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Experimental analysis of turbulent boundary layer under the influence of adverse pressure gradient*)

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The paper deals with the experimental analysis of turbulent boundary layer at the flat plate for large value of Reynolds number equal $Re_\theta \approx 3000$. The adverse pressure gradient is generated by curvature of the upper wall and corresponds to the case of pressure variation in axial compressor. The analysis is concentrated on the problem of scaling of turbulent boundary layer and on the physical background behind scaling laws being compared. The results obtained suggest, that boundary layer at APG conditions requires two velocity scales, i.e. inner (imposed by inner boundary condition) and outer (imposed by outer layer) velocity scales.

Notations

β	Clauser's pressure parameter,
C_f	local drag coefficient,
δ	boundary layer thickness at $0.99U_\infty$,
δ^+	ratio of outer to inner length scales,
δ^*, θ	displacement and momentum thicknesses,
ρ	density,
H	shape factor,
L	test section length,
Λ	George–Castillo pressure parameter,
Λ_θ	George–Castillo pressure parameter based on momentum thickness,
P_∞	static pressure of the free stream,
P^+	dimensionless pressure parameter $P^+ = \nu/\rho \cdot u_\tau^3 (dP_\infty/dx)$,
Re_θ	Reynolds number based on momentum thickness,
Sg	dimensionless distance from inlet plane $\mathrm{Sg} = x_s/L$,
Tu	turbulence intensity of the free stream $Tu = u/U_0$,
τ_w	wall shear stresses,
u_τ	friction velocity,
u^+	dimensionless velocity $u^+ = U/u_\tau$,
u', v', w'	rms values of fluctuating velocity components in x, y and z directions,

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U_0 reference inlet velocity,
U_∞ free-stream velocity,
U, V mean velocity components in x and y directions,
ν kinematic viscosity,
x, y, z streamwise, wall-normal and spanwise coordinates,
x_s longitudinal distance from inlet plane,
$\overline{y}$, y^+ dimensionless distance from the wall in outer/inner scaling.

Abbreviations

l.h.s.; r.h.s. left- and right-hand side of equations respectively,
TBL turbulent boundary layer,
APG adverse pressure gradient,
ZPG zero pressure gradient,
FPG favourable pressure gradient,
HWA hot-wire anemometry.

1. Introduction

The research on turbulent boundary layers is being carried out for more than a century and as it has been pointed out in recent review by W. K. George [1], "*. . . a little more than a decade ago the basic characteristics of turbulent boundary layer. . . were widely believed to be well understood. . . and it bothered only a few. . . that real shear stress measurements. . . differed consistently from (theoretical) results. . . and instead of re-examination of the theory it became common wisdom that there was something wrong with the experimental techniques. . .*". During the last decade this problem started to be re-examined again, the more so that the practical motivation for better understanding of TBL has appeared from the aeronautical industry, which needs more accurate modelling tools for the design of aircraft body. The modelling of TBL is of particular importance as more than a half of drag of modern aircraft originates from wall friction, which is a phenomenon directly governed by the behaviour of TBL. However, the current CFD tools reflect the physics of TBL to the extent which is presently understood and the need to improve the knowledge of TBL physics was behind the establishing of the joint EU project Wallturb [2], which is aimed at generating and analyzing new data on near-wall turbulence and extracting physical understanding of TBL from the new experiments. This in turn should enable to improve modelling of boundary layers, especially in near-wall regions.

Among the crucial issues of the subject is the problem of TBL scaling, or rather the physical background behind these scaling laws. The search for scaling laws, which was a valuable analytical tool for decades, today leads to improvement of turbulence closure models which reflect the current understanding of TBL physics. This issue becomes more complex when one considers the TBL at large Reynolds numbers with the presence of pressure gradient, where the lack of reliable data is particularly evident. As it has been pointed out by STANISLAS [2], the Adverse Pressure Gradient (APG) TBL is particularly problematic, because of the lack of proper experimental data for sufficiently large Re numbers and for experimental conditions corresponding to real engineering applications. This is why the experimental analysis of APG TBL was selected as the aim of the present research. The experiment has been performed for the conditions representative for practical turbomachinery flows, that determined not only the distribution of pressure but also required a sufficiently high value of the Reynolds number ($\mathrm{Re}_\theta \approx 3\,000$).

2. Similarity analysis of turbulent boundary layer

The idea of multi-zone structure of TBL is well established, with the separation between inner (i.e. consisting of viscous, buffer and logarithmic sublayers) and outer (i.e. the wake-law sublayer) layers. For the inner region of TBL there is a common agreement about the universality of Prandtl's scaling, given by the formula:

$$\frac{U}{u_\tau} = f(y^+;\delta^+) \tag{2.1}$$

where δ^+, which may be regarded as the equivalent of local Reynolds number [1], is defined as the ratio of outer to inner length scales, i.e.:

$$\delta^+ = \frac{\delta \cdot \tau_w}{\nu}. \tag{2.2}$$

The inner layer is usually insensitive to upstream conditions and the only possible argument concerning this issue is the extent of the inner region, where the Prandtl's scaling is valid.

The scaling of outer region, which covers over 90% of TBL thickness, is a matter of controversy and so far three different methods have been proposed for outer TBL velocity profile scaling. The first proposal was formulated by von KÁRMÁN [3] as early as in 30' in the following form:

$$\frac{U - U_\infty}{u_\tau} = f(\overline{y};\delta^+). \tag{2.3}$$

The above equation was formulated on empirical basis as the analogy to outer scaling velocity for fully developed pipe and channel flows, which are homogeneous in streamwise direction. As it has been shown by WOSNIK *et al.* [4], the l.h.s. of momentum equation vanishes for these flows and the remaining balance between pressure gradient and viscous stress on control volume dictates that friction velocity u_τ is the proper scaling velocity also for outer regions of pipe and channel flows. However, the outer region of TBL has two velocity scales, i.e. the U_∞ imposed by outer boundary condition on the velocity and the u_τ imposed by inner boundary condition from constant shear stress layer. It was the reason why Clauser [5] discovered that using Eq. (2.3) he could not obtain the similarity and invariance of Reynolds number, especially for TBL with pressure gradient. Clauser defined therefore the so-called equilibrium boundary layer with pressure gradient as the one, where pressure parameter β given by:

$$\beta = \frac{\delta^*}{u_\tau}\frac{dP_\infty}{dx} \tag{2.4}$$

was constant and the velocity deficit $(U - U_\infty)$ profile normalized with friction velocity u_τ was independent of the streamwise coordinate, i.e. the particular profiles collapsed into a single curve. However, most TBL's with pressure gradient did not satisfy these conditions, especially TBL's with APG approaching separation, where wall stress value was close to zero.

Similar methodology (i.e. inner scaling) was followed by Bradshaw [6], who showed that for equilibrium TBL the contribution of pressure gradient to the growth of momentum deficit should be a constant multiple of the contribution brought by wall shear stress, which in fact was identical with Clauser's pressure parameter β. The more stringent treatment of Towsend [7] based on single velocity scale did not extend the applicability of inner scaling. The same conclusion must be stated about the analysis performed by Rotta [8], even if he introduced separate length and velocity scales. The subsequent stream of research was based on another proposal of Townsend [7], who formulated the criterion for the equilibrium APG TBL in the form:

$$U_\infty = \alpha(x - x_0)^m \tag{2.5}$$

which was valid for negative values of exponent m. The results of East and Sawyer [9] as well as data of Skare and Krogstadt [10] revealed a substantial disagreement concerning the value of exponent m for which the universality of TBL with pressure gradient should exist.

A substantial progress was brought into the field by George and Castillo [11] who proposed a treatment, which converged to Reynolds invariant solution of RANS equations in outer region of TBL. The resulting scaling law for outer part of TBL has the form:

$$\frac{U - U_\infty}{U_\infty} = f(\overline{y};\delta^+) \tag{2.6}$$

which originally was proposed for zero pressure gradient (ZPG) TBL. The importance of this proposal results from several aspects. First, the George–Castillo proposal assumes that inner and outer velocity scales are different, what in turn implies that the overlap region between inner and outer regions of TBL is governed by power law. It is in contrast with inner scaling given by Eq. (2.3) which assumes the same velocity scale for inner and outer regions, what implies the overlap region to be logarithmic. The second aspect that must be mentioned is the asymptotic behaviour of scaling laws, in particular George [1] showed that the von Kármán law (Eq. (2.3)) gives in the limit:

$$\delta^+ \to \infty; \qquad H \to 1, \tag{2.7}$$

while George–Castillo proposal (Eq. (2.6)) results in the relation:

$$\delta^+ \to \infty; \qquad H \to \text{const} > 1 \tag{2.8}$$

which is more consistent with experimental data. The most important however is the relation between the scaling law and RANS equations, which are so far the only engineering tool capable to model the TBL (LES treatment of wall flows is too expensive as far as computational resources are concerned). The RANS momentum equation in streamwise direction (with viscous term omitted for brevity) may be written as:

$$\overline{U}\frac{\partial \overline{U}}{\partial x} + V\frac{\partial \overline{U}}{\partial y} = \frac{\partial}{\partial y}[-\overline{uv}] + \frac{\partial}{\partial x}[\overline{v^2} - \overline{u^2}], \tag{2.9}$$

where the last term at r.h.s. is the streamwise gradient of the normal stress difference, part of which comes from integrating the y momentum equation across the flow and using it to substitute for the pressure gradient. As it has been shown in [11], the George–Castillo scaling law is the only one which accounts for this important term (even if being of the second order compared to others). As it was stated earlier, Eq. (2.6) was formulated for ZPG TBL and to extend its applicability, Castillo and George found that TBL with pressure gradient could still be in equilibrium, provided that pressure parameter defined as:

$$\Lambda = \frac{\delta}{\rho U_\infty^2 d\delta/dx} \cdot \frac{dp_\infty}{dx} \tag{2.10}$$

was constant in streamwise direction and in this case only three values of pressure parameter Λ were needed, i.e. $\Lambda = 0.22$ for APG, $\Lambda = -1.92$ for favorable pressure gradient (FPG) and $\Lambda = 0$ for ZPG, to scale properly the outer region of TBL. Later on, Castillo and Wang [13] showed that applicability of Eq. (2.6) may be extended to nonequilibrium TBL's with sudden changes of the external pressure gradient, provided that the flow is in local equilibrium. The criterion for local equilibrium was the pressure parameter defined as:

$$\Lambda_\theta = \frac{\theta}{\rho U_\infty^2 d\theta/dx} \cdot \frac{dp_\infty}{dx} = -\frac{\theta}{U_\infty d\theta/dx} \cdot \frac{dU_\infty}{dx} \tag{2.11}$$

which ought to be constant in each region. The results presented in [15] for flow around airfoil revealed, that for FPG region the pressure parameter varied in the range $\Lambda_\theta = (-0.8) - (-0.44)$, for APG $\Lambda_\theta \approx 0.22$ and in ZPG $\Lambda_\theta \approx 0$ (in two latter cases these values were roughly the same as in equilibrium TBL).

At the same time the third possible treatment was proposed, which was based on purely empirical grounds by Zagarola and Smits [14, 15]. Originally it was a proposal for developed pipe flow [14] and then it was extended for outer region of TBL [15]. The scaling law proposed by Zagarola and Smits was formulated as:

$$\frac{U - U_\infty}{U_\infty} = \frac{\delta^*}{\delta} f(\overline{y}; \delta^+) \tag{2.12}$$

and the above formula collapsed mean velocity profiles from most experiments remarkably well. As it was shown later by George [1], the Zagarola–Smits law reduces in the limit of infinite local Reynolds number either to von Kármán (Eq. (2.3)) or to George–Castillo (Eq. (2.6)) formulas. In particular, when $\delta/\delta^* \to u_\tau/U_\infty$ then Eq. (2.12) becomes asymptotic to Eq. (2.3), while for $\delta/\delta^* \to$ const. the George–Castillo (Eq. (2.6)) formula becomes the asymptote. The true reason for successful performance of Zagarola–Smits scaling is the δ/δ^* scaling factor, which reflects the variation of upstream conditions and therefore Eq. (2.12) provides the means to remove this influence.

Summing up the above-mentioned ideas one has to conclude, that there is still an open question concerning the physics of turbulent boundary layer. The inner scaling and the resulting log-law of the overlap region seem to be well recognized, while the arguments presented by George and his co-workers seem to be convincing, at least as far as the physical behaviour of TBL suggests the need for introducing separate scales. Unfortunately, the most convincing difference between the logarithmic or power law character of overlap region, is indistinguishable concerns the zero-pressure boundary layers and the only chance to look for the evidence concerns the turbulent boundary layers with non-zero pressure gradient (the APG being the most promising perspective).

3. Experimental apparatus and conditions

The experiment was performed in an open-circuit wind tunnel shown in Fig. 1, where the turbulent boundary layer developed along the flat plate, which was 2807 mm long and 250 mm wide (the details of experimental rig and measuring procedures may be found in [16]).

The upper wall was shaped according to the assumed distribution of pressure gradient shown in Fig. 2, which corresponded to the conditions encountered in stator passages of turbomachinery. Location of 18 measuring planes is shown

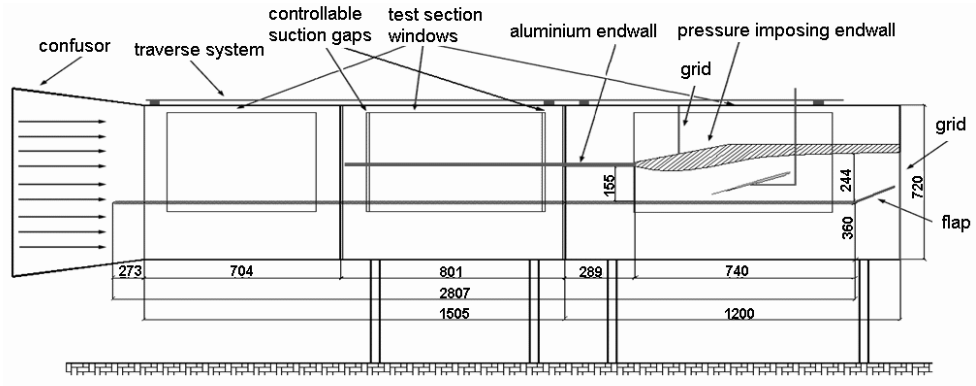

Fig. 1. Schematic view of the wind tunnel and measuring test section.

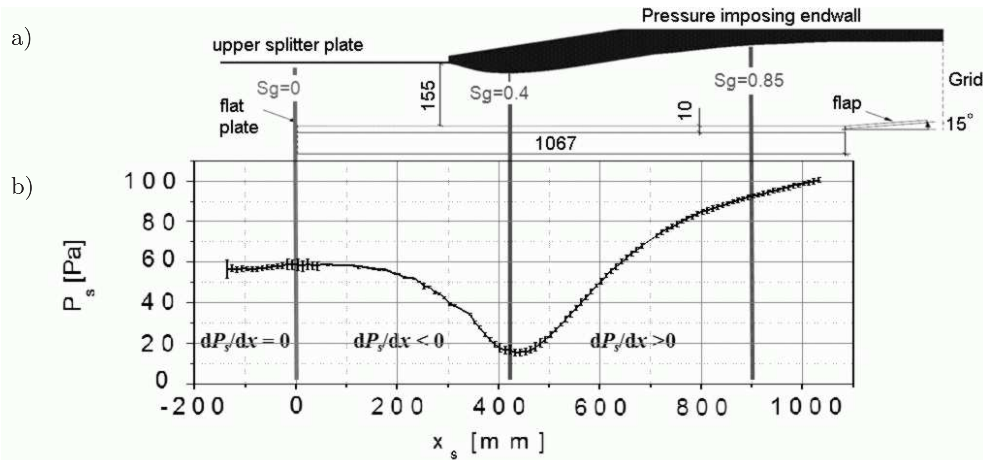

Fig. 2. The shape of upper wall (a) and the corresponding static pressure distribution (b) along the flat plate.

in Fig. 2a. The distances of planes from inlet plane and the corresponding non-dimensional coordinates S_g are given in Table 1.

The static pressure distribution was measured at the flat plate with DATA INSTRUMENTS DCXL01DN pressure transducer connected to KULITE D486 amplifier. The measuring accuracy of pressure is documented by error bars shown along the pressure distribution in Fig. 2b, the mean relative error of pressure measurements was equal to 2.16% for ZPG area (cross-sections 1) and 2.67% for APG (cross-sections 2–18). Velocity profiles were measured with single and X-wire HWA probes, the latter one had to be specially calibrated due to high turbulence intensity in TBL (for details of calibration procedures see [16]).

Table 1. Location of measuring planes shown in Fig. 2a.

No.	Distance from inlet plane x_s [mm]	Relative distance Sg
1	0	0.000
2	427	0.400
3	457	0.428
4	487	0.456
5	517	0.484
6	547	0.512
7	577	0.540
8	607	0.569
9	637	0.597
10	667	0.625
11	697	0.653
12	727	0.681
13	757	0.709
14	787	0.737
15	817	0.765
16	847	0.793
17	877	0.822
18	907	0.850

Figure 3 shows the mean velocity and velocity fluctuations measured at the inlet plane with single and X-wire HWA probes; one may notice the excellent agreement of results obtained by both techniques. It must be noticed however, that

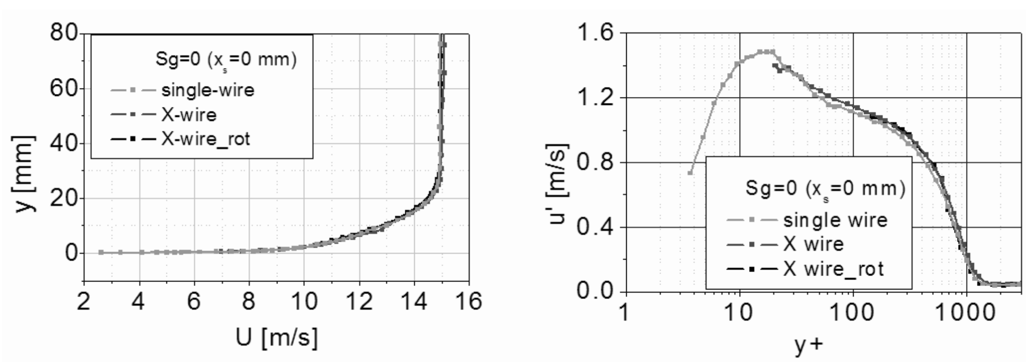

Fig. 3. The comparison of mean velicity (a) and rms of velocity fluctuations (b), measured with single and XC-wire probes at the inlet plane Sg = 0.62.

due to larger size the X-wire can not penetrate the boundary layer as close as the single probe. The comparison of results obtained with single and X-wire probes performed in 18 cross-sections allowed to estimate the uncertainty of velocity measurements given in Table 2, which confirms the quality and accuracy of the measurements reported here.

Table 2. Comparison of uncertainty of HWA measurements with literature data.

Probe:	Quantity:	Uncertainty [%]						
		Achieved accuracy for inlet plane				ITM TUCz [19]	Tutu and Chevray [19]	DISA [19]
		Layer:						
		viscous	buffer	Log-law	Wake-law			
I	U	1.5	5	2	2	0.6	1÷6	2.5
	u'	1.5	5	1	20	3	3	15
X	U	–	1.5	1.5	1.5	0.6	1÷6	2.5
	V	–	1.5	1.5	1	–	–	–
	u'	–	3	1.5	10	3	3	15
	v'	–	3	1.5	6	4	5÷12	10

The mean velocity at the inlet plane outside the boundary layer was $U_\infty = 15$ [m/s], the turbulence intensity of the undisturbed flow was equal to $Tu \approx 0.4\%$, the distribution of mean and fluctuating velocity shown in Fig. 3 confirms that the boundary layer was fully turbulent. In particular it may be noticed that the distribution of u' reveals a single peak located at $y^+ \approx 20$, which is typical for turbulent boundary layers at ZPG conditions.

The integral parameters of boundary layer at the inlet plane shown in Table 3 give additional evidence that the flow at the inlet plane is a fully developed turbulent boundary layer. One may also notice that tripping of boundary layer at the leading edge of a flat plate (for details see [16]), allowed to obtain a relatively high value of characteristic Reynolds number equal to $\mathrm{Re}_\theta \approx 3\,000$.

Table 3. Parameters of boundary layer at the inlet plane.

δ	δ^*	Θ	H	U_∞	$U_{0.99}$	C_f	τ_w	u_τ
[mm]	[mm]	[mm]	[–]	[m/s]	[m/s]	[–]	[N/m^2]	[m/s]
25.73	3.51	2.75	1.28	14.83	14.68	0.0035	0.4436	0.623

4. Results and discussion

The measurements performed in several measuring planes shown in Fig. 2 allowed to determine the evolution of boundary layer along the flat plate.

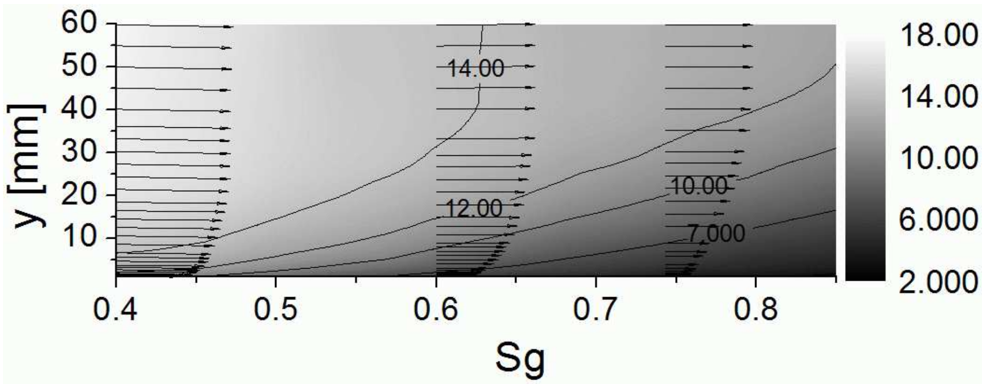

Fig. 4. Evolution of mean velocity field along the flat plate at APG region.

The sample distribution of mean velocity field in the area of APG (Sg = 0.4–0.85) shown in Fig. 4 reveals a gradual increase of boundary layer thickness. One may also notice that velocity vectors superimposed on that picture are deflected away from the wall in APG area, the maximum deflection is observed for Sg ≈ 0.74, where maximum value of pressure gradient occurs. Fig. 5 presents the downstream evolution of shape parameter H which is typical of TBL approaching separation under the APG conditions; the value of shape parameter as well as isolines of mean velocity (Fig. 4) reveal however, that the TBL analyzed has not been separated yet.

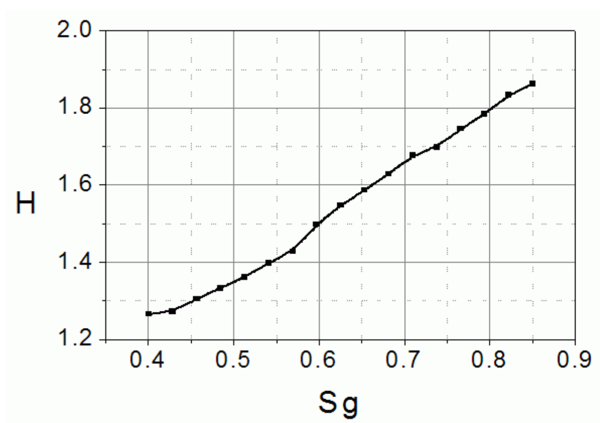

Fig. 5. Downstream evolution of shape parameter at APG region.

The more detailed insight into the TBL structure near the wall may be obtained from Fig. 6, which presents mean velocity profiles in consecutive measuring planes. All these profiles were presented in universal coordinates $u^+; y^+$, which is the inner region Prandtl's scaling given by Eq. (2.1). One may notice the viscous sublayer ($y^+ = 2$–6) which then is transformed into the buffer zone ($y^+ = 6$–30) and then the area of log-law which extends up to $y^+ \approx 300$ for initial cross-sections. The location of all these areas agrees very well with theoretical predictions and literature data [17, 18]. The wake-law region visible as the deflection of velocity profile from log-law distribution accompanies the appearance of APG. One may notice that in the cross-sections most distant from inlet plane, the deviation of velocity profile from log-law appears as early as $y^+ < 100$ (see Sg = 0.850 at Fig. 6).

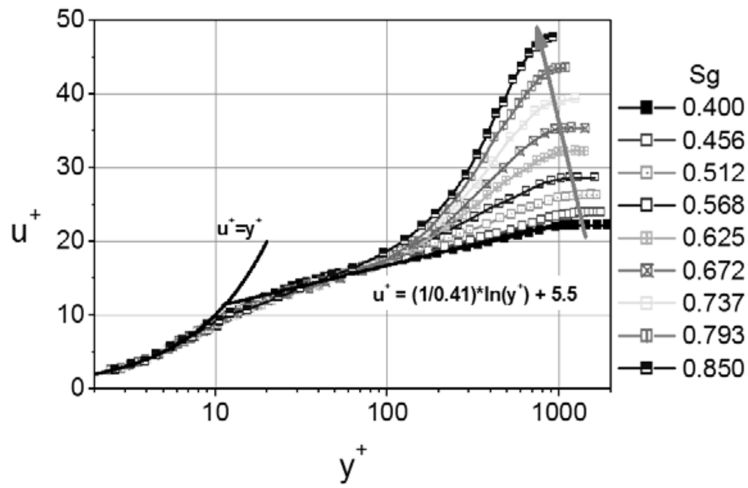

FIG. 6. Mean velocity profiles in TBL under APG conditions in universal coordinates.

The analysis of TBL outer region scaling laws began from transformation of mean velocity profiles with the von Kármán scaling proposal given by Eq. (2.3) and the results are given in Fig. 7. The Δ in this figure is a Clauser–Rotta length scale defined as $\delta^* U_\infty / u_\tau$. One may notice that measuring points do not collapse on the common curve what proves that friction velocity u_τ, which is an inner velocity scale, does not perform satisfactorily in outer TBL region. The further explanation for failure of inner scaling is the distribution of Clauser parameter β given by Eq. (2.4), which is by no means constant as can be seen from Fig. 8. One may conclude that it agrees with the statement of CASTILLO and GEORGE [12] that equilibrium boundary layers, which would fulfil the Clauser's requirements, are nearly impossible to generate and maintain, what in turn explains the failure of von Kármán scaling.

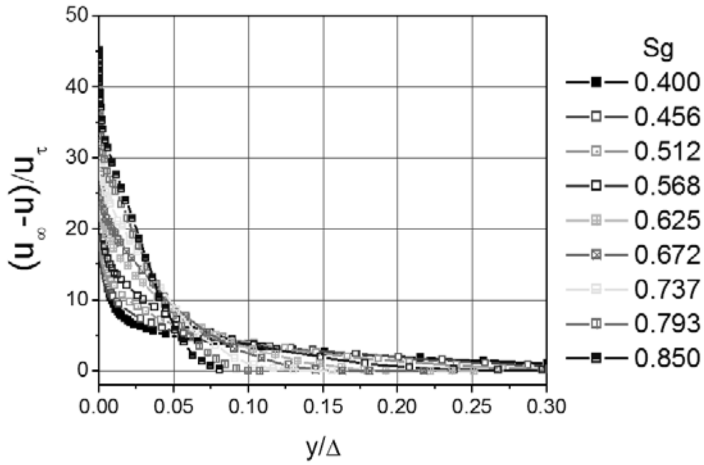

Fig. 7. Mean velocity profile in TBL under APG conditions in von Kármán scaling acc. to Eq. (2.3).

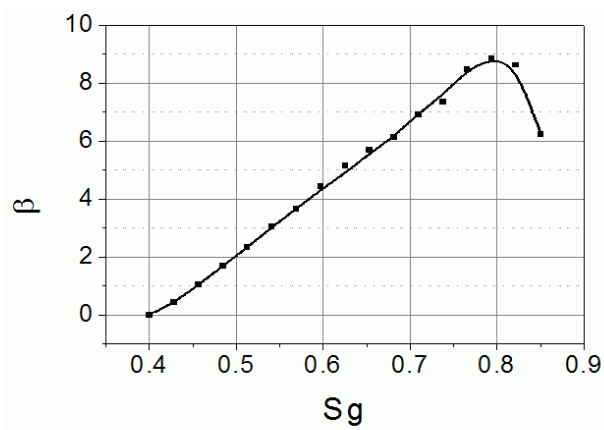

Fig. 8. Distribution of Clauser pressure parameter β calculated acc. to Eq. (2.4) along the measuring region of TBL.

The next attempt was directed towards the verification of the George–Castillo outer scaling law given by Eq. (2.6), that is summarized in Fig. 9. One may notice that also in this case, the measuring points do not collapse on the common curve, however the behaviour of particular velocity profiles measured at consecutive cross-sections is more consistent than in the case of the von Kármán scaling. Again the failure of this concept may be explained by the behaviour of Λ and Λ_θ parameters (Eqs. (2.10) and (2.11)) which, according to [12, 13], should be constant to enable the successful application of scaling given by Eq. (2.6). It can be seen from Figs. 10 and 11, that distributions of Castillo–George pressure

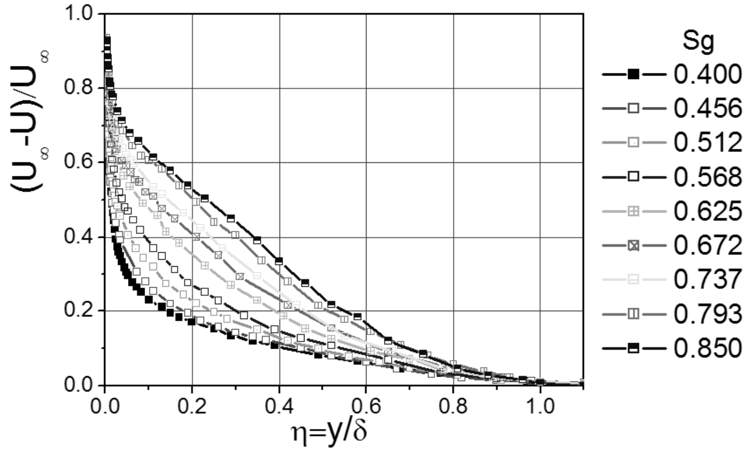

Fig. 9. Mean velocity profile in TBL under APG conditions in George-Castillo (outer) scaling acc. to Eq. (2.6).

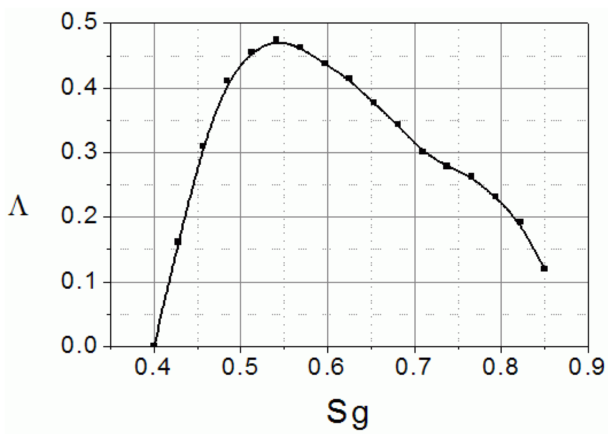

Fig. 10. Distribution of pressure parameter Λ calculated acc. to Eq. (2.10) along the measuring region of TBL.

parameters Λ and Λ_θ are not constant along the measuring region what in turn explains the failure of the George–Castillo outer scaling law.

The scaling performed according to Zagarola–Smits proposal given by Eq. (2.12) gave the most satisfactory results as can be seen from Fig. 12, which reveals that measuring points collapse at the common curve for all the measuring cross-sections.

One may conclude therefore, that the outer scaling originally proposed by Zagarola and Smits is in agreement with the two-layer approach and that Zagarola–Smits scaling is the most suitable for the mean-velocity profile, even for very strong APGs. This conclusion is in agreement with the findings of Indinger

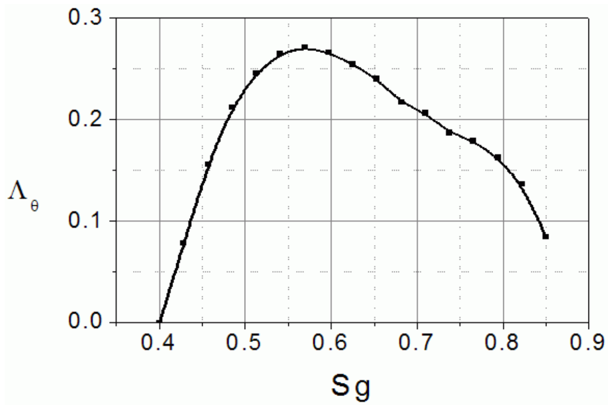

Fig. 11. Distribution of pressure parameter Λ_θ calculated acc. to Eq. (2.11) along the measuring region of TBL.

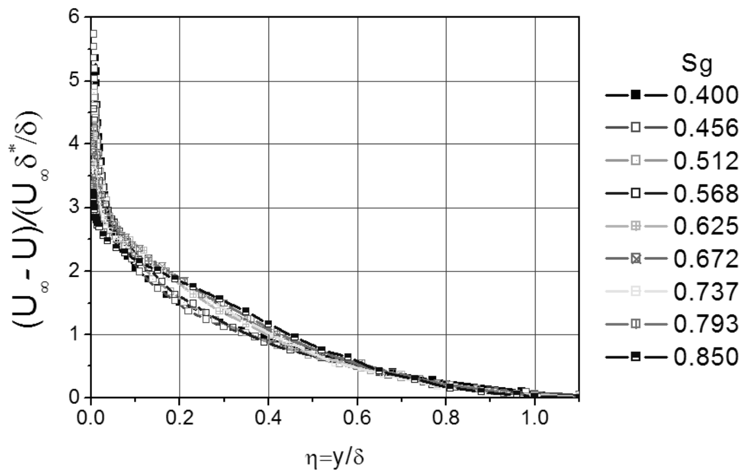

Fig. 12. Mean velocity profile in outer TBL region at APG conditions in Zagarola-Smits (outer) scaling acc. to Eq. (2.12).

et al. [17] who found also that scaling proposed by Castillo and George fails very close to separation, due to the effect of backflow.

The controversy of the applicability of inner and outer scaling is in fact a controversy of a more fundamental issue concerning the physics of TBL and in particular, the contribution of particular zones to downstream growth of TBL thickness. As it was pointed out by George [1] *"it is commonly (and erroneously) believed that because the main contribution to θ comes from near the wall, then the main contribution to $d\theta/dx$ must also come from near the wall"*. In fact, the near-wall region grows much more slowly than the outer part of the boundary layer what in turn implies, that most of the contribution to $d\theta/dx$

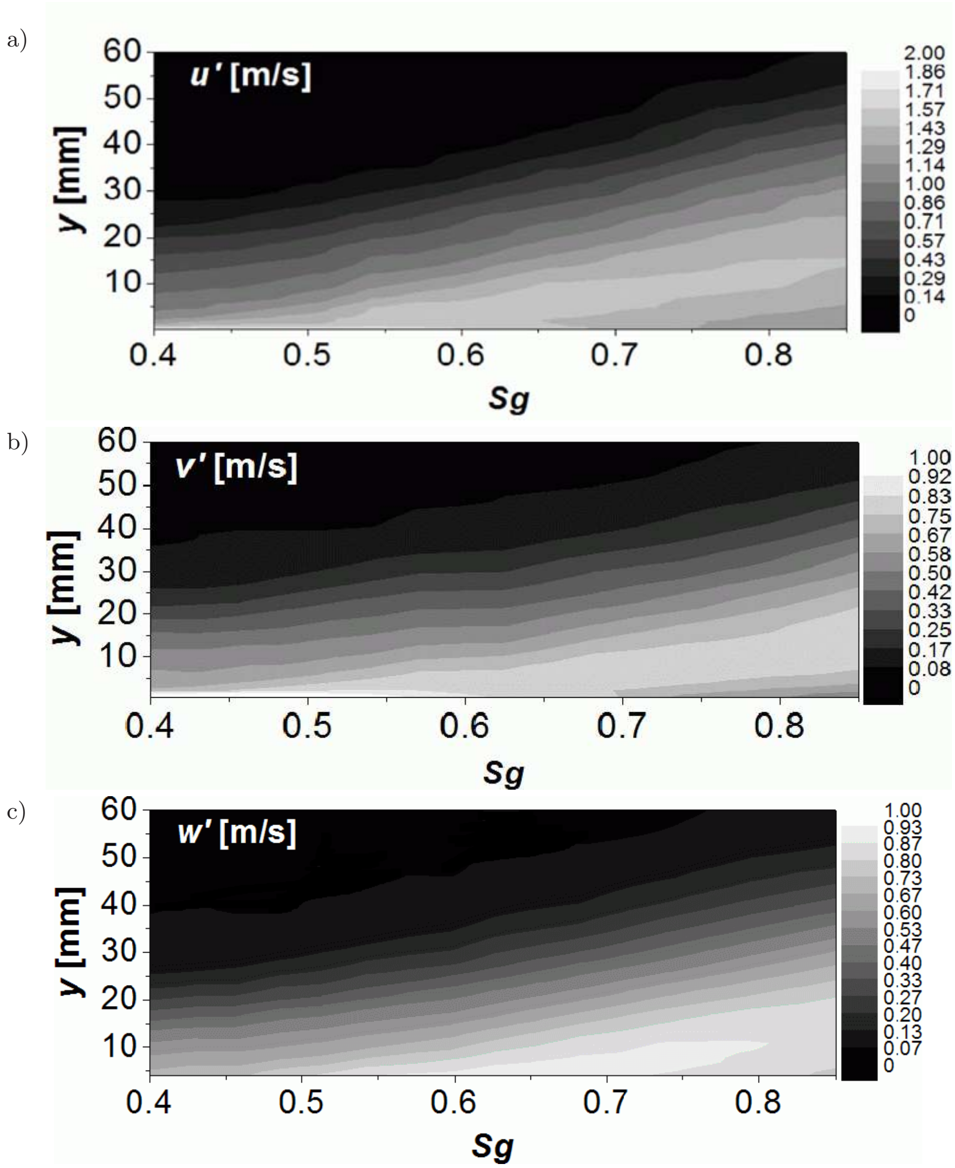

Fig. 13. Evolution of longitudinal (a), transversal (b) and normal (c) velocity fluctuations along the flat plate in APG region.

originates from areas located far away from the wall. It may be assessed from the distribution of intensity of velocity fluctuations, which may be treated as the indirect indication of intensity of turbulent transport. The distributions of longitudinal, transversal and normal to the wall velocity fluctuations u'; v'; w' shown at Figs. 13a, b and c, reveal in the initial region ($\mathrm{Sg} = 0.4 \div 0.5$) a single maximum located in the immediate vicinity of the wall. In the downstream area, beginning from Sg ≈ 0.6, a second maximum of $u'; v'; w'$ located in the outer zone of TBL appears.

It means therefore that in the presence of APG, the appearance of second peak of turbulent velocity fluctuations confirms the more pronounced contribution of outer region to the downstream development of TBL. The same conclusions are supported by sample profiles of longitudinal velocity fluctuations u' along the flat plate. The inner scaling u^+; y^+ has been applied at Fig. 14 due to its better resolution, that allows to show the distribution of particular curves more clearly. One may notice that the first maximum of u' which exists in $y^+ \approx 20$ at the beginning of the measuring area (Sg = 0.4) gradually decays, and at the same time the outer maximum located at $y^+ \approx 300$ appears and becomes more pronounced, the further downstream is located in the measuring

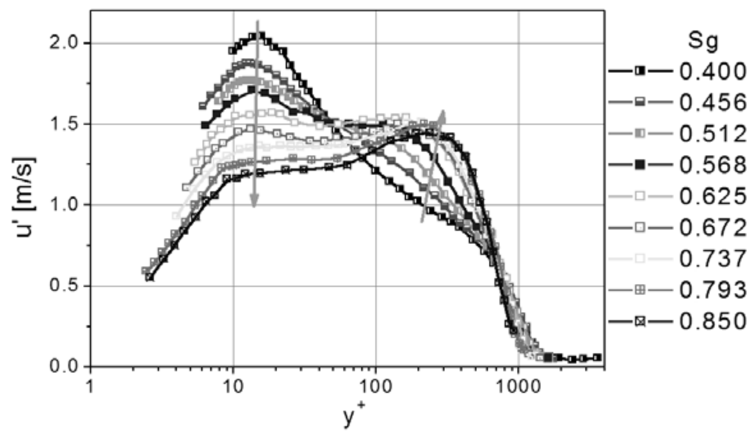

FIG. 14. Evolution of longitudinal velocity fluctuations along the flat plate at APG region.

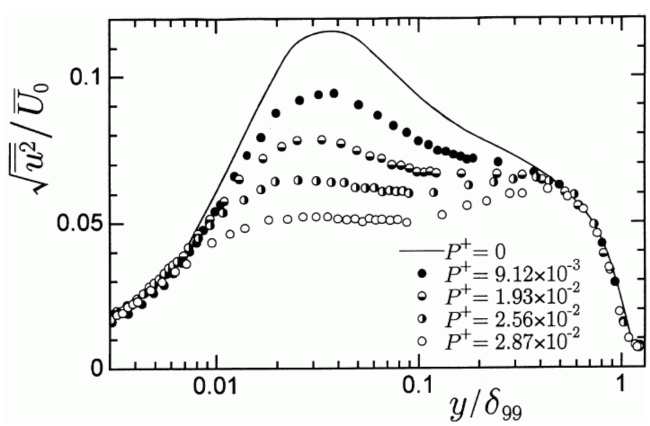

FIG. 15. Evolution of longitudinal velocity fluctuations at APG region obtained in [18]; solid line for ZPG.

cross-section (arrows in Fig. 14 denote the decay and appearance of inner and outer maxima respectively).

The appearance of the second maximum in profiles of velocity fluctuations agrees with the literature data, which document the existence of peak in turbulence production in the outer region of TBL at APG conditions [10]. For comparison, the study of Nagano *et al.* [18] was selected, because these data were obtained for almost identical Reynolds number $\mathrm{Re}_\theta \approx 3\,000$) and for similar values of the pressure gradient. The pressure gradient in their experiment was denoted P^+. As it may be seen in Fig. 15, with an increasing APG effect the reduction in turbulence intensities in the wall region with the accompanying development of outer peak is observed.

5. Conclusions

The results obtained suggest that turbulent boundary layer at APG conditions requires two velocity scales, i.e. the inner (imposed by inner boundary condition from constant shear stress layer) and outer (imposed by outer layer) velocity scales. Among the scaling proposals published in literature so far, the Zagarola–Smits scaling seems to be the most suitable for the mean-velocity profile even for very strong APGs, bearing in mind the experimental data obtained during the present research. The results concerning the turbulent velocity fluctuations confirm the basic physics behind the idea of outer scaling, in particular the appearance of second peak of velocity fluctuations confirms the more pronounced contribution of outer region to the downstream development of TBL, what in turn suggests that physical reasoning advocated by George [1] seems to be in agreement with experimental evidence. The uncertainty analysis and comparison with available literature data confirm a good quality of experimental results and support the validity and trustworthiness of conclusions presented in the paper.

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