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Brief Note

Stability of stratified elastico-viscous Walters' (Model B′) fluid in the presence of horizontal magnetic field and rotation in porous medium

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Combined effect of magnetic field and rotation is considered on the stability of stratified visco-elastic Walters' (Model B′) fluid in porous medium. In contrast to the Newtonian fluids, the system is found to be unstable at stable stratification for low values of permeability or high values of kinematic viscoelasticity. Magnetic field is found to stabilize the small wavelength perturbations for unstable stratification. Variation of growth rate in the case of unstable stratification is depicted graphically with the variation in viscosity, viscoelasticity and permeability. It has been found that the growth rate increases with the increase in kinematic viscosity and permeability, whereas it decreases with the increase in kinematic viscoelasticity.

Notations

$\mathbf{\Omega}$ $(\Omega, 0{,}0)$	rotation vector having components $(\Omega,0,0)$,
ρ	density of fluid,
μ	coefficient of viscosity,
μ'	coefficient of viscoelasticity,
μ_e	magnetic permeability,
ε	medium porosity,
∂	curly operator,
∇	del operator,
β	a constant,
π	constant value,
δ	perturbation in the respective physical quantity,
ϕ	the angle between the horizontal component of wave number k_x and wave number k,
ν	kinematic viscoelasticity (μ/ρ),

ν'	kinematic viscoelasticity (μ'/ρ),
ι	square root of (-1),
k_1	medium permeability,
p	pressure,
$\mathbf{g}\ (0,0,-g)$	acceleration due to gravity,
$\mathbf{H}\ (H,0,0)$	magnetic field vector having components $(H,0,0)$,
$\delta\rho$	perturbation in density $\rho(z)$,
δp	perturbation in pressure $p(z)$,
$\mathbf{q}\,(u,v,w)$	perturbations in fluid velocity $\mathbf{q}\,(0,0,0)$,
$\mathbf{h}\,(h_x,\ h_y,\ h_z)$	perturbations in magnetic field $\mathbf{H}\,(H,0,0)$,
$k_x,\ k_y$	wave numbers in x and y directions respectively,
$k=(k_x^2+k_y^2)^{1/2}$	wave number of the disturbance,
n	growth rate of the disturbance,
V_A^2	square of the Alfve'n velocity $(V_A^2=\mu_e H^2/4\pi\rho)$,
d	depth of the fluid layer,
$\rho_0,\ \mu_0,\ \mu_0',\ \varepsilon_0, k_{10},\ \nu_0,\ \nu_0'$	constants,
m	an integer,
D	derivative with respect to $z\,(=d/dz)$,
(u,v,w)	components of velocity after perturbation.

1. Introduction

A comprehensive account of the Rayleigh–Taylor instability has been given by Chandrasekhar [1], wherein the effects of uniform rotation and uniform magnetic field, separately, have also been studied. Reid [2] studied the effects of surface tension and viscosity on the stability of two superposed fluids. Oldroyd [3] proposed a theoretical model for a class of viscoelastic fluids. The problem of convective instability in a rotating viscoelastic fluid has been considered by Bhatia and Steiner [4]. Sharma and Kumari [5] studied thermosolutal instability in a Maxwellian viscoelastic fluid in a porous medium. There are many elastico-viscous fluids which cannot be characterized by Maxwell's or Oldroyd's constitutive relations. One such class of fluids is Walters'(Model B$'$) fluid. Walters [6] has proposed a theoretical model for such elastico-viscous fluids. Sharma and Kumar [7] have studied the stability of two superposed Walters' (Model B$'$) viscoelastic liquids.

The flow through porous medium has been of considerable interest in recent years, particularly in geophysical fluid dynamics. When we consider flow in a porous medium, some additional complexities arise which are principally due to the interactions between the fluid and the porous material. Here, we will consider those fluids for which Darcy's law is applicable, which states that the gross effect, as the fluid slowly percolates through the pores of rock, is that the usual viscous term in the equation of elastico-viscous fluid motion will be replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\mathbf{q}\right]$, where μ and μ' are the coefficients of viscosity and viscoelasticity of the Walters' (Model B$'$) fluid, k_1 is the medium permeability and $\mathbf{q}$ is the Darcian (filter) velocity of the fluid. The stability of flow of a single component fluid through porous medium, taking into account the Darcy's resistance, has been studied by LAPWOOD [8] and WOODING [9]. The physical properties of comets and meteorites strongly suggest the importance of porosity in astrophysical context (McDONNEL [10]). SHARMA [11] has studied the effect of uniform magnetic field and uniform rotation on the stability of two superposed fluids in a porous medium. SHARMA and KUMAR [12] have studied the steady flow and heat transfer of Walters' (Model B$'$) fluid through a porous pipe of uniform circular cross-section with small suction.

Another application of the result of flow through porous medium in the presence of magnetic field is in the geothermal region. The rotation of the Earth distorts the boundaries of a hexagonal convection cells in fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Keeping in mind the growing importance of non-Newtonian fluids in modern technology, industry, chemical technology and dynamics of geophysical fluids and considering the conflicting tendencies of magnetic field and rotation while acting together, we are motivated to study the stability of stratified elastico-viscous Walters' (Model B$'$) fluid in a porous medium in the presence of magnetic field and rotation.

2. Formulation of the problem and perturbation equations

The initial stationary state whose stability we wish to examine is that of an incompressible, infinitely conducting Walters' (Model B$'$) fluid of variable density, kinematic viscosity and kinematic viscoelasticity, arranged in horizontal strata in a porous medium of variable porosity and medium permeability. We are considering little unusual configuration in which not only the magnetic field is parallel to the layer but rotation is also oriented in the same direction, i.e. the elastico-viscous fluid is acted on by gravity force $\mathbf{g}(0, 0, -g)$, a uniform horizontal rotation $\mathbf{\Omega}$ $(\Omega, 0, 0)$ and a uniform horizontal magnetic field $\mathbf{H}, (H, 0, 0)$. The same type of configuration has been considered by SHARMA [13] while studying the stability of a stratified fluid in porous medium in the presence of horizontal magnetic field and rotation. CHANDRASEKHAR [1] also discussed in brief the problem of thermal instability for the case when rotation parameter $\mathbf{\Omega}$ and acceleration due to gravity $\mathbf{g}$ act in different directions.

Consider an infinite horizontal layer of thickness d bounded by the planes $z = 0$ and $z = d$. The character of the equilibrium of this stationary state is determined by supposing that the system is slightly disturbed and then, following its further evolution.

Let ρ, μ, μ', p and $\mathbf{q}\,(u,v,w)$ denote, respectively, the density, the viscosity, the viscoelasticity, the pressure and the filter velocity of fluid (initially zero). $\in$ is the medium porosity and μ_e is the magnetic permeability. Then the hydromagnetic equations relevant to the problem are

$$\frac{\rho}{\in}\left[\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\in}(\mathbf{q}.\nabla)\,\mathbf{q}\right] = -\nabla\, p + \mathbf{g}\rho - \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\mathbf{q} + \frac{2\rho}{\in}(\mathbf{q}\times\mathbf{\Omega}) + \frac{\mu_e}{4\pi}\left[(\nabla\times\mathbf{H})\times\mathbf{H}\right], \tag{2.1}$$

$$\nabla.\,\mathbf{q} = 0, \tag{2.2}$$

$$\in\frac{\partial\rho}{\partial t} + (\mathbf{q}.\nabla)\,\rho = 0, \tag{2.3}$$

$$\nabla.\,\mathbf{H} = 0, \tag{2.4}$$

$$\in\frac{\partial\mathbf{H}}{\partial t} = (\mathbf{H}\,.\,\nabla)\,\mathbf{q}\,. \tag{2.5}$$

Equation (2.3) represents the fact that the density of a particle remains unchanged as we follow it with its motion.

Let $\delta\rho$, δp, $\mathbf{q}\,(u,v,w)$ and $\mathbf{h}\,(h_x,\,h_y,\,h_z)$ denote, respectively, the perturbations in density ρ(z), pressure $p(z)$, velocity (0, 0, 0) and horizontal magnetic field $\mathbf{H}$ (H, 0, 0). Then the linearized perturbation equations become

$$\frac{\rho}{\in}\frac{\partial \mathbf{q}}{\partial t} = -\nabla\delta p + \mathbf{g}\delta\rho - \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\mathbf{q} + \frac{2\rho}{\in}(\mathbf{q}\times\mathbf{\Omega}) + \frac{\mu_e}{4\pi}\left[(\nabla\times\mathbf{h})\times\mathbf{H}\right], \tag{2.6}$$

$$\nabla.\,\mathbf{q} = 0, \tag{2.7}$$

$$\in\frac{\partial\delta\rho}{\partial t} + (\mathbf{q}.\nabla)\,\rho = 0, \tag{2.8}$$

$$\nabla.\mathbf{h} = 0, \tag{2.9}$$

$$\in\frac{\partial\mathbf{h}}{\partial t} = (H\,.\,\nabla)\;\mathbf{q}. \tag{2.10}$$

Analyzing the disturbances in normal modes, we seek solutions whose dependence on x, y, z and time t is given by

$$f(z)\exp(ik_x x + ik_y y + nt), \tag{2.11}$$

where $f(z)$ is some function of z and k_x, k_y are the wave numbers in the x and y directions, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and n is the growth rate of the disturbance which is, in general, a complex constant.

Equations (2.6)–(2.10), using expression (2.11) in the Cartesian coordinates become

$$\frac{\rho}{\in} n u = -i k_x \delta p - \frac{1}{k_1} \left(\mu - \mu' n\right) u, \tag{2.12}$$

$$\frac{\rho}{\in} n v = -i k_y \delta p - \frac{1}{k_1} \left(\mu - \mu' n\right) v + \frac{\mu_e H}{4\pi} \left(i k_x h_y - i k_y h_x\right) + \frac{2}{\in} \rho \Omega w, \tag{2.13}$$

$$\frac{\rho}{\in} n w = -D \delta p - \frac{1}{k_1} \left(\mu - \mu' n\right) w + \frac{\mu_e H}{4\pi} \left(i k_x h_z - D h_x\right) - \frac{2}{\in} \rho \Omega v - g \delta \rho, \tag{2.14}$$

$$i k_x u + i k_y v + D w = 0, \tag{2.15}$$

$$\in n \delta \rho + w \left(D \rho\right) = 0, \tag{2.16}$$

$$i k_x h_x + i k_y h_y + D h_z = 0, \tag{2.17}$$

$$\in n h_x = i k_x H u, \tag{2.18}$$

$$\in n h_y = i k_x H v, \tag{2.19}$$

$$\in n h_z = i k_x H w, \tag{2.20}$$

where D stands for d/dz.

Eliminating u, v and $\delta\, p$ from Eqs. (2.12)–(2.14) and using Eqs. (2.15)–(2.20), after little algebra, we get

$$\begin{aligned} \rho \left[n^2 + \frac{\in n}{k_1} \left(\nu - \nu' n\right) + k_x^2 V_A^2 \right] D^2 w + n^2 \left(D\rho\right) \left(Dw\right) \\ - \left[k^2 \left(n^2 + \frac{\in n}{k_1} \left(\nu - \nu' n\right) + k_x^2 V_A^2 \right) \rho \right. \\ \left. + \frac{4 \rho\, n^2 \Omega^2 k_x^2}{n^2 + \frac{\in n}{k_1} \left(\nu - \nu' n\right) + k_x^2 V_A^2} - g\, k^2 \left(D\rho\right) \right] w \\ + 2\, i\, n\, k_y \Omega \left(D\rho\right) w = 0, \end{aligned} \tag{2.21}$$

where $\nu = \mu/\rho$, $\nu' = \mu'/\rho$ and $V_A^2 = \mu_e H^2 / 4\pi\rho$ (square of the Alfve'n velocity).

Equation (2.21) is the general equation to consider the stability of stratified Walters' (Model B$'$) fluid in a porous medium in the presence of horizontal magnetic field and uniform rotation. In the absence of viscoelasticity i.e. ($\nu' = 0$), Eq. (2.21) reduces to the result by Sharma [11].

3. The case of exponentially varying stratifications

Let us assume the stratifications in density, viscosity, viscoelasticity, medium porosity and medium permeability of the forms

$$\rho = \rho_0 e^{\beta z}, \quad \mu = \mu_0 e^{\beta z}, \quad \mu' = \mu'_0 e^{\beta z}, \quad \in = \in_0 e^{\beta z}, \quad k_1 = k_{10} e^{\beta z}, \tag{3.1}$$

where ρ_0, μ_0, μ'_0, $\in_0$, k_{10} and β are constants. Equation (3.1) implies that the kinematic viscosity $\nu_0 \left(= \frac{\mu}{\rho} = \frac{\mu_0}{\rho_0} \right)$ and the kinematic viscoelasticity $\nu'_0 \left(= \frac{\mu'}{\rho} = \frac{\mu'_0}{\rho_0} \right)$ are constant everywhere. Using stratifications of the form (3.1), Eq. (2.21) transforms to

$$\begin{aligned} &\left[n^2 + \frac{\in_0 n}{k_{10}} \left(\nu_0 - \nu'_0 n \right) + k_x^2 V_A^2 \right] D^2 w + n^2 \beta \left(D w \right) \\ &\qquad - \left[\left(n^2 + \frac{\in_0 n}{k_{10}} \left(\nu_0 - \nu'_0 n \right) + k_x^2 V_A^2 \right) k^2 \right. \\ &\qquad \left. + \frac{4 \Omega^2 n^2 k_x^2}{n^2 + \frac{\in_0 n}{k_{10}} \left(\nu_0 - \nu'_0 n \right) + k_x^2 V_A^2} - g \beta k^2 \right] w = 0. \end{aligned} \tag{3.2}$$

The general solution of Eq. (3.2) is given by

$$w = A_1 e^{q_1 z} + A_2 e^{q_2 z}, \tag{3.3}$$

where A_1, A_2 are two arbitrary constants and q_1, q_2 are the roots of the equation

$$\begin{aligned} &\left[n^2 + \frac{\in_0 n}{k_{10}} \left(\nu_0 - \nu'_0 n \right) + k_x^2 V_A^2 \right] q^2 + n^2 \beta q \\ &\qquad - \left[\left(n^2 + \frac{\in_0 n}{k_{10}} \left(\nu_0 - \nu'_0 n \right) + k_x^2 V_A^2 \right) k^2 \right. \\ &\qquad \left. + \frac{4 \Omega^2 n^2 k_x^2}{n^2 + \frac{\in_0 n}{k_{10}} \left(\nu_0 - \nu'_0 n \right) + k_x^2 V_A^2} - g \beta k^2 \right] = 0. \end{aligned} \tag{3.4}$$

Here we consider the fluid to be confined between two rigid planes at $z = 0$ and $z = d$.

The boundary conditions for the case of two rigid surfaces are

$$w = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = d. \tag{3.5}$$

The vanishing of w at $z = 0$ is satisfied by the choice

$$w = A\,(e^{q_1 z} - e^{q_2 z})\,, \tag{3.6}$$

while the vanishing of w at $z = d$ requires $e^{(q_1 - q_2)\,d} = 1$, i.e.

$$(q_1 - q_2)\;d = 2\,i\,m\,\pi, \tag{3.7}$$

where m is an integer. Solving Eq. (3.4) we get,

$$q_{1,2} = \frac{1}{2}\left[\frac{-n^2\beta}{L_2} \pm \left\{\frac{n^4\,\beta^2}{L_2^2} + 4\left(k^2 + \frac{4n^2\,\Omega^2\,k_x^2}{L_2^2} - \frac{g\,\beta\,k^2}{L_2}\right)\right\}^{1/2}\right], \tag{3.8}$$

where $L_2 = \left(n^2 + \dfrac{\in_o\,n}{k_{1o}}\,(\nu_o - \nu_o' n) + k_x^2 V_A^2\right)$.

Inserting the values of q_1, q_2 from Eq. (3.8) in Eq. (3.7), we obtain

$$\begin{aligned} n^4 &\left[\beta^2 + 4\left(1 - \frac{2\in_0\nu'_0}{k_{10}}\right)\left(k^2 + \frac{m^2\pi^2}{d^2}\right) + 4\frac{\in_0^2\,\nu_0'^2}{k_{10}^2}\left(k^2 + \frac{m^2\pi^2}{d^2}\right)\right] \\ &+ 8n^3\left[\frac{\in_0\nu_0}{k_{10}}\left(1 - \frac{\in_0\nu'_0}{k_{10}}\right)\left(k^2 + \frac{m^2\pi^2}{d^2}\right)\right] + n^2\left[4\frac{\in_0^2\,\nu_0^2}{k_{10}^2}\left(k^2 + \frac{m^2\pi^2}{d^2}\right)\right. \\ &\left. + 2k_x^2V_A^2\left(k^2 + \frac{m^2\pi^2}{d^2}\right)\left(1 - \frac{4\in_0\nu_0'}{k_{10}}\right) + 4\Omega^2V_A^2 - g\beta\,k^2\left(1 - \frac{4\in_0\nu'_0}{k_{10}}\right)\right] \\ &+ n\left[4\frac{\in_0\nu_0}{k_{10}}\left\{2k_x^2V_A^2\left(k^2 + \frac{m^2\pi^2}{d^2}\right) - g\,\beta\,k^2\right]\right. \\ &+ 4\,k_x^2V_A^2\left\{k_x^2V_A^2\left(k^2 + \frac{m^2\pi^2}{d^2}\right) - g\,\beta\,k^2\right\} = 0. \end{aligned} \tag{3.9}$$

Equation (3.9) is biquadratic in n, therefore, it must give four roots and it is the dispersion relation for studying the effects of rotation and horizontal magnetic field on the stability of stratified elastico-viscous (exponentially varying density) fluid in a porous medium.

4. Results and discussion

(a) Case of stable stratification (i.e. $\beta < 0$). If $\beta < 0$ and $k_{10} > 4 \in_0 \nu_0'$, Eq. (3.9) does not admit any positive real root or complex root with positive real part; therefore, the system is stable for disturbances of all wave numbers. However, the system is unstable for $k_{10} < 4 \in_0 \nu_0'$. Thus for stable stratification, the system is stable for $k_{10} > 4 \in_0 \nu_0'$ and unstable for $k_{10} < 4 \in_0 \nu_0'$. This is in contrast to the Newtonian fluids where the system is always stable for stable stratification (CHANDRASEKHAR [1]).

(b) Case of unstable stratification (i.e. $\beta > 0$). If $\beta > 0$ and $k_{10} > 4 \in_0 \nu_0'$, the system is stable or unstable according to whether

$$k_x^2 V_A^2 \left(k^2 + \frac{m^2\pi^2}{d^2}\right) > g\,\beta\,k^2 \qquad \text{or} \qquad k_x^2 V_A^2 \left(k^2 + \frac{m^2\pi^2}{d^2}\right) < g\beta k^2.$$

The system is clearly unstable in the absence of magnetic field. However, the system can be stabilized if $V_A^2 > \dfrac{g\,\beta\,k^2}{\left(k^2 + \dfrac{m^2\,\pi^2}{d^2}\right) k_x^2}$ and $k_{10} > 4 \in_0 \nu_0'$.

Thus for unstable stratification, if $V_A^2 < \dfrac{g\,\beta\,k^2}{\left(k^2 + \dfrac{m^2\,\pi^2}{d^2}\right) k_x^2}$, Eq. (3.9) has at least one positive root and thus the system is unstable for all wave-numbers satisfying the inequality

$$k^2 < \frac{g\beta \sec^2\phi}{V_A^2} - \frac{m^2\pi^2}{d^2}, \tag{4.1}$$

where ϕ is the angle between k_x and k (i.e. $k_x = k\cos\phi$).

The behavior of growth rates with respect to kinematic viscosity ν_0, kinematic viscoelasticity ν'_0 and permeability k_{10} satisfying Eq. (3.9) has been examined numerically using the Newton–Raphson method through the software Mathcad. Figure 1 shows the variation of growth rate n_r (positive real value of n) with respect to the wave number k satisfying Eq. (3.9) for fixed permissible values of

$$\beta = 2, \quad m = 1, \quad d = 6 \text{ cm}, \quad \Omega = 6 \text{ rotations/minute}, \quad k_{10} = 6, \quad \nu_0 = 4,$$
$$g = 980 \text{ cm/sec}^2, \quad V_A^2 = 15, \quad k_x = k\cos 45^\circ \quad \text{and} \quad \in_0 = 0.5,$$

for four values of $\nu_0' = 1, 2, 3$ and 4 respectively. The various parameter values satisfy the inequality (4.1), which provides the wave-number range for which the system is unstable. These values are the permissible values for the respective parameters and are in good agreement with the corresponding values used by CHANDRASEKHAR [1] while describing various hydrodynamic and hydromagnetic stability problems. The graph shows that for fixed wave-numbers, the growth rate decreases with the increase in kinematic viscoelasticity ν_0', which indicates the stabilizing effect of viscoelasticity for the given range of wave-numbers. Figure 2 shows the variation of growth rate n_r (positive real value of n) with respect to wave-number k for fixed permissible values of

$$\beta = 2, \quad m = 1, \quad d = 6 \text{ cm}, \quad \Omega = 6 \text{ rotations/minute}, \quad k_{10} = 6, \quad \nu_0' = 1,$$
$$g = 980 \text{ cm/sec}^2, \quad V_A^2 = 15, \quad k_x = k\cos 45^\circ \quad \text{and} \quad \in_0 = 0.5,$$

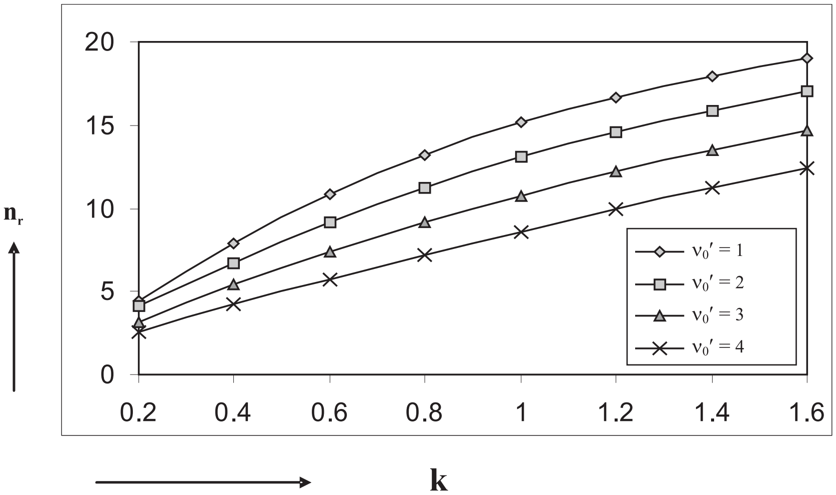

FIG. 1. Vibration of n_r (positive real part of n) with wave-number k for fixed permissible values of $\beta = 2$, $m = 1$, $d = 6$ cm, $\varepsilon_0 = 0.5$, $k_{10} = 6$, $\nu_0 = 4$, $\Omega = 6$ revolutions per minute, $g = 980$ cm/sec^2, $k_x = k \cos 45^\circ$ and $V_A^2 = 15$, for four values of $\nu_0' = 1, 2, 3$ and 4.

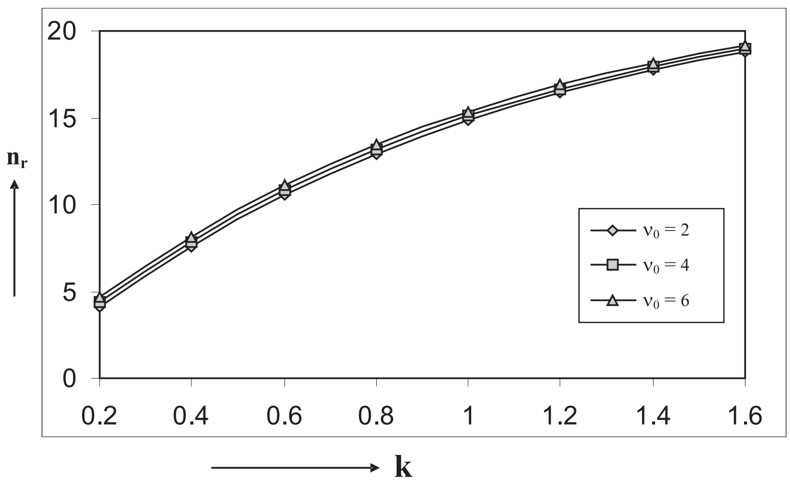

FIG. 2. Variation of n_r (positive real part of n) with wave-number k for fixed permissible values of $\beta = 2$, $m = 1$, $d = 6$ cm, $\varepsilon_0 = 0.5$, $k_{10} = 6$, $\nu_0' = 1$, $\Omega = 6$ revolutions per minute, $g = 980$ cm/sec^2, $k_x = k \cos 45^\circ$ and $V_A^2 = 15$, for three values of $\nu_0' = 2, 4$ and 6.

for three values of $\nu_0 = 2, 4$ and 6 respectively. The graph shows that for fixed wave-numbers, the growth rate increases with the increase in kinematic viscosity ν_0 which indicates the destabilizing influence of kinematic viscosity for the given range of wave-numbers. Figure 3 shows the variation of growth rate n_r (positive real value of n) with respect to wave-number k for fixed permissible values of

$$\beta = 2, \quad m = 1, \quad d = 6 \text{ cm}, \quad \Omega = 6 \text{ rotations/minute}, \quad \nu_0 = 4, \quad \nu_0' = 2,$$
$$g = 980 \text{ cm/sec}^2, \quad V_A^2 = 15, \quad k_x = k \cos 45° \quad \text{and} \quad \in_0 = 0.5,$$

for four values of k_{10} = 4,6,8 and 12 respectively. The graph shows that for fixed wave-numbers, the growth rate increases with the increase in medium permeability k_{10} which indicates the destabilizing influence of medium permeability for the given range of wave-numbers.

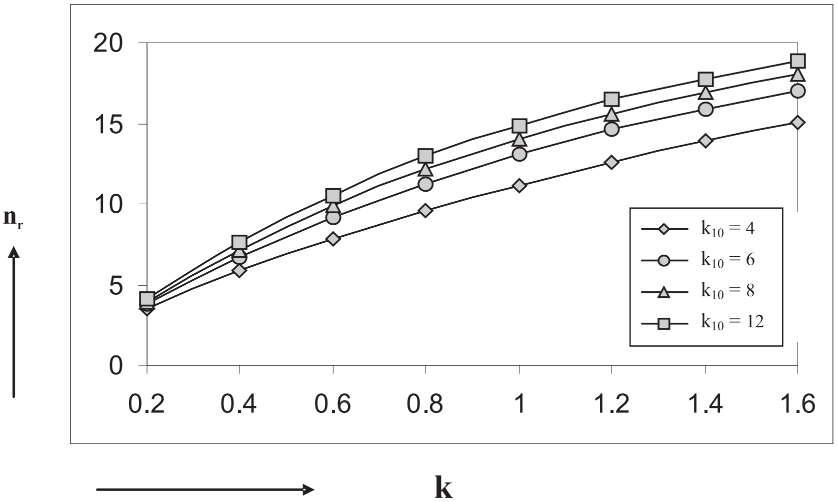

FIG. 3. Variation of n_r (positive real part of n) with wave-number k for fixed permissible values of $\beta = 2$, $m = 1$, $d = 6$ cm, $\varepsilon_0 = 0.5$, $\nu_0' = 2$, $\nu_0 = 4$, $\Omega = 6$ revolutions per minute, $g = 980$ cm/sec^2, $k_x = k$ cos 45° and $V_A^2 = 15$, for four values of $k_{10} = 4, 6, 8$ and 12.

References

1. S. CHANDRASEKHAR, *Hydrodynamic and hydromagnetic stability*, Dover Publications, New York 1981.

2. W. H. REID, *The effects of surface tension and viscosity on the stability of two superposed fluids*, Proc. Camb. Phil. Soc., **57**, p. 415, 1961.

3. J. G. OLDROYD, *Non-Newtonian effects in steady motion of some idealized elastico-viscous liquid*. Proc. Roy. Soc., **A245**, p. 278, London 1958.

4. P. K. BHATIA AND J. M. STEINER, *Convective instability in a rotating viscoelastic fluid layer*, Z. Angew. Math. Mech., **52**, p. 321, 1972.

5. R. C. SHARMA AND V. KUMARI, *Thermosolutal convection in a Maxwellian viscoelastic fluid in porous medium*, Czechoslovak Journal of Physics, **31**, 1, 1993.

6. K. WALTERS, *The motion of an elastico-viscous liquid contained between coaxial cylinders*, Quart. J. Mech. Appl. Math., **13**, p. 444, 1960.

7. R. C. SHARMA AND P. KUMAR, *Study of the stability of two superposed Walters' (Model B′) visco-elastic liquids*, Czechoslovak Journal of Physics, **47**, 2, p. 197, 1997.

8. E. R. LAPWOOD, *Convection of a fluid in porous medium*, Proc. Camb. Phil. Soc., **44**, p. 508, 1948.

9. R. A. WOODING, *Rayleigh instability of a thermal boundary layer in flow through a porous medium*, J. Fluid Mechanics, **9**, p. 183, 1960.

10. J. A. M. MCDONNEL, *Cosmic dust*, John Wiley and Sons, p. 330, Toronto 1978.

11. R. C. SHARMA, *M H D instability of rotating superposed fluids through porous medium*, Acta Physica Hungarica, **42**, p. 21, 1977.

12. P. R. SHARMA AND H. KUMAR, *Steady flow and heat transfer of Walters' (Model B′) fluid through porous pipe of uniform circular cross-section with small suction*, Proc. Nat. Acad. Sci. India, **65(A)I**, p. 75, 1995.

13. R. C. SHARMA, *Stability of stratified fluid in porous medium in the presence of horizontal magnetic field and rotation*, Jnanabha, **21**, p. 67, 1991.

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