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Thermal convection of micropolar fluid in the presence of suspended particles in rotation

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A study has been made of the convection of micropolar fluids heated from below in the presence of suspended particles (fine dust) and uniform vertical rotation $\mathbf{\Omega}$ $(0, 0, \mathbf{\Omega})$. The effect of Coriolis forces on the stability is chosen along the direction of the gravitational field. It is found that the presence of coupling between thermal and micropolar effects, rotation parameter and suspended particles may introduce overstability in the system. Using the Boussinesq approximation, the linearized stability theory and normal mode analysis, the exact solutions are obtained for the case of two free boundaries. Graphs have been plotted by giving numerical values to the parameters accounting for rotation $\mathbf{\Omega}$ $(0, 0, \mathbf{\Omega})$ and the dynamic microrotation viscosity κ and coefficient of angular viscosity γ' to depict the stability characteristics, for both the cases of stationary convection and overstability. It is found that Rayleigh number for the case of overstability and stationary convection increases with increase in rotation parameters and decreases with increase in micropolar coefficients, for a fixed wave number, showing thereby the stabilizing effect of rotation parameters and destabilizing effect of micropolar coefficients on the thermal convection of micropolar fluids. Thus there is a competition between the stabilizing effect of rotation parameters and destabilizing effect of micropolar coefficients and the suspended particles. It is also found from the graphs that the Rayleigh number for the case of overstability is always smaller than the Rayleigh number for the case of stationary convection, for a fixed wave number.

Notations

$\mathbf{\Omega}$ $(0, 0, \mathbf{\Omega})$	rotation vector having components,
ρ	density of fluid,
$\mathbf{v}$	velocity of fluid,
ϑ	spin,
p	pressure,
T	temperature,
$\mathbf{g}$	acceleration due to gravity,
k_T	thermal conductivity,
$\mathbf{u}$	particle velocity,
c_{pt}	heat capacity of particles,
$\hat{e}_z$	unit vector in z-direction,

δ the coefficient giving account of coupling between spin and heat flux,
υ kinematic viscosity of the fluid,
j_1 microinertial constant,
c_v specific heat at constant volume,
$\epsilon', \beta', \gamma'$ coefficients of angular viscosity,
κ dynamic microrotation viscosity,
N number density,
mN the mass of suspended particles per unit volume,
K Stoke's drag coefficient,
r' particle radius,
π constant value,
ρ_0 reference density,
T_0 reference temperature,
α coefficient of thermal expansion,
∂ curl operator,
∇ del operator,
β uniform temperature gradient,
$\mathbf{v}(u, v, w)$ perturbations in fluid velocity $\mathbf{v}(0, 0, 0)$,
$\mathbf{u}(\ell, r, s)$ perturbations in particles velocity $\mathbf{u}(0, 0, 0)$,
ω perturbations in spin ϑ,
$\delta\rho$ perturbations in density ρ,
θ perturbations in temperature T,
$\partial/\partial t$ convective or material derivative,
κ_T thermal diffusity,
ξ_z z-component of current density,
ζ_z z-component of vorticity,
R dimensionless Rayleigh number,
p_1 thermal Prandtl number,
k wave number of the disturbance,
n growth rate of the disturbance,
d depth of the layer,
A, ℓ_1, L_1, L_2, b constants,
D derivative with respect to z $(= d/dz)$.

1. Introduction

Micropolar theory was introduced by Eringen [1] in order to describe some physical systems which do not sastisfy the Navier–Stokes equations. Micropolar fluids are able to describe the behaviour of colloidal solutions, liquid crystals, animal blood etc.The equations governing the flow of micropolar fluid theory involve a spin vector and a microinertia tensor, in addition to the velocity vector.A generalization of the theory including thermal effects has been developed by Kazakia and Ariman [2] and Eringen [3]. The stability investigations of the Bénard problem in the framework of various external force fields, assume importance not only on account of being a meaningful mathematical extensions of the problem, but also because of its importance in the problem of meteorology, oceanography and various other fields of practical importance. The effects of the action of a uniform vertical magnetic field and a uniform vertical rotation acting individually or simultaneously on the Bénard problem, have been investigated by Chandrasekhar [4] and others, in which it is shown that in some respect their individual/combined effects are remarkably alike, namely they both inhibit the onset of instability and elongate the cells which appear at the marginal stability for certain ranges of values of the parameters involved. Another interesting point brought out by Chandrasekhar's analysis, which is in general qualitative agreement with the experimental results of Nakagawa [5, 6], Fultz, Nakagawa and Frenzen [7] and others is that, in both the problems, the marginal state could either be stationary or oscillatory in character for which sufficient conditions are obtained.

Micropolar fluid stabilities have become an important field of research these days. Ahmadi [8] and Pe'rez-Garcia et al. [9] have studied the effects of the microstructures on the thermal convection and have found that in the absence of coupling between thermal and micropolar effects, the principle of exchange of stabilities may not be fulfilled and consequently, the micropolar fluids introduce oscillatory motions. The existence of oscillatory motions in micropolar fluids has been depicted by Lekkerkerker in liquid crystals [10, 11], Bradley in dielectric fluids [12] and Laidlaw in binary mixture [13]. In the study of problems of thermal convection, it is a frequent practice to simplify the basic equations by introducing an approximation which is attributed to Boussinesq [14]. In geophysical situations, the fluid is often not pure but contains several suspended particles. Motivation for the study of certain effects of particles immersed in the fluid such as particle heat capacity, particle mass fraction and thermal force, is due to the fact that the knowledge concerning fluid — particles mixtures is not commensurate with their industrial and scientific importance. Saffman [15] has considered the stability of laminar flow of a dusty gas. Sharma *et al.* [16] have considered the effect of suspended particles on the onset of Bénard convection in hydromagnetics and found that the critical Rayleigh number was reduced because of the heat capacity of particles, thereby destabilizing the system. On the other hand, multiphase fluid systems are concerned with the motion of liquid or gas containing immiscible inert identical particles of all multiphase fluid systems observed in nature, blood flow in arteries, flow in rocket tubes, dust-in-gas cooling system to enhance heat transfer processes, movement of inset solid particles in atmosphere, and sand or other particles on sea or ocean beaches are the most common examples of multiphase fluid systems.

Generally, the suspended particles number density has a destabilizing effect on the thermal convection of the fluids. From the physical point of view, the effect of rotation on the micropolar fluids in the presence of suspended particles is interesting because there is a competition between the large enough stabilizing effect of rotation and the destabilizing effect of suspended particles. Moreover, rotation introduces Coriolis acceleration which plays an important role in the stability of the system and a centrifugal force which is neglected due to its small magnitude. The rotating fluid also finds its application in meteorphysics and oceanography. Sharma and Kumar [17] have studied the stability of micropolar fluids heated from below in the presence of suspended particles (fine dust) and have found that suspended particles number density has a destabilizing effect on the convection of micropolar fluids. Keeping in mind the importance and relevance of thermal convection with suspended particles and rotation, the present paper deals therefore with the thermal convection of micropolar fluid in the presence of suspended particles in rotation.

2. Formulation of the problem and perturbation equations

Consider the stability of an infinite, horizontal layer of an incompressible micropolar fluid of thickness d permeated with suspended particles (or fine dust). A uniform vertical rotation $\mathbf{\Omega}(0, 0, \Omega)$ pervades the system. This fluid-particles layer is heated from below but convection sets in when the temperature gradient between the lower and upper boundaries exceeds a certain critical value. The critical temperature gradient depends upon the bulk properties and boundary conditions of the fluid.

Let $\mathbf{v}, \vartheta, \mathbf{p}, \rho, T, g, k_T, c_{pt}, c_v, \hat{e}_z, \mathbf{u}, \delta, \upsilon$ and j_1 denote the velocity, the spin, the pressure, the density, the temperature, the acceleration due to gravity, the thermal conductivity, the heat capacity of particles, the specific heat at constant volume, the unit vector in z-direction, the particle velocity, the coefficient giving account of coupling between spin and heat flux, kinematic viscosity of the fluid and microinertial constant, respectively. $\epsilon', \beta', \gamma'$ are the coefficients of angular viscosity and κ is the dynamic microrotation viscosity and $\overrightarrow{r_1} = (x, y, z)$. Let N, mN denote respectively, the number density and the mass of suspended particles per unit volume. If $K = 6\pi\mu r'$, r' being the particle radius, is the Stoke's drag coefficient then the mass, momentum, internal angular momentum, internal energy balance equation, using the Boussinesq approximation, are

$$\nabla \cdot \mathbf{v} = \mathbf{0}, \tag{2.1}$$

$$\rho_0 \frac{d\mathbf{v}}{dt} = -\nabla\left(p - \frac{\rho_0}{2}|\mathbf{\Omega} \times \mathbf{r}_1|^2\right) + (\mu + \kappa)\nabla^2\mathbf{v} + \kappa\nabla \times \vartheta - \rho g\hat{e}_z + KN(\mathbf{u} - \mathbf{v}) + 2\rho_0(\mathbf{v} \times \mathbf{\Omega}), \tag{2.2}$$

$$\rho_0 j_1 \frac{d\vartheta}{dt} = (\epsilon' + \beta')\nabla(\nabla\vartheta) + \gamma'\nabla^2\vartheta + \kappa\nabla \times \mathbf{v} - 2\kappa\vartheta, \tag{2.3}$$

$$\rho_0 c_v \left(\frac{\partial}{\partial t} + \mathbf{v}.\nabla\right)T + mNc_{pt}\left(\frac{\partial}{\partial t} + \mathbf{u}.\nabla\right)T = k_T\nabla^2 T + \delta(\nabla \times \vartheta).\nabla T \tag{2.4}$$

and the equation of state is given by

$$\rho = \rho_0[1 - \alpha(T - T_0)], \tag{2.5}$$

where ρ_0, T_0 are reference density, reference temperature at the lower boundary and α is the coefficient of thermal expansion. In the Eq. (2.2), the term $2(\mathbf{\Omega}\times\mathbf{v})$ represents the Coriolis acceleration and the term $\frac{1}{2}(\mathrm{grad}\,|\mathbf{\Omega}\times\mathbf{r}_1|^2)$ represents the centrifugal force (which is of very small magnitude).

If we assume the dust particles to be of uniform particle size, spherical shape and small relative velocities between the two phases (fluid and particles), then the net effect of the particles on the fluid is equivalent to an extra body-force term per unit volume $KN(\mathbf{u}-\mathbf{v})$, as has been taken in Eq. (2.2). This force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. The distance between the particles is assumed to be so large compared with their diameter that interparticle reactions can be ignored. The equations of motion and continuity for the particles, under these restrictions, are

$$\begin{aligned} mN\left[\frac{\partial}{\partial t} + \mathbf{u}.\nabla\right]\mathbf{u} &= KN(\mathbf{v}-\mathbf{u}),\\ \frac{\partial \mathbf{N}}{\partial t} + \nabla.(N.\mathbf{u}) &= 0. \end{aligned} \tag{2.6}$$

The steady-state solution of the governing equations (2.1)–(2.6) is given by

$$\begin{aligned} &\mathbf{v}=0, \qquad \mathbf{u}=0, \qquad \vartheta=0, \qquad T=T_0-\beta(z), \qquad \rho=\rho_0(1+\alpha\beta z),\\ &p = p_0 - g\rho_0\left[z+\frac{\alpha\beta z^2}{2}\right], \end{aligned} \tag{2.7}$$

where p_0 is the pressure at $z=0$ and $\beta=(T_0-T_1)/d$ $(T_0>T_1)$ is the magnitude of uniform temperature gradient.

Let the initial stationary state, as described by (2.7), be slightly perturbed. Let $\mathbf{v}(u,v,w)$, $\mathbf{u}(\ell,r,s)$, ω, N, δp, $\delta\rho$, θ denote respectively the perturbations on fluid velocity $\mathbf{v}(0,0,0)$, particles velocity $\mathbf{u}(0,0,0)$, spin ϑ and particles number density N_0, pressure p, density ρ, temperature T. Then Eqs. (2.1)–(2.6) yield the perturbation equations

$$\nabla\cdot\mathbf{v}=0, \tag{2.8}$$

$$\begin{aligned} \rho_0\frac{d\mathbf{v}}{dt} = -\nabla\delta p + (\mu+\kappa)\nabla^2\mathbf{v} + \kappa\nabla\times\omega + \alpha\rho_0 g\hat{e}_z\\ + KN_0(\mathbf{u}-\mathbf{v}) + 2\rho_0(\mathbf{v}\times\mathbf{\Omega}), \end{aligned} \tag{2.9}$$

$$\rho_0 j_1\frac{d\omega}{dt} = (\epsilon'+\beta')\nabla(\nabla.\omega) + \gamma'\nabla^2\omega + \kappa\nabla\times\mathbf{v} - 2\kappa\omega, \tag{2.10}$$

$$(2.11)\quad H_1\frac{d\theta}{dt}=\beta(w+h_1s)+k_T\nabla^2\theta+\frac{\delta}{\rho_0c_v}[\nabla\theta.(\nabla\times\omega)-(\nabla\times\omega)_z.\beta],$$

$$(2.12)\quad mN\left(\frac{\partial}{\partial t}+\mathbf{u}.\nabla\right)\mathbf{u}=KN_0(\mathbf{v}-\mathbf{u}),$$

$$(2.13)\quad \frac{\partial M}{\partial t}+\nabla.u=0,$$

where $H_1=1+h_1$, $h_1=fc_{pt}/c_v$, $f=mN_0/\rho_0$, $M=N/N_0$, $d/dt=(\partial/\partial t+\mathbf{v}.\nabla)$ is the convective or material derivative.

Using the non-dimensional numbers,

$$z=z^*d,\qquad \theta=\beta d\theta^*,\qquad t=\frac{\rho_0d^2}{\mu}t^*,\qquad \mathbf{v}=\frac{\kappa_T}{d}\mathbf{v}^*,\qquad \mathbf{\Omega}=\frac{\mu}{\rho_0d^2}\Omega^*,$$
$$\mathbf{u}=\frac{\kappa_T}{d}\mathbf{u}^*,\qquad p=\frac{\mu\kappa_T}{d^2}p^*,\qquad \omega=\frac{\kappa_T}{d^2}\omega^*,$$

and then removing the stars for convenience, the non-dimensional forms of Eqs. (2.8)–(2.13) become

$$(2.14)\quad \nabla\cdot\mathbf{v}=0,$$

$$(2.15)\quad \frac{d\mathbf{v}*}{dt}=-\nabla\delta p+(1+K_1)\nabla^2\mathbf{v}+K_1\nabla\times\omega+\hat{e}_zR\theta+N_2(\mathbf{u}-\mathbf{v})+2(\mathbf{v}\times\mathbf{\Omega}),$$

$$(2.16)\quad \bar{j}_2\frac{d\omega}{dt}=C_1'\nabla(\nabla.\omega)-C_0'\nabla\times(\nabla\times\omega)+K_1(\nabla\times\mathbf{v}-2\omega),$$

$$(2.17)\quad H_1p_1\frac{d\theta}{dt}=\beta(w+h_1s)+\nabla^2\theta+\bar{\delta}[\nabla\theta.(\nabla\times\omega)-(\nabla\times\omega)_z],$$

$$(2.18)\quad \left(a\frac{d}{dt}+1\right)\mathbf{u}=\mathbf{v}.$$

The new dimensionless coefficients are

$$(2.19)\quad \begin{aligned}&K_1=\frac{\kappa}{\mu},\qquad \bar{j}_2=\frac{j_1}{d^2},\qquad \bar{\delta}=\frac{\delta}{\rho_0c_vd^2},\qquad C_0'=\frac{\gamma'}{\mu d^2},\\ &C_1'=\frac{\epsilon'+\beta''+\gamma'}{\mu d^2},\qquad N_2=KN_0\frac{d^2}{\mu},\qquad a=\frac{m}{Kd^2}\frac{\mu}{\rho_0},\end{aligned}$$

and the dimensionless Rayleigh number R, thermal Prandtl number p_1, are

$$(2.20)\quad R=\frac{g\alpha\beta d^4}{\mu\kappa_T},\qquad p_1=\frac{\mu}{\kappa_T},$$

where $\kappa_T=k_T/\rho_0c_v$ is the thermal diffusivity.

Eliminating $\mathbf{u}$ between (2.15) and (2.18), we obtain

$$L'_1 = L'_2[-\nabla\delta p + (1+K_1)\nabla^2\mathbf{v} + K_1\nabla\times\omega + R\theta\hat{e}_z + 2(v\times\Omega)], \tag{2.21}$$

where

$$L'_1 = a\frac{d^2}{dt^2} + F\frac{d}{dt}, \qquad L'_2 = a\frac{d}{dt} + 1 \qquad \text{and} \qquad F = f+1.$$

Elimination of s from Eq. (2.17), with the help of (2.18), yields

$$L'_2\left[H_1 p_1\frac{d}{dt} - \nabla^2\right]\theta = \left(a\frac{d}{dt} + H_1\right)\beta w + L'_2[\nabla\theta.(\nabla\times\omega) - (\nabla\times\omega)_z]. \tag{2.22}$$

Both the boundaries are considered to be free. Since the surfaces are fixed and are maintained at fixed temperature, we must have $w = 0 = \theta$ at $z = 0$ and $z = d$. Further, tangential stresses do not act on free surfaces. The conditions to be satisfied are $T_{xz} = 0$, $T_{yz} = 0$, which yield

$$\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0 \qquad \text{and} \qquad \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0. \tag{2.23}$$

Now, as w vanishes for all x and y on the boundaries, it follows from the Eqs. (2.23) that $\partial u/\partial z = \partial v/\partial z = 0$.

Differentiating Eq. (2.14) with respect to z and using $\partial u/\partial z = \partial v/\partial z = 0$, we conclude that $\partial^2 w/\partial z^2 = 0$ on free surfaces.

Thus the boundary conditions appropriate to the problem are

$$w = 0, \quad \frac{\partial^2 w}{\partial z^2} = 0, \quad \frac{\partial\zeta}{\partial z} = 0, \quad \xi_z = 0, \quad \theta = 0 \quad \text{at } z = 0 \text{ and } z = d, \tag{2.24}$$

where $\xi_z = (\nabla\times\omega)_z$, $\zeta_z = (\nabla\times\mathbf{v})_z$ are the z-component of current density, vorticity respectively.

3. Linear theory: dispersion relation

Since the perturbations applied to the system are assumed to be very small, under the linearized theory, second and higher-order perturbations are neglected and only the linear terms are retained. Accordingly, the non-linear terms $(\mathbf{v}.\nabla)\mathbf{v}$, $(\mathbf{v}.\nabla)\theta$, $(\mathbf{v}.\nabla)\omega$, $\nabla\theta.(\nabla\times\omega)$ in Eqs. (2.15)–(2.17) are neglected.

Applying the curl operator twice to Eq. (2.22) and linearizing, we obtain

$$L_1\left[H_1 p_1\frac{d}{dt} - \nabla^2\right]\theta = \left(a\frac{\partial}{\partial t} + H_1\right)\beta w - L_2\bar{\delta}\xi_z. \tag{3.1}$$

Applying the curl operator twice to Eq. (2.15) and taking the z-component, we get

$$L_1\nabla^2 w = L_2\left[R\nabla_1^2\theta + (1+K_1)\nabla^4 w + K_1\nabla^2\xi_z - 2\Omega\frac{\partial \xi_z}{\partial z}\right]. \tag{3.2}$$

Applying the curl operator to Eqs. (2.15) and (2.16) taking z-component, we get

$$L_2\frac{\partial}{\partial t}\zeta_z + n_1\zeta_z(L_2-1) = (1+K_1)\nabla^2\zeta_z L_2 + 2\Omega\frac{\partial w}{\partial z}, \tag{3.3}$$

$$\bar{j}_2\frac{\partial \xi_z}{\partial t} = C_0'\nabla^2\xi_z - K_1(\nabla^2 w + 2\xi_z), \tag{3.4}$$

where K_1 and C_0' accounts for coupling between vorticity and spin effects and spin diffusion, respectively. Here

$$\begin{aligned} \nabla_1^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \qquad L_1 = a\frac{\partial^2}{\partial t^2} + F\frac{\partial}{\partial t}, \qquad L_2 = a\frac{\partial}{\partial t} + 1, \\ \nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \end{aligned} \tag{3.5}$$

Combine the disturbances into a complete set of normal modes, and then examine the stability of each of these modes individually. For the system of Eqs. (3.1)–(3.4), the analysis can be made in terms of two-dimensional periodic waves of assigned wave numbers. Thus we ascribe to all quantities describing the perturbation dependence on x, y and t in the form

$$[w, \zeta_z, \xi_z, \theta] = [W(z), Z(z), G(z), \Theta(z)]\exp(ik_x + ik_y y + nt), \tag{3.6}$$

where $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number, k_x and k_y are real constants and n is the stability parameter which is, in general, a complex constant. The solution of the stability problem requires the knowledge of specifications of the state for each k.

For solutions having dependence of the form (3.6), Eqs. (3.1)–(3.4) take the form

$$(an+1)[H_1 p_1 n - (D^2-k^2)]\Theta = (an+H_1)W - (an+1)\bar{\delta}G, \tag{3.7}$$

$$\begin{aligned} (D^2-k^2)[(an^2+Fn) - (an+1)(1+K_1)(D^2-k^2)]W \\ = (an+1)[-Rk^2\Theta + K_1(D^2-k^2)G - 2\Omega DZ], \end{aligned} \tag{3.8}$$

$$[(an^2+Fn) - (an+1)(D^2-k^2)(1+K_1)]Z = 2\Omega DW, \tag{3.9}$$

$$[\ell_1 n + 2A - (D^2-k^2)]G = -A(D^2-k^2)W, \tag{3.10}$$

where

$$A = \frac{K_1}{C_0'}, \qquad \ell_1 = \bar{j}_2 \frac{A}{K_1}, \qquad D = \frac{d}{dz}, \qquad \frac{\partial}{\partial t} = n,$$

$$L_2 = a\frac{\partial}{\partial t} + 1 = an + 1, \qquad L_1 = a\frac{\partial^2}{\partial t^2} + F\frac{\partial}{\partial t} = an^2 + Fn.$$

The boundary conditions (2.24) transform to

$$W = D^2W = D^2G = \Theta = 0 \qquad \text{at} \qquad z = 0 \quad \text{and} \quad z = 1. \tag{3.11}$$

Eliminating Θ, Z and G from (3.7)–(3.10), we get

$$\begin{aligned}(D^2-k^2)[(an^2+Fn)-(an+1)(1+K_1)(D^2-k^2)]^2\\ \times[H_1p_1n-(D^2-k^2)][\ell_1n+2A-(D^2-k^2)]W\\ = -Rk^2[(an^2+Fn)-(an+1)(1+K_1)(D^2-k^2)]\\ \times\{(an+H_1)(\ell_1n+2A-(D^2-k^2))+\bar{\delta}A(D^2-k^2)(an+1)\}W\\ -K_1A(D^2-k^2)^2[H_1p_1n-(D^2-k^2)][(an^2+Fn)\\ -(an+1)(1+K_1)(D^2-k^2)]W-4\Omega^2[H_1p_1n-(D^2-k^2)]\\ \times[\ell_1n+2A-(D^2-k^2)]D^2W[H_1p_1n-(D^2-k^2)]\\ \times(an+1)[\ell_1n+2A-(D^2-k^2)]D^2W.\end{aligned} \tag{3.12}$$

Using the boundary conditions (3.5), we can show that all the even-order derivatives of W must vanish on the boundaries, therefore the proper solution for W characterizing the lowest mode is

$$W = A\sin\pi z, \tag{3.13}$$

where A is a constant.

Substituting this solution in Eq. (3.12), leads to the dispersion relation

$$\begin{aligned}b[(an^2+Fn)+(an+1)(1+K_1)b]^2[H_1p_1n+b][\ell_1n+2A+b]\\ = Rk^2[(an^2+Fn)+(an+1)(1+K_1)b]^2\{(an+H_1)(\ell_1n+2A+b)\\ -\bar{\delta}Ab(an+1)\}+K_1Ab^2[H_1p_1n+b][(an^2+Fn)+(an+1)(1+K_1)b]\\ -4\Omega^2\pi^2[H_1p_1n+b][\ell_1n+2A+b],\end{aligned} \tag{3.14}$$

where $b = \pi^2 + k^2$.

4. Case of overstability

Here we consider the possibility whether instability can arise as oscillations of increasing amplitude, i.e. as overstability. We discuss the possibility of whether the instability may occur as overstability. Put $n = in_i$, it being remembered that n may be complex. Since for overstability we wish to determine the critical Rayleigh number for the onset of overstability, it suffices to find conditions for which (3.8) will admit a solution with n_i real.

Substituting $n = in_i$ in Eq. (3.14), the real and imaginary parts of (3.14), yield

$$Rk^2C_1 = C_2 \tag{4.1}$$

and

$$Rk^2C_3 = C_4, \tag{4.2}$$

where the terms represented by $(C_1 - C_4)$ are given in the Appendix.

Eliminating R between Eqs. (4.1) and (4.2), we get

$$n_i^8A_5 + n_i^6A_4 + n_i^4A_3 + n_i^2A_2 + A_1 = 0, \tag{4.3}$$

where the coefficients (A_1)–(A_5) are given in the Appendix.

It is evident from the Eq. (4.3) that oscillatory modes will not be present for all values of parameters. For example, in the absence of coupling between spin and heat flux ($\bar{\delta} = 0$), rotation ($\mathbf{\Omega} = 0$) and in the absence of suspended particles ($a = 0 = f = h_1$), Eq. (4.3) allows only $n_i = 0$ and so overstable solution will not take place if

$$K_1p_1 < 2.$$

The presence of suspended particles, coupling between spin and heat flux and rotation bring overstability in the system.

For stationary convection i.e. $n_i = 0$ and in the presence of coupling between spin and heat fluxes ($\bar{\delta} \neq 0$), Eq. (4.1) reduces to

$$R = \frac{b^4(1+K_1)^2 + A(2+K_1)b^3(1+K_1)^2 + 4\pi^2\Omega^2 b + 8\pi^2\Omega^2 A}{k^2(1+K_1)\{2H_1A + b(1-\bar{\delta}A)}. \tag{4.4}$$

In the absence of rotation ($\mathbf{\Omega} = 0$) and that of between spin and heat fluxes ($\bar{\delta} = 0$), Eq. (4.4) further reduces to

$$R = \frac{(1+K_1)^2b^4 + A(2+K_1)(1+K_1)b^3}{k^2(1+K_1)\{2b + H_1A)}, \tag{4.5}$$

the result derived by Sharma and Kumar [17].

In the absence of rotation ($\bar{\delta} = 0$), and in the absence of suspended particles ($a = 0 = f = h_1$), Eq. (4.3) further reduces to

$$R = \frac{(1+K_1)^2 b^4 + b^3 A(2+K_1)(1+K_1)}{k^2(2A+b)}, \tag{4.6}$$

the result derived by Pe'rez Garcia and Rubi [18].

For Newtonian viscous fluid and in absence of the suspended particles i.e. $\bar{\delta} = 0 = K_1 = C_0' = a = f = h_1 = 0$, Eq. (4.6) further reduces to

$$R = \frac{b^3}{k^2}, \tag{4.7}$$

which is in good agreement with earlier result by Chanrasekhar [4].

5. Discussion and conclusions

Equation (4.3) has been examined numerically using the Newton–Raphson method through the Fortran 77. Then, we have plotted the variation of Rayleigh number with wave numbers using Eq. (4.1) satisfying Eq. (4.3) for overstable case, and Eq. (4.4) for stationary case for the fixed permissible values of the dimensionless parameters $A = 0.5$, $\bar{\delta} = 1$, $p_1 = 5$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$ and $\ell_1 = 1$. Figures 1 and 2 correspond to three different values of the rotation parameter i.e. $\mathbf{\Omega} = 20$, 16 rev. per minute, respectively.

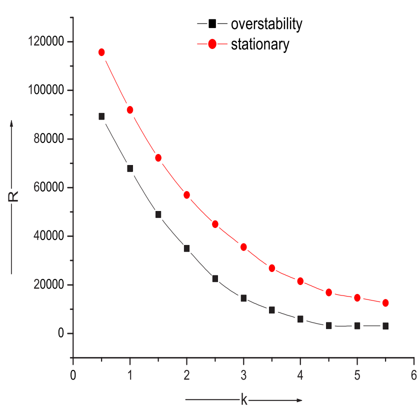

Fig. 1. The variation of Rayleigh number with wave number for $A = 0.5$, $\bar{\delta} = 1$, $p_1 = 5$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$, $l_1 = 1$ and $\Omega = 20$ rev/min.

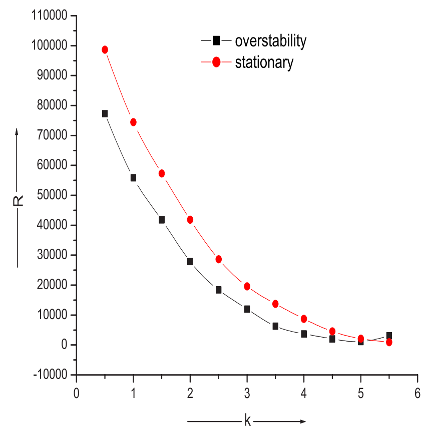

Fig. 2. The variation of Rayleigh number with wave number for $A = 0.5$, $\bar{\bar{\delta}} = 1$, $p_1 = 5$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$, $l_1 = 1$ and $\Omega = 16$ rev/min.

Figures 1, 2 show that Rayleigh number increases with increase in the rotation parameter, depicting thereby the stabilizing effect of rotation parameter.

Figures 3, 4 correspond to two values of micropolar coefficient $\kappa = 0.5$ and 1.0, respectively, accounting for dynamic microrotation viscosity. Figures 3 and 4 show that the Rayleigh number for the stationary convection and for the case of overstability, decrease with the increase in micropolar coefficient κ implying thereby the destabilizing effect of dynamic microrotation viscosity.

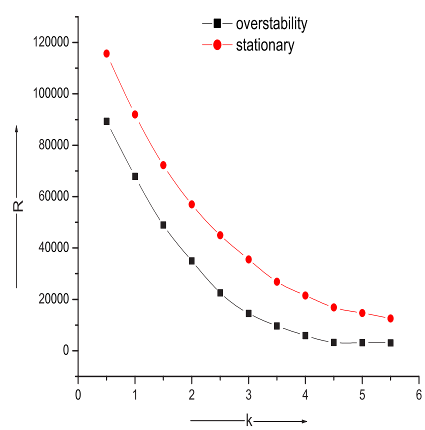

Fig. 3. The variation of Rayleigh number with wave number for $\bar{\bar{\delta}} = 1$ $p_1 = 5$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$, $l_1 = 1$, $\Omega = 20$ rev/min and $\kappa = 0.5$.

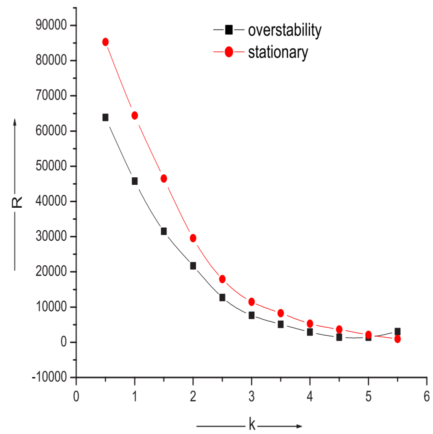

FIG. 4. The variation of Rayleigh number with wave number for $\bar{\bar{\delta}} = 1$, $p_1 = 5$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$, $l_1 = 1$, $\Omega = 20$ rev/min and $\kappa = 1.0$.

Figures 5, 6 correspond to two values of micropolar coefficient $\gamma' = 1.0$ and 1.4, respectively, accounting for the coefficient of angular viscosity, which show that the Rayleigh number for the stationary convection and for the case of overstability, decrease with the increase in micropolar coefficient γ', implying thereby the destabilizing effect of the coefficient of angular viscosity. Thus there is a competition between the large enough stabilizing effect of rotation parameter and the destabilizing effect of the micropolar coefficients. The presence of coupling between thermal and micropolar effects, rotation parameter and sus-

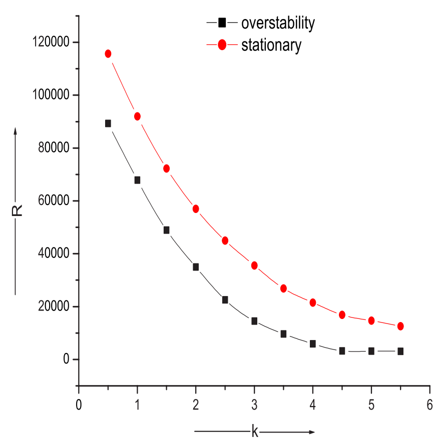

FIG. 5. The variation of Rayleigh number with wave number for $A = 0.5$, $\bar{\bar{\delta}} = 1$, $p_1 = 5$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $l_1 = 1$, $\Omega = 20$ rev/min and $\gamma' = 1.0$.

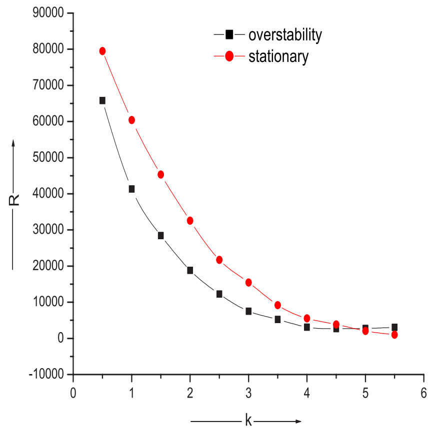

Fig. 6. The variation of Rayleigh number with wave number for $A = 0.5$, $\bar{\delta} = 1$, $p_1 = 5$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $l_1 = 1$, $\Omega = 20$ rev/min and $\gamma' = 1.4$.

pended particles, may bring overstability in the system. It is also noted from the Fig. 1, that the Rayleigh number for overstability case is always less than the Rayleigh number for the stationary convection, for a fixed wave numbers. However, the reverse may also happen for certain wave numbers, what has been depicted in Figures 2, 3, 4, 5 and 6 for $\Omega = 16$, $\kappa = 0.5$, 1.0 and $\gamma' = 1.0, 1.4$, respectively.

Appendix

$$C_1 = an_i^4\ell_1 - n_i^2\{a^2b^2(1-\bar{\delta}A) + a^2b^2(1+K_1) + H_1\ell_1 ab(1+K_1) + abF(1-\bar{\delta}A)\} + \{b^2(1+K_1) - \bar{\delta}AH_1b^2(1+K_1) + H_1b^2A(1+K_1)\},$$

$$C_2 = -H_1p_1\ell_1a^2n_i^6 + n_i^4(2a^2b^3 + H_1p_1\ell_1b\{H_1p_1F(1+K_1)2A\}) - n_i^2\big(ab^5(1+K_1) + b^4\{(1+K_1)2A + 4a(\ell_1+H_1p_1)\} - b^3\{H_1p_1\ell_1(2+K_1) + H_1p_1a2A(1+K_1) + 2aH_1p_1\ell_1\} - K_1Ab^3\{H_1p_1 + a(1+H_1p_1)\} + b^2\{H_1p_1\ell_1(1+K_1) + 2AF - 4\pi^2\Omega^2H_1p_1\ell_1\}\big) + b^5(1+K_1)^2 + Ab^4(2+K_1) - (1+K_1) + 4\pi^2\Omega^2b^2 + 8\pi^2\Omega^2Ab,$$

$$C_3 = \langle -2a^2n_i^2A - an_i^2b - H_1\ell_1an_i^2 + \bar{\delta}Abn_i^2a^2 - aFn_i^2\ell_1 + 2FH_1A + H_1bF - \bar{\delta}AbF - a^2n_i^2\ell_1b + 2H_1abA + H_1ab^2 - \bar{\delta}Aab^2 - a^2n_i^2\ell_1bK_1 + 2H_1AabK_1ab^2H_1K_1 - \bar{\delta}Ab^2K_1 + 2abA + ab^2 + H_1\ell_1b - \bar{\delta}aAb^2 + 2abAK_1 + ab^2K_1 + H_1\ell_1K_1 - \bar{\delta}Ab_1^K - H_1p_1n_i^2K_1Ab^2H_1p_1K_1Ab^3 + H_1p_1K_1^2Ab^3 + FK_1Ab^3 + aK_1Ab^4 - 8\pi^2\Omega^2AH_1p_1 - 4H_1p_1b\pi^2\Omega^2 - 4\pi^2\Omega^2b\ell_1\rangle,$$

$$C_4 = -2H_1p_1Fn_i^2A - H_1p_1Fn_i^2b^2 - Fb^2\ell_1n_i^2 + 2H_1p_1n_i^4a^2bA + H_1p_1n_i^4a^2b^2 + \ell_1n_i^4a^2b^2 + 2H_1p_1\ell_1n_i^4aFb - 4Aab^2n_i^2 - 2aFb^3n_i^2 - 2H_1p_1b^3n_i^2A - H_1p_1n_i^2ab^2 - n_i^2\ell_1n_i^2ab^4 - 2H_1p_1n_i^2AaK_1^2b^3 - H_1p_1n_i^2b^4aK_1^2 - ab^4\ell_1n_i^2K_1^2 - H_1p_1n_i^2b^34AK_1 - 2H_1p_1n_i^2b^4K_1 + 2H_1p_1n_i^2b^3A + H_1p_1b^4 + b^4\ell_1 + H_1p_1AK_1^2b^3 + H_1p_1K_1^2b^4 + 4H_1p_1K_1b^3A + 2H_1p_1b^4 + 2K_1b^4\ell_1 - H_1p_1\ell_1K_1b^3 + 4H_1p_1\ell_1K_1b^3an_i^2 + 4Ab^4 + 2b^5a + 8Aab^4K_1 + 4ab^5K_1 - 2H_1p_1\ell_1b^2n_i^2a + 4Aab^3 + 2ab^4 - H_1p_1\ell_1K_1b^22an_i^2 + 4Aab^3K_1 + 2b^4aK_1 + 4H_1p_1b^2A + 2H_1p_1b^3 + 2b^3\ell_1 + 4H_1p_1b^2AK_1 + 2H_1p_1b^3K_1 + 2b^3K_1\ell_1,$$

$$A_5 = -a^2\ell_1\{1 + H_1p_1(1 + K_1)\} + H_1p_1\bar{\delta}A + bH_1p_1\ell_1^2(1 + K_1),$$

$$A_4 = b^2\{2\ell_1a^2(\ell_1 + H_1p_1\bar{\delta}A) + F(1 - \bar{\delta}A)\} + b^3\{2H_1p_1\ell_1^2(1 + K_1) + 1(1 + K_1)\langle H_1p_1(1 - \bar{\delta}A)\ell_1F\bar{\delta}A\rangle + 2AH_1(2 - \bar{\delta}A) - K_1A\{H_1p_1(1 - \bar{\delta}A) - \ell_1F\bar{\delta}A\} + 2Aa(F - aK_1)\} + b^22Aa(1 + K_1)\{H_1p_1(F - aK_1) + a\ell_1\{\ell_1 + H_1p_1F\} + b\{\ell_1 + H_1p_1(1 - \bar{\delta}A) + aF\ell_1(H_1 - 1)\}\} + b\{H_1p_1\ell_1^2(1 - K_1)^3H - Hp_1a(1 + K_1) + 4\pi^2\Omega^2\ell_1F(H_1p_1(F\bar{\delta} - \bar{\delta} - aK_1) - \ell_1)\} + \{H_1p_1\ell_1^2F - \ell_1(\ell_1F + aH_1p_1\bar{\delta}A) + 4\Omega^2a^2H_1p_1\ell_1^2(1 - K_1)\},$$

$$A_3 = b^6\{\ell_1F(\ell_1a + H_1(1 - \bar{\delta}A)) + 2a^2(1 - \bar{\delta}A)F\ell_1\} + b^5\{2aF\ell_1^2(1 + K_1)\{H_1p_1(1 - \bar{\delta}A) - aF\ell_1\bar{\delta}A\} - 2K_1A\{H_1p_1(1 - \bar{\delta}A) - F\ell_1\}\} + b^4\{(F\ell_1 + H_1p_1\bar{\delta}A)(2F\ell_1(1 + K_1)^2 - \ell_1F(\ell_1 + H_1p_1\bar{\delta}A)) + 4AH_1p_1(1 + K_1)\{(2 - \bar{\delta}A) + (1 + K_1)^2F\ell_1(1 - \bar{\delta}A)\} - 4K_1A^2H_1p_1\{F\ell_1(H_1p_1(2 - \bar{\delta}A) - \bar{\delta}A) + (H_1 - 1) + (\ell_1^2F + (1 - \bar{\delta}A)a)\}\} + b^3\{(F\ell_1(1 + K_1)(F\ell_1 + H_1p_1\bar{\delta}A) + a\ell_1F(1 + K_1)(F\bar{\delta}A) - H_1p_1(F\bar{\delta} - \bar{\delta} - aK_1)) - \frac{K_1AH_1p_1}{2}F\ell_1 - H_1p_1(1 - \bar{\delta}A) - K_1A(1 + K_1)^2\{H_1p_1(1 - \delta A) - F\ell_1\} + 4\pi^2\Omega^2\ell_1\{-(2(H_1p_1 + F\ell_1) + H_1p_1a\ell_1) - (1 - \bar{\delta}A)F\}\} + b^2\{2H_1p_1(1 + K_1)^2(2 - \bar{\delta}A)F2A^2 (1 + K_1)^2(2 - K_1p_1) + 4A^2H_1F\ell_1(H_1 - 1) + 4\pi^2\Omega^2\{H_1p_1(1 + K_1)(1 - \bar{\delta}A) - F\ell_1\bar{\delta}A(1 + K_1)\}\} + \{4A^2H_1p_1a(1 + K_1)^2 + 4\Omega^2\pi^2a^2(F\ell_1p_1(\ell_1 + H_1p_1) + (1 + K_1)2H_1p_1A((2 - \bar{\delta}A) - \ell_1ap_1)) + 16\Omega^2\pi^2A^2H_1p_1(1 + K_1)\},$$

$$A_2 = b^7\{(1 - \bar{\delta}A)aH_1p_1\} + b^6\{F\ell(1 + K_1)\{H_1p_1(1 - \bar{\delta}A) + \ell_1F\bar{\delta}A\} + 4AH_1p_1(1 - \bar{\delta}A)\} + b^5\{2aF\ell_1^2(1 + K_1)\{H_1p_1(1 - \bar{\delta}A) - aF\ell_1\bar{\delta}A\} - 2K_1A\{H_1p_1(1 - \bar{\delta}A) - F\ell_1\}\} + b^5\{2Fa(1 + K_1)^2(1 - \bar{\delta}A) + 2AH_1p_1\ (1 + K_1)(2 - \bar{\delta}A) + F\ell_1(1 + K_1)^2(a\ell_1 + H_1p_1) + 2A^2H_1\ell_1(2 - K_1p_1) + 4A^2H_1F(1 + K_1)\bar{\delta}A\} + b^4\{2a(1 + K_1)^2\{H_1p_1(1 - \bar{\delta}A)\ell_1F\bar{\delta}A\} + 4A(1 + K_1)^2(2 - \bar{\delta}A) + 2AH_1p_1F(1 + K_1)^2(1 - \bar{\delta}A) + H_1p_1\ell_1^2F(1 + K_1)^2 + 4A^2\ell_1FH_1p_1(1 + K_1)(1 + K_1)^22\ell_1 + H_1p_1 - \bar{\delta}A - 4\pi^2\Omega^2\{2(1 - \bar{\delta}A) + aF\ell_1(\ell_1 + H_1p_1\bar{\delta}A)\}\}$$

$$
\begin{aligned}
&+ b^3\{H_1p_1\ell_1^2 a(1+K_1)^2(2-K_1p_1)H_1p_1\ell_1(1+K_1)^2 + \{a\ell_1^2 - (2-\bar{\delta}A)\}\\
&+ \{\ell_1(F\bar{\delta} - \bar{\delta} - aK_1)a + (1-\bar{\delta}A)\} + H_1p_1(H_1-1)(1-\bar{\delta}A) - H_1p_1\ell_1(1+K_1)(3-\bar{\delta}A)\\
&- H_1p_1A(1+K_1)\{3\ell_1 - 2\{1-\bar{\delta}A\}\} - K_1Aa\{-(1+K_1)(2F\ell_1(1+H_1p_1)\\
&+ H_1p_1(1-\bar{\delta}A)) + H_1p_1a\ell_1\{F\ell_1 + (1-\bar{\delta}A)(\ell_1F + (1+K_1))\}\\
&+ (F\ell_1^2 + (1-\bar{\delta}A)) + 4A^2(H_1p_1 + F\ell_1) + H_1p_1A((1-\bar{\delta}A) - H_1p_1)\\
&+ 4\Omega^2\pi^2\{2AF\ell_1(2\ell_1 + 2H_1p_1a) - 2\bar{\delta}Aa - 2a\ell_1(1+K_1)\bar{\delta}A\\
&+ H_1p_1(1+K_1)(F\ell_1 + 2(1-\bar{\delta}A))\}\}\} + b^2\{8A^2H_1p_1\{H_1p_1(2Aa + F\ell_1)\\
&- H_1p_1a(1-\bar{\delta}A)\} + (1+K_1)^2H_1A\{H_1p_1a(1-\bar{\delta}A) - F\ell_1(H_1p_1 + F\ell_1)\}4A(1+K_1)\\
&+ H_1p_1(1-K_1)^2(2+F\ell_1) + (1-K_1)^2(2F\ell_1+1) + (\bar{\delta} - aK_1)H_1F\ell_1\\
&- 4A^2(1-K_1)^2FA\ell_1 - 4(1+K_1) - K_1A\{4AH_1p_1H_1p_1(1-K_1) + H_1p_1\\
&+ (1-\bar{\delta}A) + 2Aa(1+K_1)^2(2-\bar{\delta}A)(H_1-1) + 4\pi^2\Omega^2\{2A(1+K_1)a(2-\bar{\delta}A)(2+H_1p_1)\\
&- F\ell_1 + H_1p_1\ell_1a(H_1-1)(1-\bar{\delta}A) + F\ell_1^2\}\}\} + b\{H_1p_1Aa(1-K_1) + 4\Omega^2H_1A^2F\ell_1\},
\end{aligned}
$$

and

$$
\begin{aligned}
A_1 = {} & b^7\{(1-K_1)(1-\bar{\delta}A)H_1F\} + b^6\{2A(1+K_1)^2H_1a(2-\bar{\delta}A)(H_1-1)\\
&+ (1+K_1)\{H_1p_1\ell_1(1-\bar{\delta}A)a - F\ell_1\bar{\delta}A\} + H_1F\ell_1(2Aa + \ell_1\bar{\delta}A)\\
&- K_1A(1+K_1)^2(H_1p_1(1-\bar{\delta}A) - a\ell_1F) - 4\pi^2\Omega^2(1-\bar{\delta}A)Fa\}\\
&+ b^5\{H_1p_1F(1-K_1)\{H_1p_1 + \ell_1 - F\ell_1\} + 2AH_1p_1(1-K_1)^2(2-\bar{\delta}A)\\
&+ 2A^2(1-K_1)^2(2-K_1p_1) + 2AH_1F\{(2-\bar{\delta}A) - K_1F\ell_1\{H_1p_1(1-\bar{\delta}A) - F\ell_1\}\}\\
&+ 4\pi^2\Omega^2\{(1-K_1)^2a((1-\bar{\delta}A)(H_1p_1-1) - Fa\ell_1)\}\} + b^4\{H_1p_12Aa(1+K_1)^2(H_1-1)\\
&+ 4A^2H_1p_1(1-K_1)(2-\bar{\delta}A) + F\ell_12Aa^2(2-\bar{\delta}A) + H_1F\ell_1(1-K_1)\{H_1p_1(1-\bar{\delta}A)\}\\
&+ H_1F(2A)\ell_1^2(1+K_1)(1-\bar{\delta}A) - (H_1p_1(1-\bar{\delta}A) - F\ell_1) + H_1F\ell_1p_1A^2(2-K_1p_1)\\
&+ H_1p_1 + (H_1p_1-1) + H_1F\ell_1^2a^2(1-\bar{\delta}A)(H_1-1) + 4\pi^2\Omega^2F\ell_1(1-\bar{\delta}A) - \ell_1\bar{\delta}A\\
&+ (H_1p_1(2-\bar{\delta}A + 4A^2(1+K_1)aF))\} + b^3\{H_1p_1Aa(1+K_1)\\
&+ \{4AH_1p_1(1+K_1) + H_1p_1 + (1-\bar{\delta}A)\} + (2-\bar{\delta}A)(H_1-1)\\
&+ 4\pi^2\Omega^22A(1-K_1)a\} + b^2\{H_1p_1Aa(1+K_1)^2(2+F\ell_1) + (1+K_1)^2(2F\ell_1+1)\}.
\end{aligned}
$$

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