

23. W. Nowacki, *Dynamical problems of thermoelastic diffusion in solids II*, Bull. Acad. Pol. Sci. Ser. Sci. Tech., **22**, 129–135, 1974.

24. W. Nowacki, *Dynamical problems of thermoelastic diffusion in solids III*, Bull. Acad. Pol. Sci. Ser. Sci. Tech., **22**, 266–274, 1974.

25. H.H. Sherief, F. Hamza, H. Saleh, *The theory of generalized thermoelastic diffusion*, Int. J. Engrg. Sci., **42**, 591–608, 2004.

26. B. Singh, *Reflection of P and SV waves from the free surface of an elastic solid with generalized thermodiffusion*, J. Earth. Syst. Sci., **114**, 2, 159–168, 2005.

27. B. Singh, *Reflection of SV waves from the free surface of an elastic solid in generalized thermoelastic diffusion*, Journal of Sound and Vibration, **291**, 764–778, 2006.

28. W.B. Ewing, W.S. Jardetzky, F. Press, *Elastic Waves in Layered Media*, McGraw-Hill, New York, p. 76, 1957.

29. A. Ben-Menhamen, S.J. Singh, *Seismic Waves and Sources*, Springer, New York, pp. 89–95, 1981.

Received July 30, 2008; revised version March 26, 2009.