

## Flow of a micropolar fluid on a continuous moving surface

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THE PRESENT PAPER deals with the analysis of steady boundary layer flow and heat transfer of a micropolar fluid on an isothermal continuously moving plane surface. It is assumed that the microinertia density is variable and not constant, as in many other published papers. Also, the viscous dissipation effect is taken into account. The basic partial differential equations are reduced to a system of nonlinear ordinary differential equations, which is solved numerically using the Keller-box method. Numerical results are obtained for the skin friction coefficient, local Nusselt number, as well as velocity, temperature and microrotation profiles. Results are shown in graphical form and the numerical values for the skin friction coefficient and local Nusselt number are given in the form of tables. The effects of material parameter  $K$ , Prandtl number  $Pr$  and Eckert number  $Ec$  on the flow and heat transfer characteristics are discussed.

### Notations

$A$	dimensionless constant of integration,
$c_p$	specific heat at constant pressure,
$C_f$	skin friction coefficient,
$f$	dimensionless stream function,
$g$	dimensionless microinertia,
$h$	dimensionless angular velocity,
$j$	microinertia density,
$k$	thermal conductivity,
$K$	material parameter,
$n$	constant,
$N$	angular velocity or component of the microrotation vector normal to the $x - y$ plane,
$Nu_x$	local Nusselt number,
$Pr$	Prandtl number,
$q_w$	heat transfer from the plate,
$Re_w$	local Reynolds number,

$T$	fluid temperature,
$T_w$	plate temperature,
$T_\infty$	ambient temperature,
$u, v$	velocity components along the $x$ and $y$ directions, respectively,
$U_w$	plate velocity,
$x, y$	Cartesian coordinates along the surface and normal to it, respectively.

*Greek Letters*

$\alpha$	thermal diffusivity,
$\beta$	thermal expansion coefficient,
$\gamma$	spin gradient viscosity,
$\kappa$	vortex viscosity,
$\eta$	pseudo-similarity variable,
$\theta$	dimensionless temperature,
$\nu$	kinematic viscosity,
$\mu$	dynamic viscosity,
$\rho$	fluid density,
$\tau_w$	skin friction,
$\psi$	stream function.

*Subscripts*

$w$	conditions at the wall,
$\infty$	ambient conditions.

*Subscripts*

'	differentiation with respect to $\eta$ .
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## 1. Introduction

THE PRODUCTION OF SHEETING material, which includes both metal and polymer sheets, arises in a number of industrial manufacturing processes. The fluid dynamics due to a continuous moving solid surface is important in many extrusion processes, such as the aerodynamic extrusion of plastic sheets, cooling of a metallic plate in a cooling bath, the boundary layer along material handling conveyers, boundary layer along a liquid film in condensation processes, etc. Boundary layer flow and heat transfer over a fixed or moving flat plate in a viscous and incompressible fluid is well known. In view of these applications, SAKIADIS [1] initiated the theoretical study of boundary layer on a continuous semi-infinite surface moving steadily through an otherwise quiescent fluid environment. An experimental and theoretical treatment was made for the flow past a continuous flat surface by TSOU *et al.* [2] who determined the heat transfer rates for certain values of the Prandtl number. Much theoretical work has been done on this problem since the pioneering papers by SAKIADIS [1] and

TSOU *et al.* [2], and extensive references can be found in the papers by MAGYARI and KELLER [3, 4], LIAO and POP [5], and NAZAR *et al.* [6].

All the above investigations were restricted to Newtonian fluids. Due to the increasing importance in the processing industries (and elsewhere) of materials whose flow behavior in shear cannot be characterized by Newtonian relationships, a new stage in the evolution of fluid dynamics theory is in progress (see HASSANIEN *et al.* [7]). HOYT and FABULA [8], and VOGEL and PATTERSON [9] conducted experiments with fluids containing small amounts of polymeric additives. These experiments indicated that fluids with additives display a reduction in skin friction near the surface of a rigid body. The Newtonian fluid mechanics cannot explain this phenomenon. Its extensions to non-Newtonian fluids are important for the thermal design of industrial equipments dealing with molten plastics, polymeric liquids, foodstuffs or slurries (see CHAR and CHANG [10]). Hence, considerable efforts have been directed toward this coupled, non-linear boundary layer problem. The theory of micropolar fluids proposed by ERINGEN [11, 12] is capable of explaining the behavior of exotic lubricants, polymeric fluids, liquid crystals, animal bloods, colloidal and suspension solutions, etc., for which the classical Navier–Stokes theory is inadequate. A local microrotation vector together with the velocity vector describe the flow motion of such fluids. A comprehensive review of the subject and applications of micropolar fluid mechanics was given by ARIMAN *et al.* [13], ŁUKASZEWICZ [14] and ERINGEN [15].

SOUNDALGEKAR and TAKHAR [16] solved the problem of steady boundary layer flow and heat transfer of a micropolar fluid due to a continuously moving surface by considering that the microinertia density is constant. Thus, the ordinary differential equations are locally non-similar because the parameter  $G$  in their paper is not a constant but a function of the coordinate  $x$ . Therefore, we will reconsider this problem here by assuming that the microinertia density is variable so that the basic partial differential equations can be reduced to ordinary differential equations (similarity equations). In this respect, we shall follow HOSSAIN and CHOWDHURY [17], KIM [18], and KIM and KIM [19] by assuming that close to the surface, the effect of microstructure can be neglected since the suspended particles cannot get closer to the boundary than their radius (see AHMADI [20]). The only rotation is due to fluid shear and therefore we shall assume that the gyration vector may be equal to angular velocity, which is representative of weak concentrations ( $n = 1/2$ ) (see GURAM and SMITH [21]). It is worth mentioning that some aspects of the problem of fluid flow and heat transfer characteristics of a micropolar fluid flowing over a plane moving surface has also been considered by HASSANIEN *et al.* [7], and DESSEAUX and BELLALIJ [22].

## 2. Problem formulation and basic equations

Let us consider the flow situation in Fig. 1, where a flat plate emerges from the slot of an extrusion die at a constant velocity  $U_w$  and continuously moves in an incompressible micropolar fluid medium at rest. We assume that the constant temperature of the plate is  $T_w$  and that of the ambient fluid is  $T_\infty$ , where we assume that  $T_w > T_\infty$  (the plate is heated). The origin of the Cartesian coordinate system is placed at the location where the plate is drawn into the fluid medium, with the  $x$ -axis measured along the plate in the right direction and the  $y$ -axis measured normal to the plate. Neglecting external body forces and considering the viscous dissipation effect, the field equations of a steady, two-dimensional, laminar, incompressible micropolar fluid can be expressed within the boundary layer approximations as follows (see SOUNDALGEKAR and TAKHAR [16] or HOSSAIN and CHOWDHURY [17]):

$$(2.1) \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$(2.2) \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \frac{\mu + \kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial N}{\partial y},$$

$$(2.3) \quad \frac{\partial}{\partial y} \left( \gamma \frac{\partial N}{\partial y} \right) - \kappa \left( 2N + \frac{\partial u}{\partial y} \right) = 0,$$

$$(2.4) \quad u \frac{\partial j}{\partial x} + v \frac{\partial j}{\partial y} = 0,$$

$$(2.5) \quad u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{(\mu + \kappa)}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2,$$

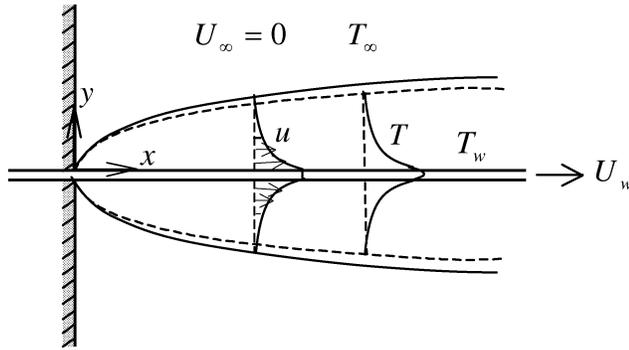


FIG. 1. Physical model and coordinate system.

subject to the boundary conditions

$$(2.6) \quad \begin{aligned} v = j = 0, \quad u = U_w, \quad T = T_w, \quad N = -n \frac{\partial u}{\partial y} & \quad \text{at } y = 0, \\ u \rightarrow 0, \quad N \rightarrow 0, \quad T \rightarrow T_\infty & \quad \text{as } y \rightarrow \infty. \end{aligned}$$

Here  $u$  and  $v$  are the velocity components along the  $x$ - and  $y$ -axes, respectively,  $N$  is the angular velocity or microrotation whose direction of rotation is in the  $x-y$  plane,  $T, T_w, T_\infty, \rho, \mu, \kappa, \gamma, \nu, j, \alpha$  and  $c_p$  are the fluid temperature, plate temperature, ambient fluid temperature, fluid density, dynamic viscosity, vortex viscosity, spin-gradient viscosity, kinematic viscosity, microinertia density, thermal diffusivity and specific heat at constant pressure, respectively, and  $n$  is a constant such that  $0 \leq n \leq 1$ . The last term in Eq. (2.5) represents the viscous dissipative heat, which must be taken into account when the Prandtl number of the fluid is large or  $(T_w - T_\infty)$  is small. It should be mentioned that the case  $n = 0$  is called strong concentration by GURAM and SMITH [21], indicates that  $N = 0$  near the wall and represents concentrated particle flows in which the microelements close to the wall surface are unable to translate or rotate (JENA and MATHUR [23]). The case  $n = 1/2$  indicates vanishing of anti-symmetrical part of the stress tensor and denotes weak concentration (AHMADI [20]). The case  $n = 1$ , as suggested by PEDDIESON [24], is used for modelling of the turbulent boundary layer flows. The condition  $n \neq 0$  means that in the neighborhood of a rigid boundary, the effect of microstructure is negligible since the suspended particles cannot get closer to the boundary than their radius. Hence, in the neighborhood of the boundary, the only rotation is due to fluid shear and therefore, the gyration vector must be equal to fluid vorticity (see BHARGAVA *et al.* [25]). However, we shall consider here only the case of weak concentration of particles at the plate ( $n = 1/2$ ).

It is known (see GORLA [26]) that  $N$  is the total spin of microstructure and fluid media in the flow field. In some cases, the microstructure effects become negligible and the flow behaves like an ordinary (Newtonian) viscous flow. Therefore, if we state that  $N =$  angular velocity is a valid solution, then this is possible only if

$$(2.7) \quad \gamma = (\mu + \kappa/2)j = \mu (1 + K/2)j,$$

where  $K = \kappa/\mu$  is the material parameter. The above equation gives a relationship between the coefficients of viscosity and microinertia. The derivation of Eq. (2.7) for micropolar fluids has been established by AHMADI [20] and it has been used by many recent authors (see REES and BASSOM [27] or REES and POP [28]). We notice that for  $K = 0$  (or  $\kappa = 0$ ) Eqs. (2.1), (2.2) and (2.5) are decoupled from Eqs. (2.3) and (2.4). Therefore, the present problem and its

results reduce to those of the classical Newtonian fluid model. We introduce now the following variables:

$$(2.8) \quad \begin{aligned} \psi &= (2\nu U_w x)^{1/2} f(\eta), & N &= (U_w^3/2\nu x)^{1/2} h(\eta), \\ \theta(\eta) &= (T - T_\infty)/(T_w - T_\infty), & j &= \frac{2\nu x}{U_w} g(\eta), \\ \gamma &= (\mu + \kappa/2) \frac{2\nu x}{U_w} g(\eta), & \eta &= \left( \frac{U_w}{2\nu x} \right)^{1/2} y, \end{aligned}$$

where  $\psi$  is the stream function defined in the usual way as  $u = \partial\psi/\partial y$  and  $v = -\partial\psi/\partial x$ .

After some algebra, we obtain the following ordinary differential equations:

$$(2.9) \quad (1 + K)f''' + ff'' + Kh' = 0,$$

$$(2.10) \quad \left(1 + \frac{K}{2}\right) (gh')' - K(2h + f'') = 0,$$

$$(2.11) \quad 2gf' - fg' = 0,$$

$$(2.12) \quad \frac{1}{\text{Pr}}\theta'' + f\theta' + \text{Ec}(1 + K)f''^2 = 0,$$

and the boundary conditions (2.6) become

$$(2.13) \quad \begin{aligned} f(0) = 0, \quad g(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad h(0) = -nf''(0), \\ f'(\infty) = 0, \quad \theta(\infty) = 0, \quad h(\infty) = 0, \end{aligned}$$

where Pr is the Prandtl number and Ec is the Eckert number, which is defined as

$$(2.14) \quad \text{Ec} = \frac{U_w^2}{c_p(T_w - T_\infty)}.$$

We notice that since we have assumed  $T_w > T_\infty$  (heated plate), it results in  $\text{Ec} > 0$ . However, for a cooled plate ( $T_w < T_\infty$ ), the parameter  $\text{Ec} < 0$ , but we will not consider this case here. Further, we assume that gyration is taken to be equal to the angular velocity at the plate, which is representative of weak concentration, i.e.  $n = 1/2$ . The solution of Eq. (2.11) satisfying the boundary conditions (2.13) is given by

$$(2.15) \quad g = A f^2,$$

where  $A$  is a dimensionless constant of integration. For convenience, we only consider the case of  $A$  equals to unity throughout this paper, though the form of the results will apply generally for other  $A$  of  $O(1)$ .

The physical quantities of interest are the skin friction coefficient and the local Nusselt number, which are defined as

$$(2.16) \quad C_f = \frac{\tau_w}{\rho U_w^2}, \quad \text{Nu}_x = \frac{x q_w}{k(T_w - T_\infty)},$$

where  $k$  is the thermal conductivity of the micropolar fluid, and the skin friction  $\tau_w$  and heat transfer from the plate  $q_w$  are given by

$$(2.17) \quad \tau_w = \left[ (\mu + \kappa) \frac{\partial u}{\partial y} + \kappa N \right]_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}.$$

Using the similarity variables (2.8), we get

$$(2.18) \quad C_f \text{Re}_w^{1/2} = \frac{1}{\sqrt{2}} (1 + K/2) f''(0),$$

$$\text{Nu}_x / \text{Re}_w^{1/2} = -\frac{1}{\sqrt{2}} \theta'(0),$$

where  $\text{Re}_w = U_w x / \nu$  is the local Reynolds number. By using (2.15) for the case  $A = 1$ , Eq. (2.10) becomes

$$(2.19) \quad \left( 1 + \frac{K}{2} \right) (f^2 h')' - K (2h + f'') = 0.$$

### 3. Results and discussion

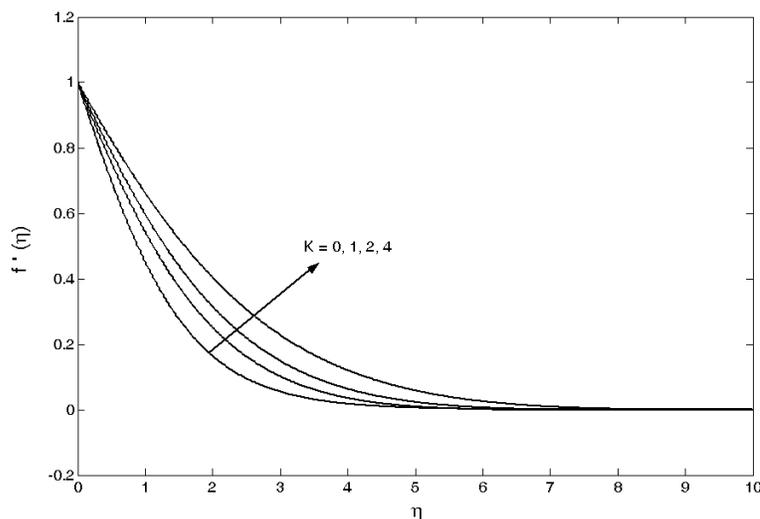
The nonlinear ordinary differential equations (2.9), (2.12) and (2.19), satisfying the boundary conditions (2.13) were solved numerically using the Keller box-method as described in the book by CEBECI and BRADSHAW [29] for several values of the parameters  $K$ ,  $\text{Ec}$  and  $\text{Pr}$ , while the constant  $A = 1$  and  $n = 1/2$  (weak concentration of fluid particles at the plate).

Figure 2 shows the resulting dimensionless velocity profiles  $f'(\eta)$  for various values of the material parameter  $K$ . It is observed that the velocity boundary layer thickness increases with increasing values of  $K$  associated with a decrease in the wall velocity gradient, and hence produces a decrease in the magnitude of the reduced skin friction  $f''(0)$ , as can be seen from Table 1. The magnitude of velocity profiles  $f'(\eta)$  is larger for micropolar fluids in comparison with Newtonian fluids ( $K = 0$ ). This may be due to the effects of microrotation that increase the velocity. Further, for a particular value of  $K$ , the velocity decreases monotonically with  $\eta$  and becomes zero at the boundary layer edge. Table 1 shows the values of the reduced skin friction  $f''(0)$  for  $K = 0$  (Newtonian fluid), 0.5, 1, 2 and 4. It is observed that the absolute values of  $f''(0)$  decrease as  $K$  increases.

This indicates that micropolar fluids display a drag reduction compared to Newtonian fluids. It is worth mentioning that  $Pr$  and  $Ec$  have no influence on the skin friction as well as velocity and angular velocity. This is because the thermal field gives no influence on the flow field since Eqs. (2.9)–(2.11) do not contain the parameters  $Pr$  and  $Ec$  as well as temperature  $\theta$ . In other words, Eqs. (2.9)–(2.11) are not coupled with Eq. (2.12).

**Table 1.** Values of  $f''(0)$  for various  $K$ .

$K$	$f''(0)$
0	−0.6276
0.5	−0.5704
1	−0.5217
2	−0.4523
4	−0.3694



**FIG. 2.** Velocity profiles for various  $K$ .

The angular velocity or microrotation profiles  $h(\eta)$  for various values of material parameter  $K$  are shown in Fig. 3. It is observed that the microrotation continuously decreases with  $\eta$  and becomes zero far away from the plate. As expected, the microrotation effects are more dominant near the wall. Also, the microrotation decreases as  $K$  increases in the vicinity of the plate but the reverse happens as one moves away from it. It is evident from Figs. 2 and 3 that both the velocity and angular velocity profiles satisfy the boundary conditions (2.13). Thus, these figures support the validity of the present results.

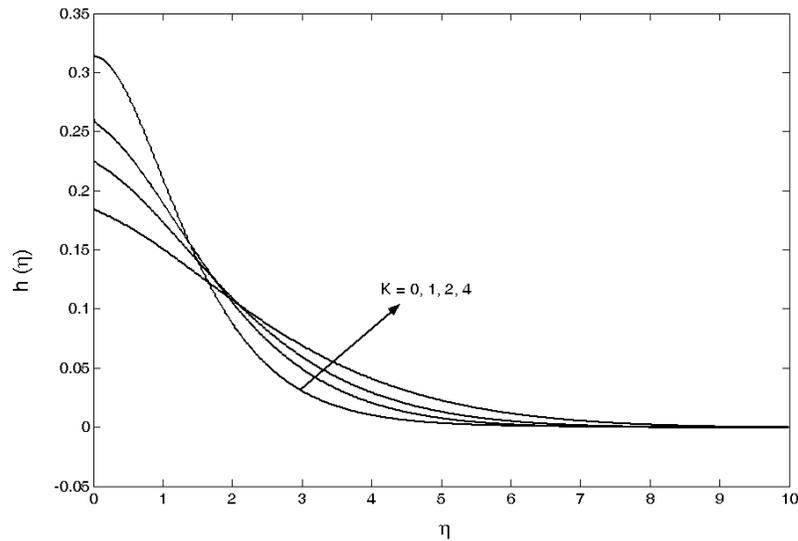


FIG. 3. Angular velocity profiles for various  $K$ .

Figures 4–6 show the resulting dimensionless temperature profiles  $\theta(\eta)$  as a function of  $\eta$  for various parameters. The temperature decreases monotonically and tends to zero at the edge of the boundary layer. It is evident from these figures that the boundary condition  $\theta(\infty) = 0$  is satisfied. It is observed from Figs. 4 and 6 that the temperature decreases with an increase in  $K$  and  $Pr$  respectively, which results in decreasing manner of the thermal boundary layer

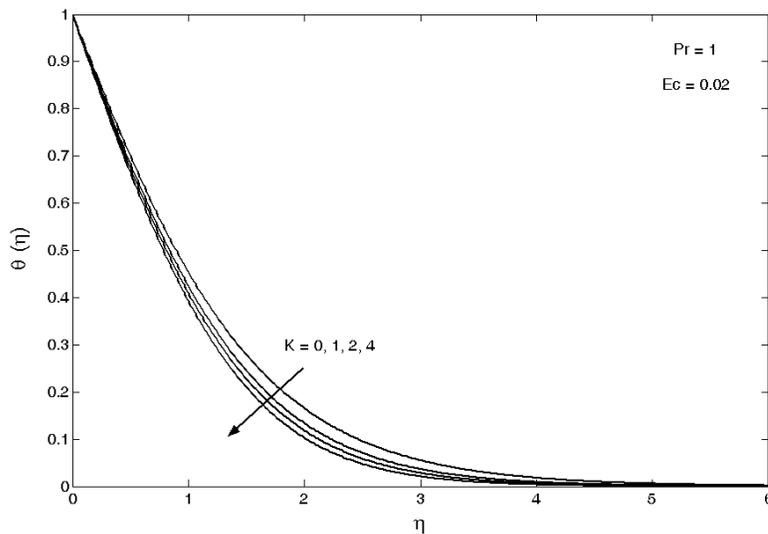


FIG. 4. Temperature profiles for various  $K$  with  $Pr = 1$  and  $Ec = 0.02$ .

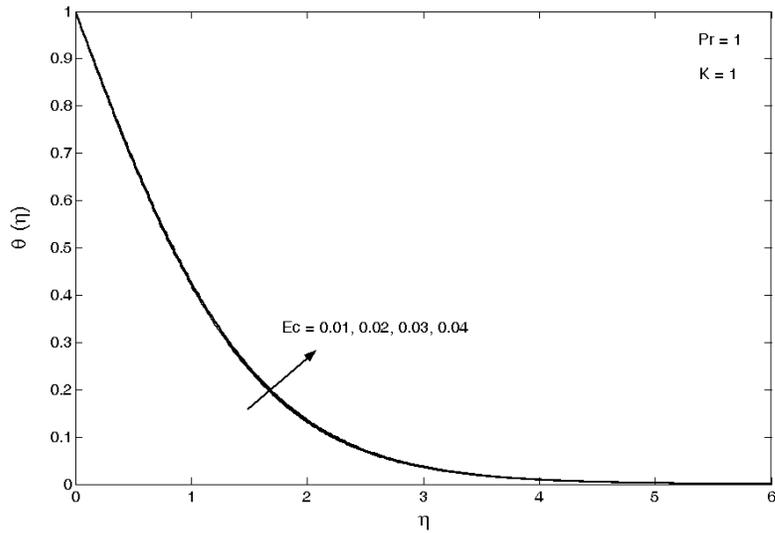


FIG. 5. Temperature profiles for various  $Ec$  with  $Pr = K = 1$ .

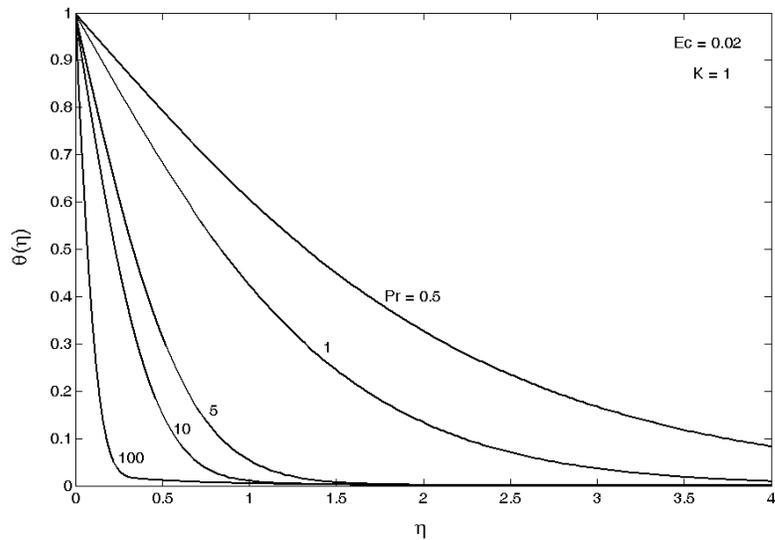


FIG. 6. Temperature profiles for various  $Pr$  with  $Ec = 0.02$  and  $K = 1$ .

thickness, associated with an increase in the wall temperature gradient  $-\theta'(0)$ , and hence produces an increase of the surface heat transfer rate. These values of  $-\theta'(0)$  are shown in Tables 2 and 3 for various  $K$  and  $Pr$  respectively. The reverse happens for increasing frictional heating parameter  $Ec$  (see Fig. 5, Table 2 and Table 3).

**Table 2.** Values of  $-\theta'(0)$  for various  $K$  and  $Ec$  when  $Pr = 1$ .

$K \setminus Ec$	0.01	0.02	0.03	0.04
0	0.6244	0.6213	0.6181	0.6150
0.5	0.6467	0.6427	0.6387	0.6347
1	0.6608	0.6562	0.6515	0.6468
2	0.6800	0.6742	0.6684	0.6627
4	0.7015	0.6943	0.6871	0.6798

**Table 3.** Values of  $-\theta'(0)$  for various  $Pr$  and  $Ec$  when  $K = 1$ .

$Pr \setminus Ec$	0.01	0.02	0.03	0.04
0.5	0.4228	0.4202	0.4175	0.4149
1	0.6608	0.6562	0.6515	0.6468
5	1.6474	1.6307	1.6141	1.5974
10	2.3783	2.3505	2.3227	2.2948
100	7.7349	7.5986	7.4624	7.3262

#### 4. Conclusions

In this paper, we have theoretically studied the problem of steady boundary layer flow and heat transfer of a micropolar fluid on an isothermal continuously moving plane surface assuming that the microinertia density is variable and not constant, as considered in the previous papers. Under this approximation, the reduced boundary layer equations are similar and they were solved numerically using the Keller-box method. The development of velocity profiles, microrotation profiles and temperature profiles have been illustrated in graphs, while the values of skin friction coefficient and local Nusselt number are given in tables. A discussion concerning the effects of the Eckert number  $Ec$ , material parameter  $K$  and Prandtl number  $Pr$  on the reduced skin friction coefficient, local Nusselt number, as well as the velocity, angular velocity and temperature profiles for the case  $n = 1/2$  (weak concentration of fluid particles at the plate), has been done. From this investigation, it may be concluded that:

- The magnitude of the reduced skin friction decreases but the local heat transfer rate increases with an increase in the value of material parameter  $K$ .
- The velocity increases but the temperature decreases with  $K$ .
- Near the wall, angular velocity decreases as  $K$  increases but the opposite trend is observed far away from the wall.

- Pr and Ec have no influence on the skin friction as well as velocity and angular velocity or microrotation profiles.
- Pr has a similar effect as  $K$  on the local Nusselt number and temperature profiles.

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