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# Effect of rotation on Rayleigh waves in an isotropic generalized thermoelastic diffusive half-space

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The present investigation is a study of the effect of rotation on the characteristics of Rayleigh waves propagation in a homogeneous, isotropic, thermoelastic diffusive half-space in the framework of different theories of thermoelastic diffusion, including the Coriolis and Centrifugal forces. The medium is subjected to stress-free, thermally insulated/isothermal and chemical potential boundary conditions and is rotating about an axis perpendicular to its plane. Secular equations of surface wave propagation in the considered media are derived. The phase velocities and attenuation coefficients of surface wave propagation have been computed by using the irreducible case of Cardano's method, with the help of DeMoivre's theorem known from the secular equations. The amplitudes of surface displacements, temperature change, concentration and the specific loss of energy are computed numerically. Rotation effect on the phase velocity, attenuation coefficient, amplitudes of surface wave propagation and specific loss of energy are presented graphically in order to illustrate and compare the analytically obtained results. Some special cases of frequency equation are also deduced from the present investigation.

**Key words:** wave propagation, isotropic, generalized thermoelastic diffusion, rotation, phase velocity, attenuation coefficient.

## 1. Introduction

The classical uncoupled theory of thermoelasticity predicts two phenomena which are not compatible with physical observations. First, the equation of heat conduction of this theory does not contain any elastic term; second, the heat conduction equation is of a parabolic type, predicting infinite speeds of propagation of the heat waves.

Biot [7] introduced the theory of coupled thermoelasticity to overcome the first shortcoming. The governing equations of this theory are coupled, eliminating the first paradox of the classical theory. However, both theories share the second shortcoming, because the heat equation for the coupled theory is of a mixed parabolic/hyperbolic type. In the Lord–Shulman theory [15], a flux rate term is incorporated (with one relaxation time) into the Fourier's law of heat conduction, and a generalized theory was formulated admitting finite speeds for thermal signals. The GREEN–LINDSAY theory [12], called temperature rate-dependent thermoelasticity in which temperature rate-dependence is included among the consecutive variables with two constants that act as two relaxation times, does not violate the classical Fourier law of heat-conduction when the body under consideration has a center of symmetry. The LORD and SHULMAN [15] theory of generalized thermoelasticity was further extended to homogeneous anisotropic heat-conducting materials recommended by DHALIWAL and SHERIEF [10]. All these theories predict a finite speed of heat propagation. CHANDERASHEKHARIAH [8] treats this wave-like thermal disturbance as a second sound. A survey article of various representative theories in the range of generalized thermoelasticity have been presented by HETNARSKI and IGNACZAK [14].

Diffusion can be defined as a random walk of an ensemble of particles from the region of high concentration to that of low concentration. Nowadays, there is a great deal of interest in the study of this phenomenon due to its application in geophysics and electronic industry. In integrated circuit fabrication, diffusion is used to introduce dopants in controlled amounts into the semiconductor substance. In particular cases, diffusion is used to form the base and emitter in bipolar transistors, integrated resistors and the source/drain regions in Metal Oxide Semiconductor (MOS) transistors, and dope poly-silicon gates in MOS transistors. In most of the applications, the concentration is calculated using what is known as Fick's law. This is a simple law which does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced, or the effect of temperature on this interaction. Study of the phenomenon of diffusion is used to improve the conditions of oil extraction (seeking ways of more efficient recovering of oil from oil deposits). These days, oil companies are interested in the process of thermodiffusion for more efficient extraction of oil from oil deposits.

Until recently, thermodiffusion in solids, especially in metals, was considered as a quantity that is independent of the body deformation. Practice however, indicates that the process of thermodiffusion could have a very considerable influence upon the deformation of the body.

The thermodiffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain, in addition to heat and mass exchange with the environment. NOWACKI [18–21] developed the theory of thermoelastic diffusion by using a coupled termoelastic model. DUDZIAK and KOWALSKI [11] and OLESIAK and PYRYEV [22], respectively, discussed the theory of thermodiffusion and coupled quasi-stationary problems of thermal diffusion in an elastic layer. They studied the influence of cross-effects arising from coupling of the fields of temperature, mass diffusion and strain, due to which the thermal excitation results in additional mass concentration and which generates additional fields of temperature.

Sherief *et al.* [29] developed the generalized theory of thermoelastic diffusion with one relaxation time, which allows the finite speeds of propagation of waves. Recently Kumar *et al.* [17] derived the basic equations for generalized thermoelastic diffusion(GL-model) and discussed the Lamb waves. Sherief and Saleh [30] investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Singh [31, 32] discussed the reflection phenomenon of waves from a free surface of an elastic solid with generalized thermodiffusion. Aouadi [2–6] investigated different types of problems in thermoelastic diffusion. Sharma [24, 26] discussed the plane, harmonic, generalized thermoelastic diffusive waves and elasto-thermodiffusive surface waves in heat-conducting solids.

Chanderashekhariah and Srinath [9] discussed the thermoelastic plane waves without energy dissipation in a rotating body. Ahmad and Khan [1] studied the thermoelastic plane waves in a rotating isotropic medium. Othman [23] investigated the effect of rotation and relaxation time on a thermal shock problem for a half-space in generalized thermo-viscoelasticity. Sharma *et al.* [27, 28] discussed the effect of rotation on Rayleigh waves in the piezothermoelastic half-space.

Keeping in mind the above applications of thermodiffusion, the present paper deals with the study of the influence of rotation on Rayleigh waves in a homogeneous, isotropic thermoelastic diffusive half-space, in the context of different theories of thermoelastic diffusion. The amplitudes of surface displacements, temperature change and concentration are determined. The phase velocity and attenuation coefficients of wave propagation have been computed by using the irreducible case of Cardano's method, with the help of DeMoivre's theorem, from the secular equations. The specific loss of energy has been also computed.

## 2. Basic equations

Following Kumar *et al.* [17], the governing equations for anisotropic homogeneous elastic solid with generalized thermoelastic diffusion (Green–Lindsay theory), in absence of body forces, heat and mass diffusion sources, are:

(i) Constitutive relations

$$\sigma_{ij} = c_{ijkm}e_{km} + a_{ij}(T + \tau_1\dot{T}) + b_{ij}(C + \tau^1\dot{C}), \tag{2.1}$$

$$\rho S = k + \frac{\rho C_E}{T_0}(T + \tau_0\dot{T}) - a_{ij}e_{ij} + a(C + \tau^0\dot{C}), \tag{2.2}$$

$$P = b_{ij}e_{ij} + b(C + \tau^1\dot{C}) - a(T + \tau_1\dot{T}). \tag{2.3}$$

(ii) Equations of motion

$$c_{ijkm}e_{km,j} + a_{ij}[(T+\tau_1\dot{T})_{,j}] + b_{ij}[(C+\tau^1\dot{C})_{,j}] = \rho\ddot{u}_i. \tag{2.4}$$

(iii) Equation of heat conduction

$$\rho C_E(\dot{T}+\tau_0\ddot{T}) + aT_0(\dot{C}+\tau^0\ddot{C}) - a_{ij}\dot{e}_{ij}T_0 = K_{ij}T_{,ij}. \tag{2.5}$$

(iv) Equation of mass diffusion

$$-\alpha^*_{ij}b_{km}e_{km,ij} - \alpha^*_{ij}b[(C+\tau^1\dot{C})_{,ij}] + \alpha^*_{ij}a[(T+\tau_1\dot{T})_{,ij}] = -\dot{C}. \tag{2.6}$$

Here $c_{ijkm}(c_{ijkm} = c_{kmij} = c_{jikm} = c_{ijmk})$ are elastic parameters. $a_{ij}$ $(= a_{ji})$, $b_{ij}$ $(= b_{ji})$ are tensors of thermal and diffusion moduli respectively. $\rho$, $C_E$ are, respectively, the density and specific heat at constant strain; $a, b$ are, respectively, coefficients describing the measure of thermoelastic diffusion effects and of diffusion effects, $T_0$ is the reference temperature assumed to be such that $|T/T_0| \ll 1$. $\tau^0, \tau^1$ are diffusion relaxation times with $\tau^1 \geq \tau^0 \geq 0$, and $\tau_0$, $\tau_1$ are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$. $u_i$ are components of the displacement vector $\mathbf{u}$. $T(x_1, x_2, x_3, t)$ is the temperature change and $C$ is the concentration. $\sigma_{ij}$ $(= \sigma_{ji})$, $K_{ij}$ $(= K_{ji})$, $e_{ij} = (u_{i,j} + u_{j,i})/2$ are the components of stress, thermal conductivity and strain tensor, respectively. $\alpha^*_{ij}$ $(= \alpha^*_{ji})$ are diffusion parameters. $P$, $S$ are the chemical potential and entropy per unit mass respectively, and $k$ is a material constant. The symbols "," and "." correspond to partial and time derivatives respectively.

If we assume

$$\begin{gathered} c_{ijkm} = \lambda\delta_{ij}\delta_{km} + \mu\delta_{im}\delta_{jk} + \mu\delta_{ik}\delta_{jm}, a_{ij} = -\beta_1\delta_{ij}, \\ b_{ij} = -\beta_2\delta_{ij}, \qquad K_{ij} = K\delta_{ij}, \qquad \alpha^*_{ij} = D\delta_{ij}, \end{gathered} \tag{2.7}$$

we obtain the basic governing equations for homogeneous, isotropic, generalized thermoelastic diffusion (Green–Lindsay theory) in absence of the body forces, heat and mass diffusion sources, as:

(i) Constitutive relations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{kk} - \beta_1(T+\tau_1\dot{T}) - \beta_2(C+\tau^1\dot{C})], \tag{2.8}$$

$$\rho T_0 S = k + \rho C_E(T+\tau_0\dot{T}) + \beta_1 T_0 e_{kk} + aT_0(C+\tau^0\dot{C}), \tag{2.9}$$

$$P = -\beta_2 e_{kk} + b(C+\tau^1\dot{C}) - a(T+\tau_1\dot{T}). \tag{2.10}$$

(ii) Equations of motion

$$\mu u_{i,jj} + (\lambda+\mu)u_{j,ij} - \beta_1(T+\tau_1\dot{T})_{,i} - \beta_2(C+\tau^1\dot{C})_{,i} = \rho\ddot{u}_i. \tag{2.11}$$

(iii) Equation of heat conduction

$$\rho C_E(\dot{T}+\tau_0\ddot{T})+\beta_1 T_0\dot{e}_{kk}+aT_0(\dot{C}+\tau^0\ddot{C})=KT_{,ii}. \tag{2.12}$$

(iv) Equation of mass diffusion

$$D\beta_2 e_{kk,ii}+Da(T+\tau_1\dot{T})_{,ii}+\dot{C}-Db(C+\tau^1\dot{C})_{,ii}=0. \tag{2.13}$$

Following Sherief *et al.* [29], the governing equations for an isotropic homogeneous elastic solid with generalized thermoelastic diffusion (Lord–Shulman theory) in absence of the body forces, heat and mass diffusion sources are:

(i) Constitutive relations

$$\sigma_{ij}=2\mu e_{ij}+\delta_{ij}[\lambda e_{kk}-\beta_1 T-\beta_2 C], \tag{2.14}$$

$$\rho T_0 S=\rho C_E T+\beta_1 T_0 e_{kk}+aT_0 C, \tag{2.15}$$

$$P=-\beta_2 e_{kk}+bC-aT. \tag{2.16}$$

(ii) Equations of motion

$$\mu u_{i,jj}+(\lambda+\mu)u_{j,ij}-\beta_1 T_{,i}-\beta_2 C_{,i}=\rho\ddot{u}_i. \tag{2.17}$$

(iii) Equation of heat conduction

$$\rho C_E(\dot{T}+\tau_0\ddot{T})+\beta_1 T_0(\dot{e}_{kk}+\tau_0\ddot{e}_{kk})+aT_0(\dot{C}+\tau_0\ddot{C})=KT_{,ii}. \tag{2.18}$$

(iv) Equation of mass diffusion

$$D\beta_2 e_{kk,ii}+DaT_{,ii}+\dot{C}+\tau_0\ddot{C}-DbC_{,ii}=0. \tag{2.19}$$

Combining Eqs. (2.8)–(2.13) and (2.14)–(2.19) in one form, we obtain the governing equations for an isotropic homogeneous elastic solid with generalized thermoelastic diffusion in absence of the body forces, heat and mass diffusion sources, which are:

(i) Constitutive relations

$$\sigma_{ij}=2\mu e_{ij}+\delta_{ij}[\lambda e_{kk}-\beta_1(T+\tau_1\dot{T})-\beta_2(C+\tau^1\dot{C})], \tag{2.20}$$

$$\rho T_0 S=k+\rho C_E(T+\alpha\dot{T})+\beta_1 T_0 e_{kk}+aT_0(C+\beta\dot{C}), \tag{2.21}$$

$$P=-\beta_2 e_{kk}+b(C+\tau^1\dot{C})-a(T+\tau_1\dot{T}). \tag{2.22}$$

(ii) Equations of motion in the rotation frame of reference are:

$$\begin{aligned}\mu u_{i,jj}+(\lambda+\mu)u_{j,ij}-\beta_1(T+\tau_1\dot{T})_{,i}-\beta_2(C+\tau^1\dot{C})_{,i}&\\ =\rho[\ddot{u}_i+\{\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u}\}_i+(2\mathbf{\Omega}\times\dot{\mathbf{u}})_i]&.\end{aligned} \tag{2.23}$$

(iii) Equation of heat conduction

$$\rho C_E(\dot{T}+\tau_0\ddot{T})+\beta_1 T_0(\dot{e}_{kk}+\varepsilon\tau_0\ddot{e}_{kk})+aT_0(\dot{C}+\gamma\ddot{C})=KT_{,ii}. \tag{2.24}$$

(iv) Equation of mass diffusion

$$D\beta_2 e_{kk,ii}+Da(T+\tau_1\dot{T})_{,ii}+(\dot{C}+\varepsilon\tau^0\ddot{C})-Db(C+\tau^1\dot{C})_{,ii}=0. \tag{2.25}$$

Here, the medium is rotating with angular velocity $\Omega=\Omega\hat{\nu}$, where $\hat{\nu}$ is the unit vector along the axis of rotation and these equations of motion include two additional terms, namely:

(i) The centripetal acceleration $\Omega\times(\Omega\times\mathbf{u})$ due to time-varying motion,

(ii) The Coriolis acceleration $(2\Omega\times\dot{\mathbf{u}})$,

where $\beta_1=(3\lambda+2\mu)\alpha_t$ and $\beta_2=(3\lambda+2\mu)\alpha_c$; $\lambda,\mu$ are the Lamé constants, $\alpha_t$ is the coefficient of linear thermal expansion and $\alpha_c$ is the coefficient of linear diffusion expansion. $k=\alpha=\beta=\gamma=\varepsilon=\tau_0=\tau^0=\tau_1=\tau^1=0$ for the Coupled Thermoelasticity (CT) model, $k=\alpha=\beta=\tau_1=\tau^1=0$, $\varepsilon=1$, $\gamma=\tau_0$ for the Lord–Shulman (LS) model, and $\alpha=\tau_0$, $\beta=\tau^0$, $\varepsilon=0$, $\gamma=\tau^0$ for the Green–Lindsay (GL) model.

## 3. Formulation of the problem

We consider a homogeneous isotropic, thermodiffusive elastic half-space initially at uniform temperature $T_0$. The origin of the coordinate system $(x_1,x_2,x_3)$ is assumed at any point on the plane horizontal surface. We take $x_3$-axis along the axis of material symmetry and pointing vertically downwards into the half-space, which is thus represented by $x_3\geq 0$. The surface $x_3=0$ is subjected to traction-free, thermally insulated or isothermal and chemical potential boundary conditions. We choose the $x_1$-axis in the direction of wave propagation, so

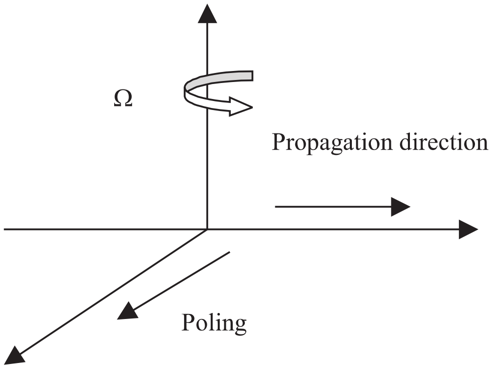

Fig. 1. Geometry of the problem.

that all particles on a line parallel to $x_2$-axis are equally displaced. Therefore, all the field quantities will be independent of the $x_2$-coordinate. Then for a two-dimensional problem,

$$\overrightarrow{u}(x_1, x_3, t) = (u_1, 0, u_3), \qquad T(x_1, x_3, t), \qquad C(x_1, x_3, t) \tag{3.1}$$

are the displacement vector, temperature change and concentration.

We define the dimensionless quantities:

$$\begin{aligned}
&x_i' = \frac{w_1^* x_i}{v_1}, \quad t' = w_1^* t, \quad u_i' = \frac{w_1^* u_i}{v_1}, \quad T' = \frac{\beta_1 T}{\rho v_1^2}, \quad C' = \frac{\beta_2 C}{\rho v_1^2},\\
&P' = \frac{P}{\beta_2}, \quad \tau_0' = w_1^* \tau_0, \quad \tau_1' = w_1^* \tau_1, \quad \tau^{0'} = w_1^* \tau^0, \quad \tau^{1'} = w_1^* \tau^1,\\
&\Omega' = \frac{\Omega}{w_1^*}, \quad h' = \frac{v_1 h}{w_1^*}, \quad \sigma_{ij}' = \frac{\sigma_{ij}}{\beta_1 T_0}, \quad v_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad w_1^* = \frac{\rho C_E v_1^2}{K}.
\end{aligned} \tag{3.2}$$

Here $w_1^*$ is the characteristic frequency of the medium, $v_1$ is the longitudinal wave velocity in the medium.

Upon introducing the quantities (3.2) in Eqs. (2.23)–(2.25), after suppressing the primes, we obtain

$$u_{1,11} + \delta_1 u_{1,33} + \delta_2 u_{3,13} - \tau_t^1 T_{,1} - \tau_c^1 C_{,1} = \ddot{u}_1 - \Omega^2 u_1 + 2\Omega \dot{u}_3, \tag{3.3}$$

$$\delta_2 u_{1,13} + \delta_1 u_{3,11} + u_{3,33} - \tau_t^1 T_{,3} - \tau_c^1 C_{,3} = \ddot{u}_3 - \Omega^2 u_3 - 2\Omega \dot{u}_1, \tag{3.4}$$

$$\nabla^2 T = \tau_t^0 \dot{T} + \zeta_1 \tau_c^0 \dot{C} + \zeta_2 \tau_e^0 \dot{e}, \tag{3.5}$$

$$q_1^* \nabla^2 e + q_2^* \tau_t^1 \nabla^2 T - q_3^* \tau_c^1 \nabla^2 C + \tau_f^0 C = 0, \tag{3.6}$$

where

$$\begin{aligned}
&\delta_1 = \frac{\mu}{\lambda + 2\mu}, \quad \delta_2 = \frac{\lambda + \mu}{\lambda + 2\mu}, \quad \zeta_1 = \frac{a T_0 v_1^2 \beta_1}{w_1^* K \beta_2}, \quad \zeta_2 = \frac{\beta_1^2 T_0}{\rho K w_1^*},\\
&q_1^* = \frac{D w_1^* \beta_2^2}{\rho v_1^4}, \quad q_2^* = \frac{D w_1^* \beta_2 a}{\beta_1 v_1^2}, \quad q_3^* = \frac{D w_1^* b}{v_1^2}, \quad e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3},\\
&\tau_t^1 = 1 + \tau_1 \frac{\partial}{\partial t}, \quad \tau_c^1 = 1 + \tau^1 \frac{\partial}{\partial t}, \quad \tau_t^0 = 1 + \tau_0 \frac{\partial}{\partial t}, \quad \tau_c^0 = 1 + \gamma \frac{\partial}{\partial t},\\
&\tau_e^0 = 1 + \varepsilon \tau_0 \frac{\partial}{\partial t}, \quad \tau_f^0 = 1 + \varepsilon \tau^0 \frac{\partial}{\partial t}, \quad \nabla^2 \equiv \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}.
\end{aligned}$$

## 4. Solution of the problem

We assume the solutions in the form

$$(u_1, u_3, T, C) = (1, W, S, R) U \exp[\iota\xi(x_1 \sin\theta + m x_3 - ct)], \tag{4.1}$$

where $c = \omega/\xi$ is the non-dimensional phase velocity, $\omega$ is the frequency and $\xi$ is the wave number. Here $\theta$ is the angle of inclination of wave normal to the axis of symmetry ($x_3$-axis); $m$ is still an unknown parameter. 1, $W$, $S$, $R$ are respectively the amplitude ratios of displacements $u_1$, $u_3$, temperature change $T$ and concentration $C$ with respect to $u_1$.

Using solutions (4.1) in Eqs. (3.3)–(3.6), we obtain

$$\delta_1 m^2 + s^2 - c^2(1+\mathit{\Gamma}^2) + (\delta_2 ms - 2\iota\mathit{\Gamma} c^2)W + \iota\omega^{-1} cs[\tau_t^{11} S + \tau_c^{11} R] = 0, \tag{4.2}$$

$$\delta_2 ms + 2\iota\mathit{\Gamma} c^2 + (\delta_1 s^2 + m^2 - c^2(1+\mathit{\Gamma}^2))W + \iota\omega^{-1} cm[\tau_t^{11} S + \tau_c^{11} R]U = 0, \tag{4.3}$$

$$\zeta_2 \tau_e^{10} sc + \zeta_2 \tau_e^{10} mcW - \iota\omega^{-1}[z(s^2+m^2) + c^2 \tau_t^{10}]S - \iota\omega^{-1} c^2 \zeta_1 \tau_c^{10} R = 0, \tag{4.4}$$

$$q_1^*(s^2+m^2)(s+mW) - \iota\omega^{-1} c(s^2+m^2)(q_2^* \tau_t^{11} S - q_3^* \tau_c^{11} R) + \omega^{-2} c^3 \tau_f^{10} R = 0, \tag{4.5}$$

where

$$\begin{gathered} \mathit{\Gamma} = \mathit{\Omega}\omega^{-1}, \qquad z = \iota\omega, \qquad s = \sin\theta, \\ \tau_t^{11} = 1 - \iota\xi\tau_1, \quad \tau_c^{11} = 1 - \iota\xi\tau^1, \\ \tau_t^{10} = 1 - \iota\xi\tau_0, \quad \tau_c^{10} = 1 - \iota\xi\gamma, \quad \tau_e^{10} = 1 - \iota\xi\varepsilon\tau_0, \quad \tau_f^{10} = 1 - \iota\xi\varepsilon\tau^0. \end{gathered} \tag{4.6}$$

The system of Eqs. (4.2)–(4.5) has a non-trivial solution if the determinant of the coefficients $[1, W, S, R]^T$ vanishes, which yields the following polynomial characteristic equation:

$$m^8 + A^* m^6 + B^* m^4 + C^* m^2 + D^* = 0. \tag{4.7}$$

The coefficients $A^*$, $B^*$, $C^*$, $D^*$ are given in Appendix A. The characteristic equation (4.7) is biquadratic in $m^2$ and hence it possesses four roots $m_p^2$, $p = 1, 2, 3, 4$. Since we are interested in surface waves only, so it is essential that motion should be confined to free surface $x_3 = 0$ of the half-space, so that the characteristic roots $m_p^2$ must satisfy the radiation conditions $\mathrm{Re}(m_p) \geq 0$. Then the formal expressions for displacements, temperature change and concentration can be written as

$$u_1 = \sum_{p=1}^{4} A_p \exp[\iota\xi(x_1 \sin\theta + \iota m_p x_3 - ct)], \tag{4.8}$$

$$u_3 = \sum_{p=1}^{4} n_{1p} A_p \exp[\iota\xi(x_1 \sin\theta + \iota m_p x_3 - ct)], \tag{4.9}$$

$$T = \sum_{p=1}^{4} n_{2p} A_p \exp[\iota\xi(x_1 \sin\theta + \iota m_p x_3 - ct)], \tag{4.10}$$

$$C = \sum_{p=1}^{4} n_{3p} A_p \exp[\iota\xi(x_1 \sin\theta + \iota m_p x_3 - ct)], \tag{4.11}$$

where $A_p$ $(p = 1, 2, 3, 4)$ are arbitrary constants.

The coupling constants $n_{1p}, n_{2p}, n_{3p}$ $(p = 1, 2, 3, 4)$ are given in Appendix A.

## 5. Boundary conditions

The non-dimensional boundary conditions at the surface $x_3 = 0$ are given by

(i) Mechanical conditions (stress-free surface)

$$\begin{aligned} \sigma_{33} &= (\delta_2 - \delta_1)u_{1,1} + u_{3,3} - \tau_t^1 T - \tau_t^1 C = 0, \\ \sigma_{31} &= \delta_1(u_{3,1} + u_{1,3}) = 0. \end{aligned} \tag{5.1}$$

(ii) Thermal conditions

$$T_{,3} + hT = 0, \tag{5.2}$$

where $h$ is the surface heat transfer coefficient. Here $h \to 0$ corresponds to thermally insulated boundaries and $h \to \infty$ refers to isothermal surfaces.

(iii) Chemical potential

$$P = u_{1,1} + u_{3,3} - n_2 \tau_c^1 C + n_1 \tau_t^1 T = 0, \tag{5.3}$$

where

$$n_1 = \frac{a(\lambda + 2\mu)}{\beta_1 \beta_2}, \qquad n_2 = \frac{b(\lambda + 2\mu)}{\beta_2^2}.$$

## 6. Derivation of the secular equations

Substituting the values of $u_1$, $u_3$, $T$ and $C$ from Eqs. (4.8)–(4.11) in the boundary conditions (5.1)–(5.3), we obtain a system of four simultaneous linear equations as

$$\sum_{p=1}^{4} P_{1p}A_p = 0, \quad \sum_{p=1}^{4} P_{2p}A_p = 0, \quad \sum_{p=1}^{4} P_{3p}A_p = 0, \quad \sum_{p=1}^{4} P_{4p}A_p = 0, \tag{6.1}$$

where

$$P_{1p} = (\delta_2 - \delta_1) + \iota m_p n_{1p} + \frac{\iota \tau_t^{11} n_{2p}}{\xi} + \frac{\iota \tau_c^{11} n_{3p}}{\xi}, \quad P_{2p} = n_{1p} + \iota m_p,$$

$$P_{3p} = 1 + \iota m_p n_{1p} + \frac{\iota \tau_c^{11} n_2 n_{3p}}{\xi} - \frac{\iota \tau_t^{11} n_1 n_{2p}}{\xi}, \; P_{4p} = (\iota m_p + h)n_{2p}, \; p = 1, 2, 3, 4.$$

The system of Eqs. (6.1) has a non-trivial solution if the determinant of the coefficients of amplitudes $A_p$, $p = 1, 2, 3, 4$ vanishes. After lengthy algebraic reductions, it leads to the secular equation:

$$P_{41}D_1 - P_{42}D_2 + P_{43}D_3 - P_{44}D_4 = 0, \tag{6.2}$$

where

$$\begin{aligned}
D_1 &= P_{12}(P_{23}P_{34} - P_{33}P_{24}) - P_{13}(P_{22}P_{34} - P_{32}P_{24}) + P_{14}(P_{22}P_{33} - P_{32}P_{23}),\\
D_2 &= P_{11}(P_{23}P_{34} - P_{24}P_{33}) - P_{13}(P_{21}P_{34} - P_{31}P_{24}) + P_{14}(P_{21}P_{33} - P_{31}P_{23}),\\
D_3 &= P_{11}(P_{22}P_{34} - P_{24}P_{32}) - P_{12}(P_{21}P_{34} - P_{31}P_{24}) + P_{14}(P_{21}P_{32} - P_{31}P_{22}),\\
D_4 &= P_{11}(P_{22}P_{33} - P_{23}P_{32}) - P_{12}(P_{21}P_{33} - P_{31}P_{23}) + P_{13}(P_{21}P_{32} - P_{31}P_{22}).
\end{aligned}$$

For thermally insulated ($h \to 0$) thermoelastic diffusion of the half-space, the secular Eq. (6.2) becomes

$$m_1 n_{21} D_1 - m_2 n_{22} D_2 + m_3 n_{23} D_3 - m_4 n_{24} D_4 = 0, \tag{6.3}$$

and in the case of isothermal ($h \to \infty$) thermoelastic diffusion of the half-space, the Eq. (6.2) reduces to

$$n_{21} D_1 - n_{22} D_2 + n_{23} D_3 - n_{24} D_4 = 0. \tag{6.4}$$

Eqs. (6.3) and (6.4) are Rayleigh surface waves secular equations for stress-free, chemical potential, thermally insulated and isothermal boundaries of generalized isotropic thermoelastic diffusive half-space in a rotating medium, respectively. These secular equations contain complete information corcerning the phase velocity, wave number and attenuation coefficient of the Rayleigh waves in such media. In general, wave number and hence the phase velocity of waves is a complex quantity, therefore the waves are attenuated in space.

If we write

$$c^{-1} = \nu^{-1} + \iota\omega^{-1}F \tag{6.5}$$

so that $\xi = E + \iota F$, where $E = \omega/\nu$, $\nu$ and $F$ are real. Also the roots of characteristic Eq. (4.7) are, in general, complex, and hence we assume that $m_p = p_p + \iota q_p$, so that the exponent in the plane wave solutions (4.8)–(4.11) becomes

$$\iota E(x_1 \sin\theta - m_p^I x_3 - \nu t) - E\left(\frac{F}{E}x_1 \sin\theta + m_p^R x_3\right), \tag{6.6}$$

where

$$m_p^R = p_p - q_p\frac{F}{E}, \qquad m_p^I = q_p + p_p\frac{F}{E}. \tag{6.7}$$

This shows that $\nu$ is the propagation velocity and $F$ is the attenuation coefficient of the wave. Upon using representation (6.5) in secular Eqs. (6.3) and (6.4), the values of propagation speed $\nu$ and attenuation coefficient $F$ of wave propagation can be obtained.

## 7. Surface displacements, temperature change and concentration

The amplitudes of surface displacements, temperature change and concentration at the surface $x_3 = 0$ during Rayleigh wave propagation in the cases of stress-free chemical potential, thermally insulated or isothermal boundaries of the half-space are:

$$\begin{aligned} u_{1S} &= G^* A \exp[\iota E(x_1 \sin\theta - \nu t)], \qquad & u_{3S} &= H^* A \exp[\iota E(x_1 \sin\theta - \nu t)], \\ T_S &= I^* A \exp[\iota E(x_1 \sin\theta - \nu t)], \qquad & C_S &= J^* A \exp[\iota E(x_1 \sin\theta - \nu t)], \end{aligned} \tag{7.1}$$

where

$$\begin{aligned} A &= A_1 \exp(-F x_1 \sin\theta), \\ G^* &= (D_1 - D_2 + D_3 - D_4)/D_1, \\ H^* &= (n_{11}D_1 - n_{12}D_2 + n_{13}D_3 - n_{14}D_4)/D_1, \\ I^* &= (n_{21}D_1 - n_{22}D_2 + n_{23}D_3 - n_{24}D_4)/D_1, \\ J^* &= (n_{31}D_1 - n_{32}D_2 + n_{33}D_3 - n_{34}D_4)/D_1. \end{aligned} \tag{7.2}$$

## 8. Specific loss

The specific loss is the ratio of energy ($\Delta W$) dissipated in leading a specimen through a stress cycle, to the elastic energy ($W$) stored in the specimen when the strain is a maximum. The specific loss is the most direct method of defining internal friction in a material. For a sinusoidal plane wave of small amplitude, Kolsky [16] shows that the specific loss $\Delta W/W$ equals $4\pi$ times the absolute value of the imaginary part of $\xi$ to the real part of $\xi$, i.e.

$$\frac{\Delta W}{W} = 4\pi\left|\frac{\mathrm{Im}(\xi)}{\mathrm{Re}(\xi)}\right| = 4\pi\left|\frac{\nu F}{\omega}\right| = 4\pi\left|\frac{F}{E}\right|. \tag{8.1}$$

## 9. Special cases

1. In absence of the rotation effect ($\Omega = 0$) in Eqs. (6.3) and (6.4), we obtain the corresponding frequency equations for isotropic generalized thermoelastic diffusive half-space.

2. Further in absence of the diffusion effect, i.e. if we take $\beta_2 = a = b = \Omega = 0$ in Eqs. (6.3)–(6.4), we obtain the frequency equations of generalized isotropic thermoelastic half-space as

$$m_1 m'_{21} G_1^* - m_2 m'_{22} G_2^* + m_3 m'_{23} G_3^* = 0, \tag{9.1}$$

$$m'_{21} G_1^* - m'_{22} G_2^* + m'_{23} G_3^* = 0, \tag{9.2}$$

where

$$m'_{1p} = -\frac{\iota\xi f_2 m_p^3 + (f_5 f_2 + \tau_t^{11} f_4 s) m_p}{\iota\xi m_p^4 + (\iota\xi f_3 + f_4 \tau_t^{11} + f_5) m_p^2 + f_3 f_5},$$

$$m'_{2p} = \frac{(f_2 - s) f_4 m_p^2 - f_3 f_4 s}{\iota\xi m_p^4 + (\iota\xi f_3 + f_4 \tau_t^{11} + f_5) m_p^2 + f_3 f_5},$$

$$H_{1p}^* = \delta_2 - \delta_1 + \iota m_p m'_{1p} + \frac{\iota \tau_t^{11} m'_{2p}}{\xi}, \qquad H_{2p}^* = m'_{1p} + \iota m_{2p}, \qquad p = 1, 2, 3,$$

$$G_1^* = H_{12}^* H_{23}^* - H_{22}^* H_{13}^*, \quad G_2^* = H_{11}^* H_{23}^* - H_{21}^* H_{13}^*, \quad G_3^* = H_{11}^* H_{22}^* - H_{21}^* H_{12}^*.$$

The above Eqs. (9.1)–(9.2) are similar to those derived by Sharma *et al.* [25, Eq. (35)] by setting $\beta_2 = a = b = \Omega = 0$.

3. In absence of the thermal, diffusion and rotation effects, we obtain the frequency equation corresponding to isotropic elastic half-space by changing the non-dimensional quantities into the physical quantities as

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 = 4\left(1 - \frac{c^2}{c_1^2}\right)^{1/2}\left(1 - \frac{c^2}{c_2^2}\right)^{1/2}, \tag{9.3}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \qquad c_2^2 = \frac{\mu}{\rho}.$$

The frequency equation (9.3) is same as that derived in Ewing, Jardetzky and Press [13].

## 10. Numerical results and discussion

Following Sherief and Saleh [30], we take the following values of relevant parameters for copper:

$$\lambda = 7.76 \times 10^{10}\ \mathrm{kg\,m^{-1}s^{-2}}, \qquad \mu = 3.86 \times 10^{10}\ \mathrm{kg\,m^{-1}}s^{-2},$$

$$T_0 = 0.293 \times 10^3\ \mathrm{K}, \qquad C_E = .3831 \times 10^3\ \mathrm{J\,kg^{-1}K^{-1}},$$

$$\alpha_t = 1.78 \times 10^{-5}\ \mathrm{K^{-1}}, \qquad \alpha_c = 1.98 \times 10^{-4}\ \mathrm{m^3\,kg^{-1}},$$

$$a = 1.2 \times 10^4\ \mathrm{m^2\,s^{-2}K^{-1}}, \qquad b = 9 \times 10^5\ \mathrm{kg^{-1}\,m^5\,s^{-2}},$$

$$D = 0.85 \times 10^{-8}\ \mathrm{kg\,s\,m^{-3}}, \qquad \rho = 8.954 \times 10^3\ \mathrm{kg\,m^{-3}},$$

$$K = 0.383 \times 10^3\ \mathrm{W\,m^{-1}\,K^{-1}}, \qquad \Omega = 1\ \mathrm{rpm},$$

$$\tau_0 = 0.01\ \mathrm{s}, \quad \tau_1 = 0.07\ \mathrm{s}, \quad \tau^0 = 0.02\ \mathrm{s}, \quad \tau^1 = 0.08\ \mathrm{s}.$$

*Phase velocity and attenuation coefficient*

Figures 2 and 3, respectively, show the variations of phase velocity and attenuation coefficient with respect to wave number, for $\theta = 75^\circ$. Figures 2 and 3 describe the rotation effect on phase velocity and attenuation coefficient in different theories(CT, LS and GL) of thermoelastic diffusion. The rotation and without rotation effects are indicated by the symbols R and WR respectively. In these figures., the solid line (—) and dashed line (- - -) correspond to the coupled-thermoelasticity theory (CT). The star and square symbols on these lines correspond to the Lord–Shulman theory (LS). Similarly, the triangle and circle symbols on these lines correspond to the Green–Lindsay theory (GL). The phase velocity with rotation effect is also shown in small diagram within the Fig. 2 to depict the effect of relaxation times.

From Fig. 2 it is evident that the values of phase velocity without the rotation effect increase initially but then decrease smoothly in all three theories (CT, LS and GL). The values of phase velocity corresponding to the CT theory lie between the values corresponding to the GL and LS theories, i.e. higher and lower than that of GL and LS theories respectively. On the other hand, the values of phase velocity with rotation effect increase first and then decrease to remain constant after that in CT and LS theories, whereas the values increase

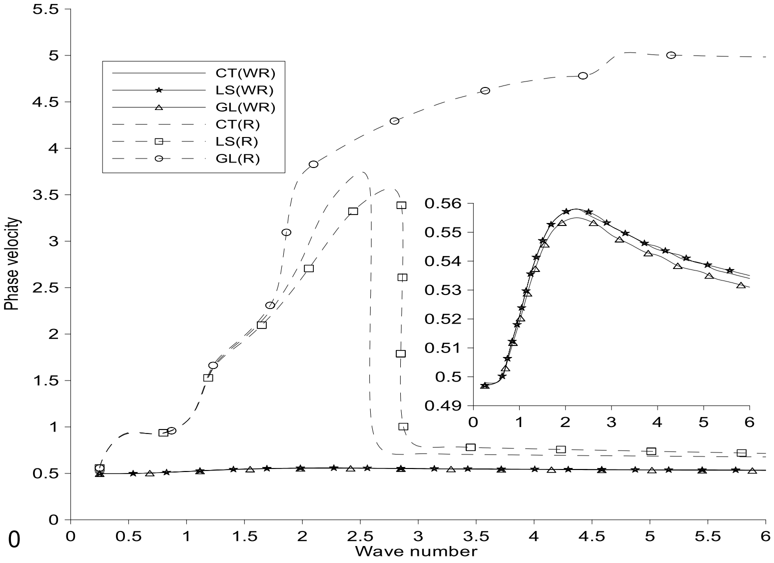

Fig. 2. Variations of phase velocity w.r.t. wave number.

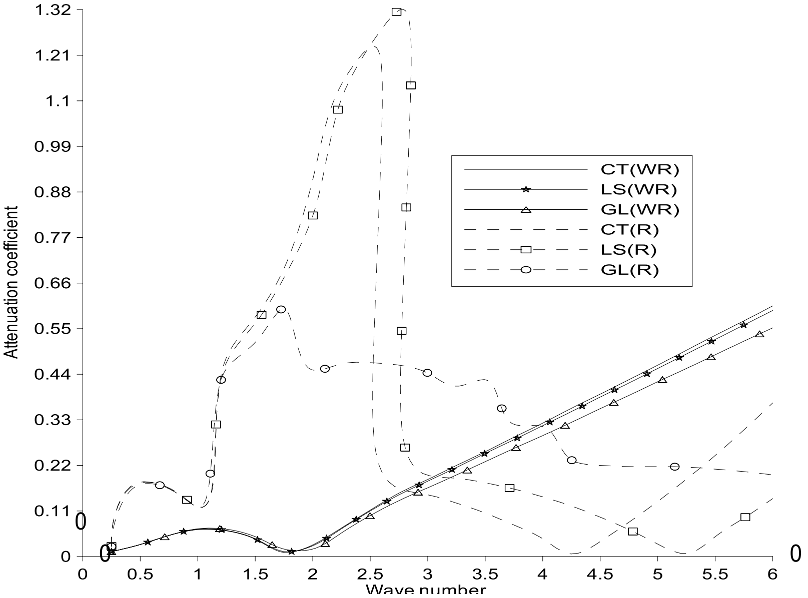

Fig. 3. Variations of attenuation coefficient w.r.t. wave. number.

and then remain constant in the GL theory. The increase in the values of phase velocity is greater when we move from CT to generalized theories (LS and GL). From Fig. 2 it is clearly seen that the values of phase velocity increase with the increase of angular speed ($\Omega$).

Figure 3 shows that without the rotation effect, values of the attenuation coefficient increase in all three theories, but the increase in the values is less from CT to LS and to GL theory. On the other hand, the values of attenuation coefficient with rotation effect increase with wave number initially and then decrease dramatically, but finally they increase smoothly. On comparing the values of the attenuation coefficient with and without the rotation effects, we find that the increase in the values is smaller with the rotation effect.

The variations of phase velocity and attenuation coefficient for Rayleigh waves have been plotted in Figs. 4 and 5 with respect to wave number for $\theta = 75°$ and $\theta = 90°$ keeping $\Omega = 1$ fixed. In these figures, the solid line (—) and dashed line (- - -) correspond to the coupled-thermoelasticity theory (CT). The star and square symbols on these lines correspond to the Lord–Shulman theory (LS). Similarly, the triangle and circle symbols on these lines correspond to the Green–Lindsay theory (GL). The phase velocity with rotation effect is also shown in small diagram within the Fig. 4 to depict the effect of relaxation times.

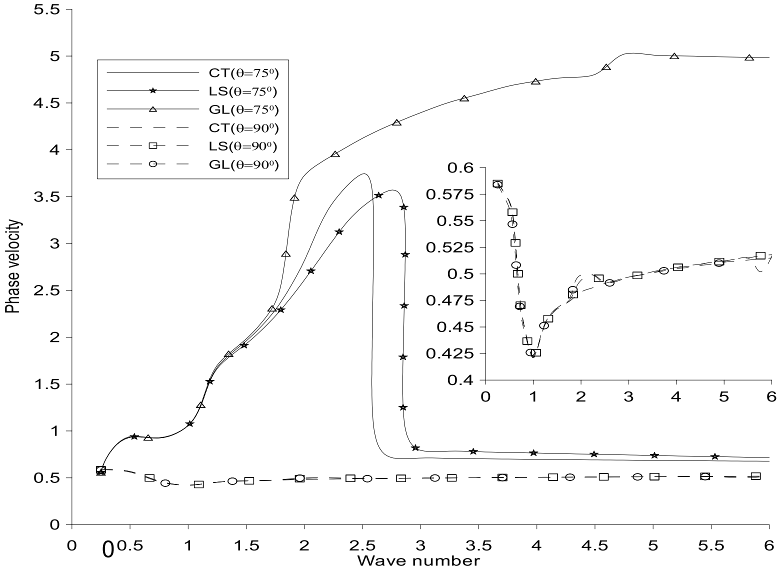

Fig. 4. Variations of phase velocity w.r.t. wave number.

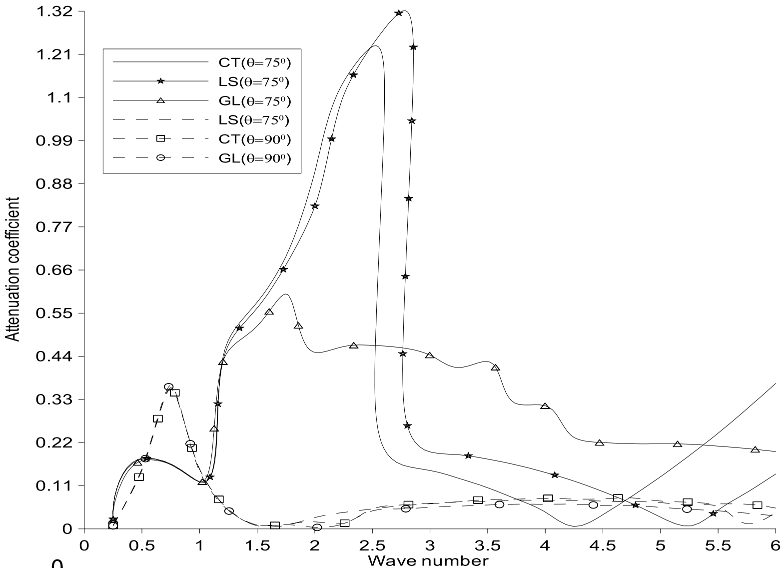

Fig. 5. Variations of attenuation coefficient w.r.t. wave.

Figure 4 clearly indicates that as the value of $\theta$ increases, the increase in the values of phase velocity is smaller, and as we move from CT to LS and then to the GL theory, the increase in the values is greater. Similar type of behavior is noticed in Fig. 5.

The variations of amplitudes of displacements $(u_1, u_3)$, temperature change $(T)$ and concentration $(C)$ with respect to wave number are plotted in Figs. 6–10 for $\theta = 75^\circ$. In these figures, the solid and dashed lines correspond to the CT theory for WR and R respectively. The star and circle symbols on these lines correspond to the LS theory respectively. Similarly, the triangle and square symbols correspond to the GL theory respectively.

Figure 6 shows that without the rotation effect, the values of $u_1$ increase slowly for CT and LS theories, but for the GL theory, the values show a decrease initially, but then increase steadily. If we compare the values of $u_1$ corresponding to different theories, we notice that the values of $u_1$ are greater in the GL theory as compared to the CT and LS theories to a certain limit, but after that the values of $u_1$ are greater in LS theory as compared to other two theories (CT and GL). On considering the rotation effect, we see that there is a sharp decrease in the values of $u_1$ and then a dramatic increase in the values of $u_1$, which remain constant after that. The values corresponding to GL theory remain higher than those of CT and LS theories. With the rotation effect, the values of $u_1$ are greater

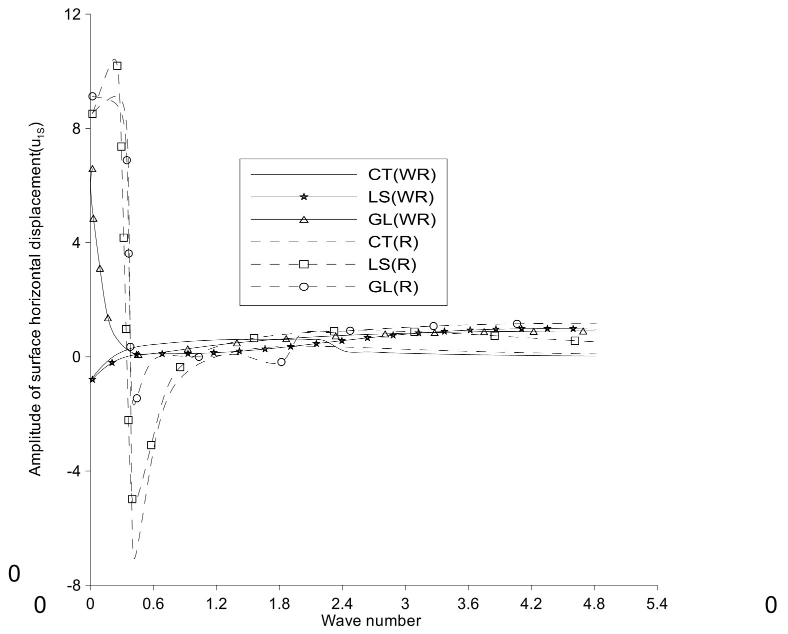

Fig. 6. Variations of amplitude of surface horizontal displacement w.r.t. wave number.

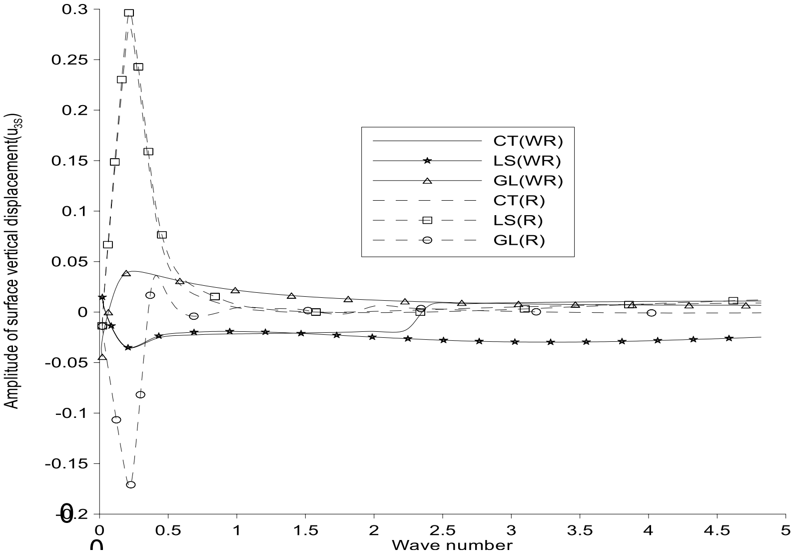

Fig. 7. Variations of amplitude of surface vertical displacement w.r.t. wave number.

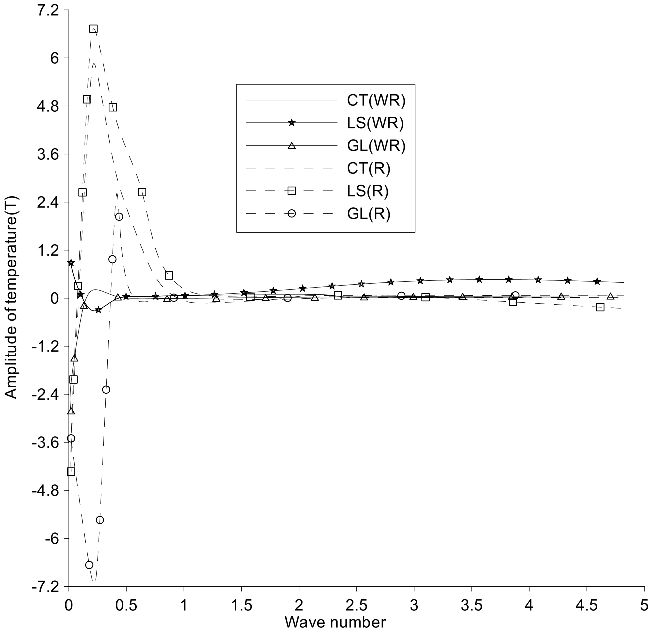

FIG. 8. Variations of amplitude of temperature change w.r.t. wave number.

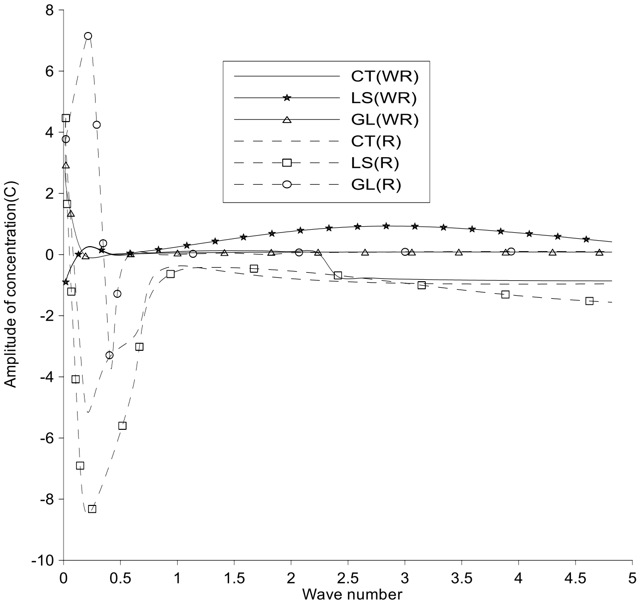

FIG. 9. Variations of amplitude of concentration w.r.t. wave number.

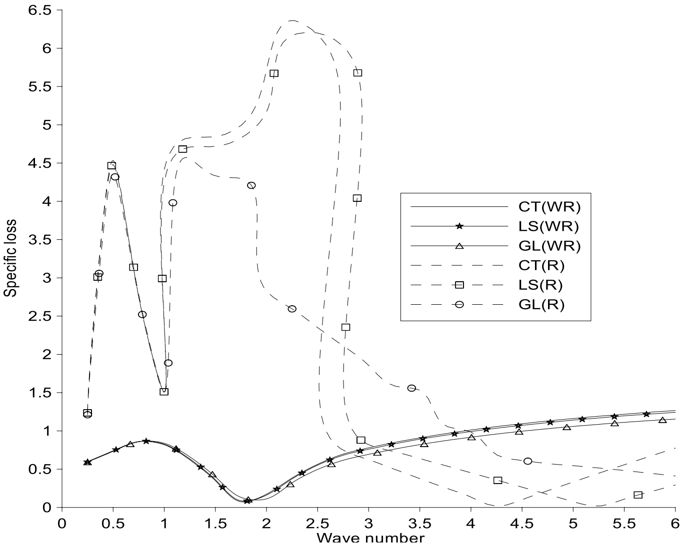

Fig. 10. Variations of specific loss w.r.t. wave number.

than the values for no rotation effects. On the other hand, the reverse behavior occurs in case of vertical surface displacement except the LS theory where the values of $u_3$ are greater with the rotation effect.

The values of $T$ show an approximately constant behavior in all theories for the case without the rotation effect. With rotation effect, the values of $T$ increase initially and then decrease to remain constant. In absence of the rotation effect, the values of $T$ are smaller compared to the values for the case without rotation effect. Similarly, the values of $C$ without the rotation effect are greater than those of without rotation effect.

Figure 10 shows the variations of specific loss with respect to the wave number for $\theta = 75°$. The values of specific loss without the rotation effect are increasing in all the theories CT, LS and GL, whereas the values of specific loss with rotation effect increase initially and then decrease sharply, but finally they increase. The values of specific loss increase with rotation effect initially, but after a certain limit, the reverse behavior occurs.

## 11. Conclusions

The propagation of Rayleigh waves in a homogeneous isotropic, rotating half-space have been investigated in the context of different theories of thermoelastic diffusion. The secular equations for surface wave propagation in the considered media are derived. The dispersion curves, attenuation coefficients, amplitudes of surface displacements, temperature change and concentration and specific loss of energy are computed and shown graphically in different theories of thermoelastic diffusion in the case of copper material. The numerically computed results are found to be in good agreement with the corresponding analytic results. The effects of relaxation times and rotation are observed on phase velocity, attenuation coefficient and amplitudes of surface displacements, temperature change, concentration and specific loss of energy. The analysis carried out will be useful in the design and construction of rotating sensors and other surface acoustic wave (SAW) devices, in addition to possible biosensing applications.

## Appendix A

$$A^* = \frac{\iota\xi(f_1I_1 - f_2I_8) + \delta_1 d_1 - d_7 s\tau_c^{11}}{\iota\xi\delta_1 I_1},$$

$$B^* = \frac{f_1d_1 - \delta_1 d_2 - f_2 d_3 - \iota\xi l_1^2 f_8 + s(d_5\tau_t^{11} - d_8\tau_c^{11})}{\iota\xi\delta_1 I_1},$$

$$C^* = \frac{f_2d_4 - l_1^2 I_3 - \delta_1 f_3 I_6 - f_1 d_2 + s(d_6\tau_t^{11} - d_9\tau_c^{11})}{\iota\xi\delta_1 I_1},$$

$$D^* = -\frac{f_3(s^2\tau_t^{11} I_{12} + f_1 I_6 - s^4\tau_c^{11} I_{17}) + l_1^2 I_6}{\iota\xi\delta_1 I_1},$$

$$f_1 = s^2 - c^2(1+\Gamma^2), \quad f_2 = \delta_2 s, \quad f_3 = \delta_1 s^2 - c^2(1+\Gamma^2), \quad f_4 = \zeta_2\tau_e^{10}c,$$

$$f_5 = \iota\xi s^2 + c\tau_t^{10}, \quad f_6 = c\zeta_1\tau_c^{10}, \quad f_7 = f_8 s^2 - \iota w^{-1}c\tau_f^{10}, \quad f_8 = q_3^*\tau_c^{11},$$

$$l_1 = 2\iota\Gamma c^2,$$

$$I_1 = q_1^*\tau_c^{11} - f_8, \quad I_2 = q_1^*\tau_c^{11}s^2 - f_3 f_8, \quad I_3 = f_5 f_8 + \iota\xi f_7 + f_6 q_2^*\tau_t^{11},$$

$$I_4 = f_6\tau_t^{11} - f_5\tau_c^{11}, \quad I_5 = (f_8 + q_2^*\tau_c^{11})\tau_t^{11}, \quad I_6 = f_5 f_7 + f_6 q_2^*\tau_t^{11}s^2,$$

$$I_7 = (f_7 + q_2^*\tau_c^{11}s^2)\tau_t^{11}, \quad I_8 = q_1^*\tau_c^{11}s - f_8 f_2, \quad I_9 = \iota\xi\tau_c^{11}s^2 - I_4,$$

$$I_{10} = f_4 f_8 + q_1^* f_6, \quad I_{11} = f_6 + f_4\tau_c^{11}, \quad I_{12} = f_4 f_7 + q_1^* f_6 s^2,$$

$$I_{13} = f_7 + f_8 f_3 - q_1^* s^2\tau_c^{11}, \quad I_{14} = I_{11}s^2 + f_3 f_6, \quad I_{15} = q_1^*(f_5 + \iota\xi s^2) - f_4 q_2^*\tau_t^{11},$$

$$I_{16} = f_5 + \iota\xi f_3 + f_4\tau_t^{11}, \quad I_{17} = f_5 q_1^* - f_4 q_2^*\tau_t^{11}, \quad I_{18} = (f_3 + s^2)q_2^* + q_1^* s^2,$$

$$d_1 = \iota\xi I_2 - I_3 - q_1^* I_4 - f_4 I_5, \quad d_2 = f_3 I_3 + I_6 + q_1^* s^2 I_4 + f_4 I_7,$$

$$d_3 = q_1^* s I_9 - f_2 I_3 - f_4 s I_5, \quad d_4 = f_2 I_6 + f_4 s I_7 + q_1^* s^3 I_4,$$

$$d_5 = f_2 I_{10} + f_4 s I_1 - q_1^* s I_{11}, \quad d_6 = f_2 I_{12} - f_4 s I_{13} - q_1^* s I_{14},$$

$$d_7 = \iota\xi q_1^*(f_2 - s), \qquad d_8 = f_2 I_{15} + f_4 s(q_1^* + q_2^*)\tau_t^{11} - q_1^* s(\iota\xi s^2 + I_{16}),$$

$$d_9 = f_2 s^2 I_{17} + f_4 s I_{18} \tau_t^{11} - q_1^* s(f_3 f_5 + s^2 I_{16}),$$

$$n_{1p} = -\frac{\iota\xi I_8 m_p^5 - \iota\xi f_8 l_1 m_p^4 + d_3 m_p^3 - l_1 I_3 m_p^2 - d_4 m_p - l_1 I_6}{\iota\xi I_1 m_p^6 + d_1 m_p^4 - d_2 m_p^2 - f_3 I_6},$$

$$n_{2p} = \frac{d_5 m_p^4 + l_1 I_{10} m_p^3 + d_6 m_p^2 + l_1 I_{12} m_p - f_3 s I_{12}}{\iota\xi I_1 m_p^6 + d_1 m_p^4 - d_2 m_p^2 - f_3 I_6},$$

$$n_{3p} = -\frac{d_7 m_p^6 + \iota\xi q_1^* l_1 m_p^5 + d_8 m_p^4 + l_1 I_{15} m_p^3 + d_9 m_p^2 + l_1 s^2 I_{17} m_p - f_3 s^3 I_{17}}{\iota\xi I_1 m_p^6 + d_1 m_p^4 - d_2 m_p^2 - f_3 I_6},$$

$p = 1, 2, 3, 4.$

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