

Application of density as a parameter in description of failure stress under uniaxial loading of softwood in LR orthotropy plane

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A DESCRIPTION OF FAILURE STRESS for tension and compression of pinewood in the LR orthotropy plane, in accordance with the Mises, Ashkenazi–Ganov and Tsai–Wu criteria, is presented in the work. The tests for pinewood has shown that strength depends on density along the L -direction. This dependence was used for the description of the pinewood failure stress. The choice of the criterion was determined by the possibility to describe correctly the failure stress, assuming that the failure stress function goes through the values of strength obtained from the tests in the L and R directions. An analysis of the above criteria leads to the choice of the Ashkenazi–Ganov criterion as the most precise criterion describing the experimental data.

1. Introduction

IN THE LR PLANE of wood the failure stresses for compression and tension are dependent on the loading direction relative to the grain. The tests have shown that failure stress for tension and compression decrease for an increasing grain angle measured from the L direction (e.g. REITERER and STANZL–TSCHEGG [1], LIU [2]). This is a result of the occurrence of different failure mechanisms for different grain angles. These mechanisms are described in the cited above [1] and TABARSA and CHUI [3], POULSEN *et al.* [4], VURAL and REVICHANDRAN [5], BYSKOV *et al.* [6]. Additionally, a significant influence of density on tensile and compression strength in the L direction was observed in the case of wood (e.g. GALICKI and CZECH [7], THIBAUT *et al.* [8], GINDL and TEISCHINGER [9]). This dependence of strength on density is a result of the varying latewood content in the tested volume of wood. Compression strength in the L direction depends on density and on the kind of failure mechanism. In the case of pine, wood has failed by kinking. Failure criteria for anisotropic bodies are applied to describe the failure stress of wood which is an orthotropic material. Among others, the MISES [10], ASHKENAZI and GANOV [11] and TSAI and WU [12] criteria were chosen to describe the pinewood failure stress. These criteria in the uniaxial

stress state take the forms, respectively:

$$(1.1) \quad (F_{1111} \cos^4 \Theta + D_1 \sin^2 2\Theta + F_{2222} \sin^4 \Theta) \sigma_\Theta^2 = 1,$$

$$(1.2) \quad (A_{1111} \cos^4 \Theta + D_2 \sin^2 2\Theta + A_{2222} \sin^4 \Theta) \sigma_\Theta = 1,$$

$$(1.3) \quad (a_{11} \cos^2 \Theta + a_{22} \sin^2 \Theta) \sigma_\Theta \\ + (a_{1111} \cos^4 \Theta + D_3 \sin^2 2\Theta + a_{2222} \sin^4 \Theta) \sigma_\Theta^2 = 1,$$

where $D_1 = 2F_{1122} + 4F_{1212}$, $D_2 = 2A_{1122} + 4A_{1212}$, $D_3 = 2a_{1122} + 4a_{1212}$, a_{ij} – components of the strength second-rank tensor, F_{ijkl} , A_{ijkl} , and a_{ijkl} – components of the strength fourth-rank tensor, σ_Θ – strength in the direction inclined to the direction L at angle Θ (Fig. 1). The strength components are expressed by strength as follows: $F_{1111} = 1/\sigma_0^2$, $F_{2222} = 1/\sigma_{90}^2$, $A_{1111} = 1/\sigma_0$, $A_{2222} = 1/\sigma_{90}$, $a_{11} = 1/\sigma_{t,0} - 1/\sigma_{c,0}$, $a_{22} = 1/\sigma_{t,90} - 1/\sigma_{c,90}$, $a_{1111} = 1/\sigma_{t,0}\sigma_{c,0}$, $a_{2222} = 1/\sigma_{t,90}\sigma_{c,90}$, where σ_t and σ_c – tension and compression strength respectively, σ_0 , σ_{90} – strength for the grain angle Θ equal to 0° and 90° respectively. The components F_{1122} , F_{1212} , A_{1122} , A_{1212} , a_{1122} and a_{1212} of the fourth rank strength tensor are determined using the least square method for other stress states.

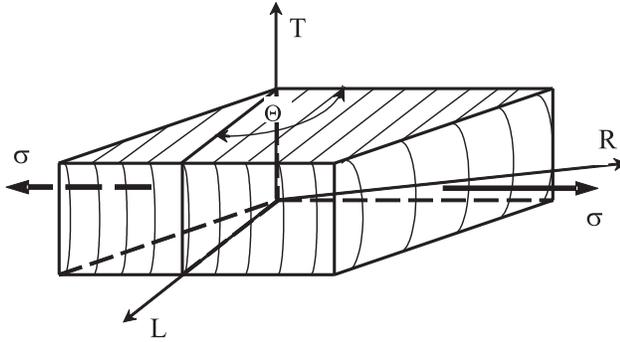


FIG. 1. Relation of the loading of wood specimens to principal axes LRT.

The results of compressive and tensile tests of pinewood (*pinus silvestris*) (GALICKI and CZECH [7, 13]) show that the failure stresses for tension and compression are functions of density ρ for different grain angles Θ . These functions and experimental values of failure stress are presented in Fig. 2. For the grain angle Θ equal to 0° a linear regression was used to determine these functions. For other angles two hypotheses were assumed – $H: r = 0$ and alternately

$K: r \neq 0$, where r is the coefficient of linear correlation. A comparison of the random values of t in the Student test for $\alpha = 0.05$ coefficient level and values of t calculated for linear correlation r for angles Θ different than 0° , allows to assume the hypothesis $H: r = 0$ to be true. Therefore, for these grain angles the compression and tensile failure stress are not correlated with density. Then the failure stresses for different angles Θ can be taken as average values. Such an approach allows to replace the experimental data with empirical functions of tensile and compressive failure stress, dependent on density for different grain angles Θ . Observation of the failure structure of wood leads to the same conclusion. Failure of wood for grain angles different from 0° takes place in early-wood where cell wall thickness is the smallest, while for an angle of 0° both early-wood and late-wood are involved in the failure process. Therefore, as soft-wood density depends mainly on the late-wood content, it is not surprising that the influence of density is observed only for $\Theta = 0^\circ$.

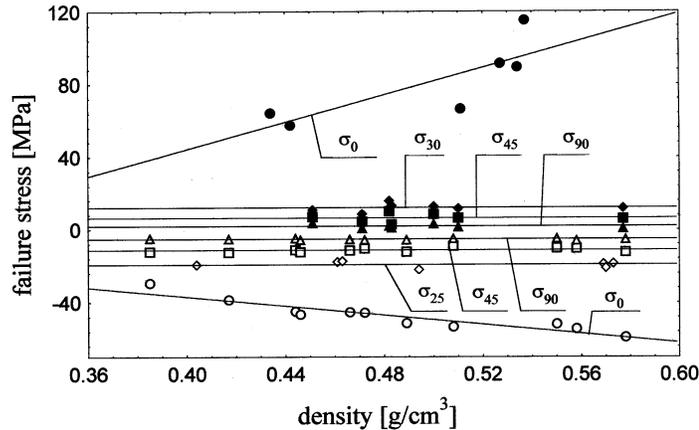


FIG. 2. Tensile and compression strength (for $\Theta = 0^\circ$ and 90°) and failure stress (for $\Theta \neq 0^\circ$ and 90°) as functions of density for different grain angles.

2. Choice of the criterion to describe pinewood strength

The description of failure stress is usually (e.g. EBERHARDSTEINER [14]) obtained by determination of the strength tensor components in Eq. (1.1), (1.2) and (1.3) using the least square method. When the Ashkenazi–Ganov criterion is used to describe compression and tensile failure stress, the sum of squared deviation takes the form:

$$(2.1) \quad \Phi = \sum_{i=1}^n [(A_{1111} \cos^4 \Theta_i + D_2 \sin^2 2\Theta_i + A_{2222} \sin^4 \Theta_i) \sigma_{\Theta_i} - 1]^2,$$

where n – number of tests. The parameters A_{1111} , D_2 and A_{2222} are calculated from a system of equations:

$$(2.2) \quad \partial\Phi/\partial A_{1111} = 0, \quad \partial\Phi/\partial D_2 = 0, \quad \partial\Phi/\partial A_{2222} = 0.$$

Additionally, the condition:

$$(2.3) \quad |d\sigma_\Theta/d\Theta| \leq 0$$

must be satisfied for a correct description of failure stress in the range from 0° to 90° of angle Θ . In general, an appropriate choice of failure criterion and a correct determination of the parameters for this criterion enable us to obtain a correct description of the failure stress. In this case, while the components F_{1111} , F_{2222} , A_{1111} , A_{2222} , a_{11} , a_{22} , a_{1111} and a_{2222} are determined on the basis of uniaxial stress states, the constant D is determined using the least square method (for example Eq. (2.2)₂ for Ashkenazi–Ganov criterion. Then the function $\sigma_\Theta = \sigma(\Theta)$ goes through the values of σ_0 and σ_{90} strengths for tension and compression. It follows from the condition (2.3) that the constants D can take different values in a defined range. For the Mises criterion: $2F_{1111} \leq D_1 \leq 2F_{2222}$ and for the Ashkenazi–Ganov criterion: $2A_{1111} \leq D_2 \leq 2A_{2222}$. Hence for values of σ_0 and σ_{90} , both the compression and tension failure curves can take different forms for different values of D . In Fig. 3 the border curves (for the lowest and the highest values of D) and experimental data for pinewood are presented. It follows from Fig. 3 that the most correct description of failure stress is obtained for the Ashkenazi–Ganov criterion, because all experimental data lie between the border failure curves both for the maximal and minimal values of density. The Mises and Tsai–Wu descriptions are possible but they are not accurate. Therefore, the Mises and Tsai–Wu criteria were not considered to describe the tensile and compressive stress failure.

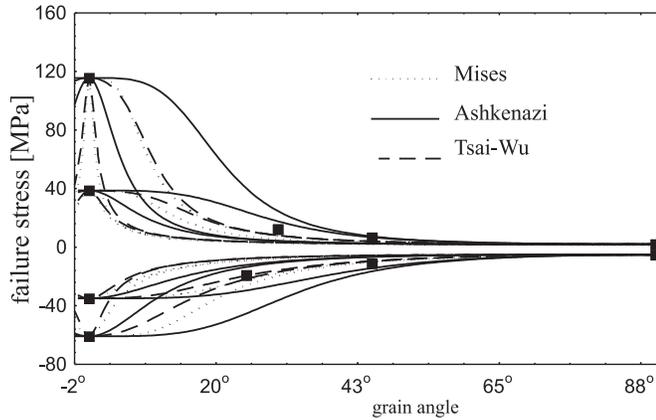


FIG. 3. Failure border curves for the Mises, Tsai–Wu and Ashkenazi criteria.

3. Methods of determination of the parameter D

3.1. D as a constant independent of density

The dependencies $\sigma_{\Theta_i} = \sigma_{\Theta_i}(\rho)$ were determined on the basis of the tension and compression tests. Then Eq. (2.1) takes the form:

$$(3.1) \quad \Phi = \sum_{j=1}^{\infty} \sum_{i=1}^m \left\{ \left[(\sigma_0^{-1}(\rho_j) \cos^4 \Theta_i + D_2 \sin^2 2\Theta_i + \sigma_{90}^{-1}(\rho_j) \sin^4 \Theta_i) \sigma_{\Theta_i}(\rho_j) - 1 \right]^2 \right\},$$

where m is the number of the grain angles Θ for which the tests were conducted and ρ_j is density from the range $(\rho_{\min}, \rho_{\max})$; ρ_{\min}, ρ_{\max} – minimal and maximal density of specimens used in the tests. According to Eq. (1.1), (1.2), (1.3) the constants D are dependent on F_{1122} and F_{1212} , A_{1122} and A_{1212} , a_{1122} and a_{1212} respectively for the assumed criterion. It follows from Eq. (3.1) that D can be determined as constant on the assumption that the above strength tensor components are constants. However, the influence of density on the failure stress is obvious. For the Ashkenazi–Ganov criterion, D_2 is equal to

$$(3.2) \quad D_2 = \lim_{n \rightarrow \infty} \left[\frac{\sum_{j=1}^n \sum_{i=1}^m \sigma_{\Theta_i}(\rho_j) \sin^2 2\Theta_i - \sum_{j=1}^n \sum_{i=1}^m \frac{\sigma_{\Theta_i}^2(\rho_j)}{\sigma_0(\rho_j)} \cos^4 \Theta_i \sin^2 2\Theta_i}{\sum_{j=1}^n \sum_{i=1}^m \sigma_{\Theta_i}^2(\rho_j) \sin^4 2\Theta_i} - \frac{\sum_{j=1}^n \sum_{i=1}^m \frac{\sigma_{\Theta_i}^2(\rho_j)}{\sigma_{90}(\rho_j)} \sin^4 \Theta_i \sin^2 2\Theta_i}{\sum_{j=1}^n \sum_{i=1}^m \sigma_{\Theta_i}^2(\rho_j) \sin^4 2\Theta_i} \right],$$

where n is a natural number. On the basis of experimental results displayed in Fig. 2 and assuming

$$(3.3) \quad \sum_{j=0}^n \frac{1}{\sigma_0(\rho_j)} = \frac{1}{\sigma_0(\rho_{\min})} + \sum_{j=1}^n \frac{1}{\sigma_0 \left(\rho_{\min} + j \frac{\rho_{\max} - \rho_{\min}}{n} \right)},$$

where j is an integer, it follows that Eq. (3.2) takes the form presented below for σ_{Θ_i} ($\Theta_i \neq 0^\circ$) being constants

$$(3.4) \quad D_2 = \lim_{n \rightarrow \infty} \left[\frac{(n+1) \sum_{i=1}^m \sigma_{\Theta_i}(\rho_j) \sin^2 2\Theta_i}{(n+1) \sum_{i=1}^m \sigma_{\Theta_i}^2(\rho_j) \sin^4 2\Theta_i} - \frac{\sum_{j=0}^n \frac{1}{\sigma_0(\rho_j)} \sum_{i=1}^m \sigma_{\Theta_i}^2(\rho_j) \cos^4 \Theta_i \sin^2 2\Theta_i}{(n+1) \sum_{i=1}^m \sigma_{\Theta_i}^2(\rho_j) \sin^4 2\Theta_i} - \frac{\frac{(n+1)}{\sigma_{90}} \sum_{i=1}^m \sigma_{\Theta_i}^2(\rho_j) \sin^4 \Theta_i \sin^2 2\Theta_i}{(n+1) \sum_{i=1}^m \sigma_{\Theta_i}^2(\rho_j) \sin^4 2\Theta_i} \right]$$

In this way two series of $D_2 = D_2(n)$, one for tension and the other for compression, were obtained – Fig. 4. From Fig. 4 it follows that these series are convergent. Furthermore, the values of constants D_2 vary insignificantly in the range of n from 1 to 100. Therefore $n = 1$ can be taken for the calculation of D_2 from Eq. (3.4).

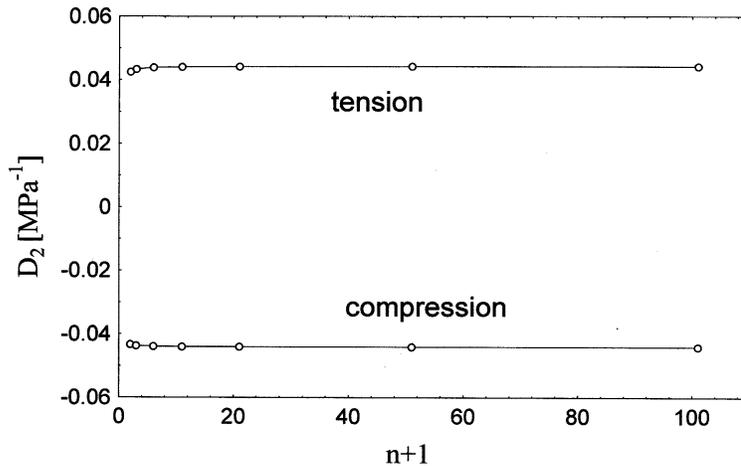


FIG. 4. Values of D_2 for increasing number n .

3.2. D as a function of density

In case when the experimental dependences between the failure stress σ_{Θ_i} and density ρ are functions $\sigma_{\Theta_i} = \sigma_{\Theta_i}(\rho)$, then it follows from Eq. (2.1) that

$$(3.5) \quad D_2(\rho) = \frac{\sum_{i=1}^m \sigma_{\theta_i}(\rho) \sin^2 2\theta_i - \sum_{i=1}^m \frac{\sigma_{\theta_i}^2(\rho)}{\sigma_0(\rho)} \cos^4 \theta_i \sin^2 2\theta_i}{\sum_{i=1}^m \sigma_{\theta_i}^2(\rho) \sin^4 2\theta_i} - \frac{\sum_{i=1}^m \frac{\sigma_{\theta_i}^2(\rho)}{\sigma_{90}(\rho)} \sin^4 \theta_i \sin^2 2\theta_i}{\sum_{i=1}^m \sigma_{\theta_i}^2(\rho) \sin^4 2\theta_i}.$$

It follows from Eq. (3.5) that the parameters (components of strength tensor) determined from the tests for an assumed criterion are dependent on density. According to Fig. 2

$$(3.6) \quad \sigma_0 = a + b\rho,$$

where a, b are coefficients. On the assumption that σ_{θ_i} and σ_{90} are constants in the entire range of ρ , $D_2 = D_2(\rho)$ takes the form

$$(3.7) \quad D_2 = \frac{\sum_{i=1}^m \sigma_{\theta_i} \sin^2 2\theta_i - \frac{1}{a + b\rho} \sum_{i=1}^m \sigma_{\theta_i}^2 \cos^4 \theta_i \sin^2 2\theta_i}{\sum_{i=1}^m \sigma_{\theta_i}^2 \sin^4 2\theta_i} - \frac{\frac{1}{\sigma_{90}} \sum_{i=1}^m \sigma_{\theta_i}^2 \sin^4 \theta_i \sin^2 2\theta_i}{\sum_{i=1}^m \sigma_{\theta_i}^2 \sin^4 2\theta_i}.$$

Dependences of tensile and compressive failure stress on grain angle and density for pinewood according to Eq. (3.7) are presented in Figs. 5 and 6. Another description of strength was obtained by using Eq. (3.4). The difference between this description and the description determined from Eq. (3.7) expressed in |MD| (mean deviation) is presented in Figs. 7 and 8. In this way the parameters of the applied criterions can be determined by means of three methods. Two methods presented above (Eq. (3.4) and Eq. (3.7)) allow to determine these parameters using the experimental curves of $\sigma_{\theta_i} = \sigma_{\theta_i}(\rho)$. Such an approach in the case of Eq. (3.7) allows to obtain the parameter D_2 as a function of density. These methods allow to determine parameters D_2 dependent on density, taking into consideration the interval $(\rho_{\max}, \rho_{\min})$. Using the third method (Eqs. (2.2)), the determination of the criterion parameters D_2 , A_{1111} and A_{2222} is based directly

on the experimental data. Although these parameters depend on density, when they are determined for a limited number of specimens then description of the failure stress can be incorrect, particularly when A_{1111} is a constant.

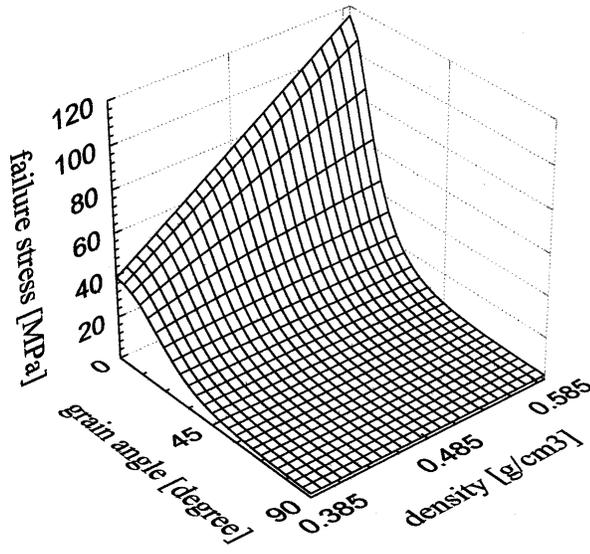


FIG. 5. Tensile failure stress as a function of density and the grain angle.

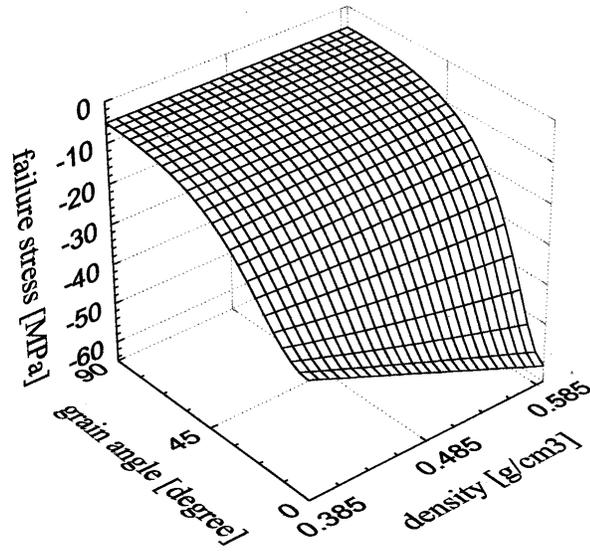


FIG. 6. Compression failure stress as a function of density and the grain angle.

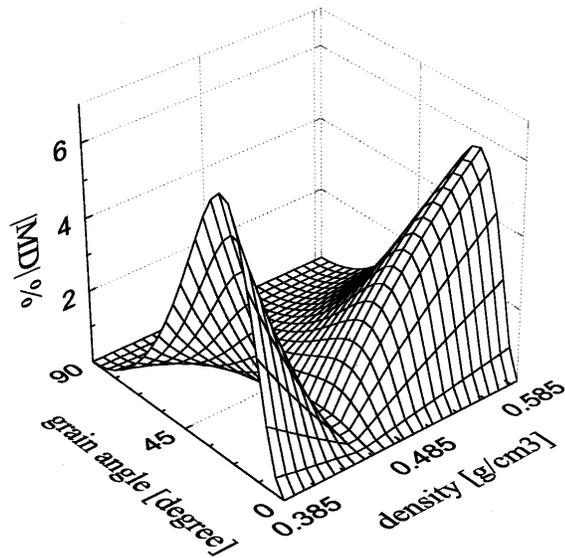


FIG. 7. Variation of $|MD|$ of tensile failure stress for $D_2 = 0.0433$ in relation to tensile failure stress for $D_2 = D_2(\rho)$.

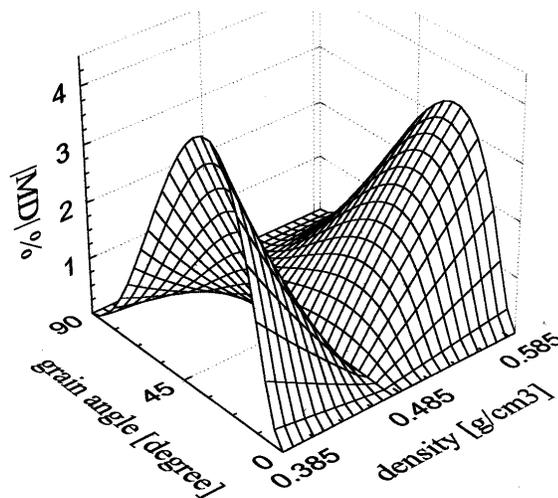


FIG. 8. Variation of $|MD|$ of compression failure stress for $D_2 = -0.0433$ in relation to compression failure stress for $D_2 = D_2(\rho)$.

4. Discussion

Equation (2.1) is often used for determination of the strength tensor components for different failure criteria. This approach can be applied when failure

stress depends only on the loading direction. In the case of pine, the tests showed that strength along the grain was linearly dependent on density (Eq. (3.6)). Therefore application of the parameters a, b, D_2 and A_{2222} is necessary to describe correctly the failure stress. Then the method expressed by Eq. (2.2) can be used to determine these parameters. However, in result of the use of the least square method for large anisotropy of the failure stress in the range $\langle 0^\circ, 90^\circ \rangle$ of Θ angle in LR orthotropy plane the large values of MD can be obtained for angle Θ equal to 90° . Thus, application of the least square method for MD is a necessary condition for correct description of the failure stress for larger values of angles Θ . This is why the least square method was used to calculate D_2 as a constant and as a function of density, on the assumption that failure surface goes through empirical straight lines for $\Theta = 0^\circ$ and 90° . Then D_2 can be determined from Eq. (3.4) or Eq. (3.7). Furthermore, such an approach seems to be correct because in the other case from the least square method applied to LT and RT planes, other values of the failure stress can be obtained than those calculated for LR plane on L and R directions. However, both the methods of determination of D_2 described above are based on the empirical functions $\sigma_{\Theta_i} = \sigma_{\Theta_i}(\rho)$. Application of these functions is important because testing such materials as wood it is difficult to obtain a proportional distribution of the experimental points in the space of failure stress, grain angle and density. In case of concentration of these points in any part of this space, the failure stress surface moves in the direction of these points. The determination of D_2 as a constant by using the functions σ_{Θ_i} excludes the influence of concentration of experimental data on the description of failure stress in the considered space. Furthermore, although these functions are determined from the conducted tests, the graphs of these functions can be insignificantly changed after applying a statistical verification. For example, for $\Theta \neq 0^\circ$ (Fig. 2) an accurate description of failure stress has the form $\sigma_{\Theta_i} = a_i + b_i\rho$, but it can be proved for density as variable in the interval $\langle \rho_{\min}, \rho_{\max} \rangle$ that failure stress is independent of density after applying the Student t tests. Therefore, these functions as constants were used to describe the failure stress. In this case a more accurate description is obtained using Eq. (3.7). Three parameters Ω^2 , MD and $\Delta\sigma$ defined as

$$(4.1) \quad \Omega^2 = \int_{\rho_{\min}}^{\rho_{\max}} \sum_i [\sigma_t(\Theta_i, \rho) - \sigma_{\text{ex}}(\Theta_i, \rho)]^2 d\rho,$$

$$(4.2) \quad \text{MD} = \frac{[\sigma_t(\Theta_i, \rho) - \sigma_{\text{ex}}(\Theta_i, \rho)]}{\sigma_t(\Theta_i, \rho)},$$

$$(4.3) \quad \Delta\sigma = \sigma_t(\Theta_i, \rho) - \sigma_{\text{ex}}(\Theta_i, \rho)$$

where $\sigma_t(\Theta_i, \rho)$ and $\sigma_{\text{ex}}(\Theta_i, \rho)$ – theoretical and experimental strength curves, were used to estimate the correctness of the two methods. Maximal and minimal values of $\Delta\sigma$ in the entire range of ρ and the values of Ω^2 for grain angles Θ_i for tensile and compressive failure stress are presented in Table 1. Functions of MD for Θ_i angles in the entire range of ρ for tension and compression are presented in Figs. 9 and 10. It follows from the analysis of $\Delta\sigma$ and Ω^2 from Table 1 that the Ashkenazi–Ganov criterion describes experimental data more accurately when D_2 is a function of density, contrary to the cases when D_2 are constant values. Then, on the basis of the experimental curves for grain angles 25° and 45° for compression and 30° and 45° for tension, it can be concluded that the maximal deviations of tensile failure stress in MPa is $\sigma_t + 1.59 \geq \sigma \geq \sigma_t - 0.91$ and compressive failure stress in MPa is $\sigma_t + 1.31 \geq \sigma \geq \sigma_t - 2.82$.

Table 1. Parameters of correctness of failure stress values.

tension						compression					
D_2 MPa ⁻¹	Ω_{30}^2	Ω_{45}^2	Ω^{2*}	$\Delta\sigma_{\text{max}}$ MPa	$\Delta\sigma_{\text{min}}$ MPa	D_2 MPa ⁻¹	Ω_{25}^2	Ω_{45}^2	Ω^{2**}	$\Delta\sigma_{\text{max}}$ MPa	$\Delta\sigma_{\text{min}}$ MPa
0.0425	0.576	0.139	0.715	2.20	-0.91	-0.0433	1.516	0.214	1.720	1.20	-4.14
0.0442	0.503	0.148	0.651	2.08	-0.94	-0.0441	1.276	0.237	1.513	1.25	-3.87
$D_2 = D_2(\rho)$	0.443	0.147	0.590	1.59	-0.91	$D_2 = D_2(\rho)$	1.036	0.245	1.281	1.31	-2.82

$$*\Omega^2 = \Omega_{30}^2 + \Omega_{45}^2, \quad **\Omega^2 = \Omega_{25}^2 + \Omega_{45}^2$$

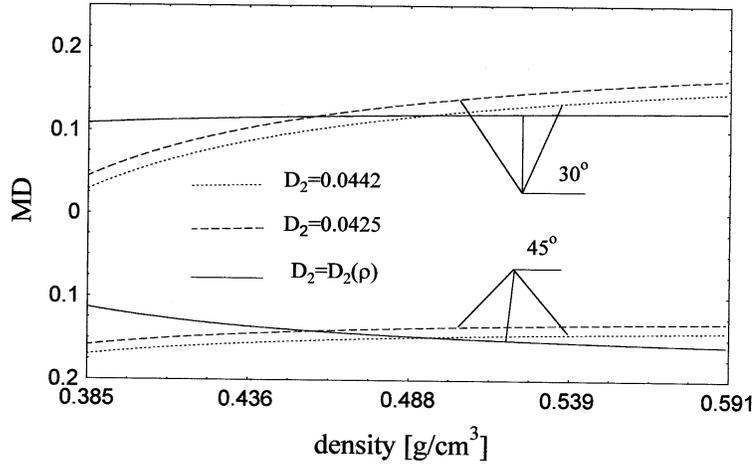


FIG. 9. Variation of MD tensile failure stress for grain angles equal to 30° and 45° .

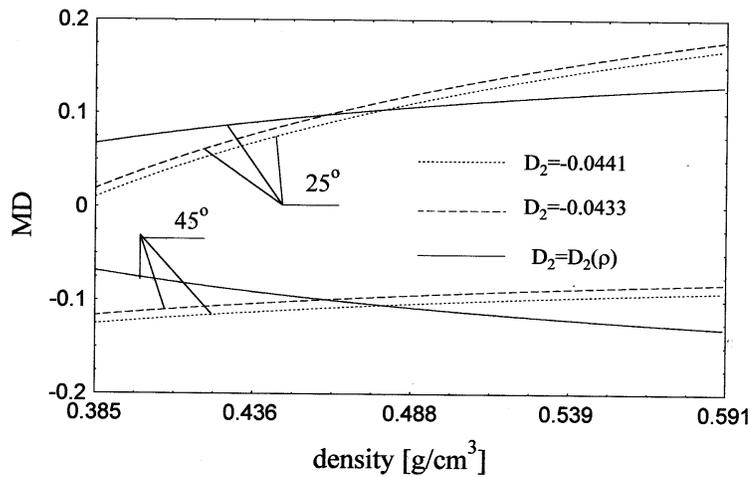


FIG. 10. Variation of MD compression failure stress for grain angles equal to 25° and 45° .

Acknowledgments

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