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Deformation due to inclined load in thermoelastic half-space with voids

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The two-dimensional deformation of homogeneous, isotropic, thermoelastic half-space with voids as a result of inclined line load is investigated by applying the Laplace and Fourier transforms. The inclined load is assumed to be a linear combination of a normal load and a tangential load. The displacements, stresses, temperature distribution and change in volume fraction field so obtained in the physical domain are computed numerically. The variations of these quantities have been depicted graphically in the Lord–Shulman (L–S) theory and Green–Lindsay (G–L) theory for an insulated boundary.

Key words: thermoelastic, concentrated force, uniformly and linearly distributed forces, generalized thermoelasticity, voids, inclined load, Laplace and Fourier transforms.

1. Introduction

The theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory has practical utility for investigating various types of geological and biological materials for which the classical elastic theory is inadequate. This theory is concerned with elastic materials having a distribution of small pores (voids) in which the void volume is included among the kinematics variables, and in the limiting case of volume tending to zero, the theory reduces to the classical theory of elasticity.

A nonlinear theory of elastic materials with voids was developed by Nunziato and Cowin [7]. Later, Cowin and Nunziato [9] developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. They considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of beams and small amplitudes of acoustic waves. Puri and Cowin [11] studied the behavior of plane waves in linear elastic materials with voids. Domain of influence theorem in the theory of elastic materials with voids was discussed by Dhaliwal and Wang [18]. Scarpetta [19] studied the well-posedness theorems for linear elastic materials with voids. Birsan [26] established the existence and uniqueness of weak solutions in the linear theory of elastic shells with voids.

Rusu [14] studied the existence and uniqueness of solutionsin thermoelastic materials with voids. Saccomandi [15] presented some remarks about the thermoelastic theory of materials with voids. Ciarletta and Scalia [17] discussed the nonlinear theory of nonsimple thermoelastic materials with voids. Ciarletta and Scarpetta [21] discussed some results concerning thermoelasticity for dielectric materials with voids. Dhaliwal and Wang [20] developed a heat-flux dependent theory of thermoelasticity with voids. Marin [22, 23] studied the uniqueness and domain of influence results for thermoelastic bodies with voids. Marin [24] presented the contributions on uniqueness in thermoelastodynamics for bodies with voids. Marin and Salca [25] obtained the relation of the Knopoff-de Hoop type in thermoelasticity of dipolar bodies with voids. Chirita and Scalia [28] studied the spatial and temporal behavior in linear thermoelasticity of materials with voids. Pompei and Scalia [29] discussed the spatial decay estimates in linear thermoelasticity of materials with voids.

When the source surface is very long in one direction in comparison with the others, the use of two-dimensional approximation is justified and consequently, calculations are simplified to a great extent and one gets analytical solutions in closed form. A very long strip-source and a very long line-source are examples of such two-dimensional sources. Love [1] obtained expressions for the displacements due to a line source in an isotropic elastic medium. Maruyama [2] obtained the displacement and stress fields corresponding to long strike-slip faults in a homogeneous isotropic half-space. Okada [12, 16] presented a compact analytic expressions for the surface deformation and internal deformation due to inclined shear and tensile faults in a homogeneous isotropic half-space. Several authors [4, 6, 27, 30] discussed the problems of inclined load in the theory of elastic solids. No attempt has been made so far to study the response to inclined load in a thermoelastic body with voids.

We study the general plane strain problem of thermoelastic half-space with voids due to different sources. The integral transform technique has been used to solve it. We have obtained the expression for displacements, stresses, temperature distribution and change in volume fraction field in a thermoelastic half-space with voids due to an inclined line load in the form of Laplace and Fourier transforms, which are converted to the original solution numerically. The deformation due to other sources such as strip loads, continuous line loads etc. can also be similarly obtained. The deformation at a point of the medium is useful to analyze the deformation field around mining tremors and drilling into the crust of the earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation fields in the entire volume surrounding the source region.

2. Basic equations

Following Lord-Shulman [3], Green–Lindsay [5] and Cowin and Nunziato [9], the field equations and constitutive relations in thermoelastic solid with voids without body forces, heat sources and extrinsic equilibrated body force can be written as:

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\,\nabla\,(\nabla.\mathbf{u}) + b\nabla\phi - \beta\nabla(T+\delta_{2k}\tau_1\dot{T}) = \rho\ddot{\mathbf{u}}, \tag{2.1}$$

$$\alpha\nabla^2\phi - b\,(\nabla.\mathbf{u}) - \xi_1\phi - \omega_0\dot{\phi} + mT = \rho\chi\ddot{\phi}, \tag{2.2}$$

$$K\nabla^2 T - \beta T_0(\nabla.\dot{\mathbf{u}} + \tau_0\delta_{1k}\nabla.\ddot{\mathbf{u}}) - mT_0\dot{\phi} = \rho c_e(\dot{T}+\tau_0\ddot{T}), \tag{2.3}$$

and

$$t_{ij} = \lambda\, u_{G,G}\,\delta_{ij} + \mu(u_{i,j}+u_{j,i}) + b\,\phi\,\delta_{ij} - \beta(T+\delta_{2k}\tau_1\dot{T})\delta_{ij}, \quad (i,j=x,y,z). \tag{2.4}$$

In Eqs. (2.1)–(2.4) we have used the notations: λ, μ – Lamé constants, α (stress times squared length), b, ξ_1 (stress), ω_0 (stress times time), χ (equilibrated inertia) which are material constants due to the presence of voids, m (stress temperature coefficient) material constant due to presence of voids and temperature, T – temperature change, $\beta = (3\lambda+2\mu)\alpha_t$, α_t – linear thermal expansion coefficient $\mathbf{u}$ – displacement vector, t_{ij} – stress tensor, ρ, c_e – density and specific heat at constant strain, respectively, K – thermal conductivity, ϕ – change in the volume fraction field, δ_{ij} – Kronecker delta, T_0 – uniform temperature; a superposed dot denotes differentiation with respect to time variable t, τ_0, τ_1 are thermal relaxation times. For the L–S theory, $\tau_1 = 0$, $\delta_{1k} = 1$ and for the G–L theory $\tau_1 > 0$, $\delta_{1k} = 0$ (i.e., $k = 1$ for the L–S theory and $k = 2$ for the G–L theory). The thermal relaxations τ_0 and τ_1 satisfy the inequality $\tau_1 \geq \tau_0 > 0$ for the G–L theory only, $\nabla = \hat{i}\dfrac{\partial}{\partial x} + \hat{j}\dfrac{\partial}{\partial y} + \hat{k}\dfrac{\partial}{\partial z}$, $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ are the gradient and Laplacian operators respectively.

3. Formulation and solution of the problem

We consider a homogeneous, isotropic, thermally conducting elastic half-space with voids in the undeformed state at uniform temperature T_0. The rectangular Cartesian coodinate system $(x,\ y,\ z)$ with z-axis pointing vertically into the medium is introduced.

Suppose that an inclined line load F_0 per unit length is acting on the y-axis and its inclination to z-direction is θ (Fig. 1). To simplify the algebra, only problems with zero initial conditions are considered.

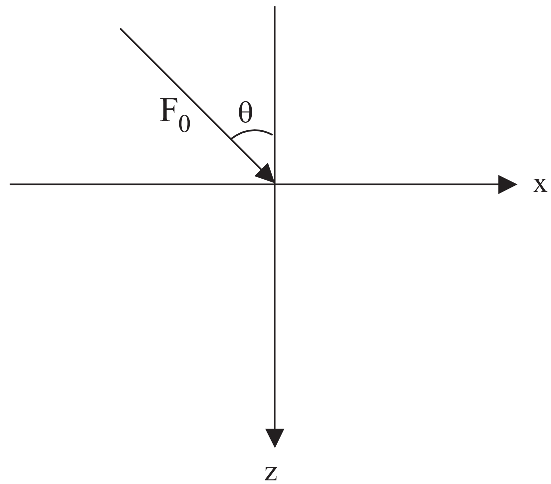

FIG. 1. Inclined load over a thermoelastic half-space with voids.

For a two-dimensional problem, we assume $\mathbf{u} = (u,\, 0,\, w)$ in Eqs. (2.1)–(2.4). We introduce the dimensionless quantities:

$$
\begin{aligned}
&x' = \frac{\omega_1^*}{c_2}x, \qquad z' = \frac{\omega_1^*}{c_2}z, \qquad t' = \omega_1^* t, \qquad u' = \frac{\omega_1^*}{c_2}u,\\
&w' = \frac{\omega_1^*}{c_2}w, \qquad T' = \frac{T}{T_0}, \qquad \phi' = \frac{\omega_1^{*2}\chi}{c_2^2}\phi,\\
&\in_1 = \frac{\beta c_2^2}{K\omega_1^*}, \qquad \tau_0' = \omega_1^*\tau_0, \qquad \tau'_1 = \omega_1^*\tau_1, \qquad a' = \frac{\omega_1^*}{c_2}a,
\end{aligned}
\tag{3.1}
$$

$$
t'_{ZZ} = \frac{t_{ZZ}}{\beta T_0}, \qquad t'_{ZX} = \frac{t_{ZX}}{\beta T_0}, \qquad h' = \frac{hc_2}{\omega_1^*},
\tag{3.2}
$$

where

$$
c_2 = \left(\frac{\mu}{\rho}\right)^{1/2} \quad \text{and} \quad \omega_1^* = \frac{\rho c_e c_2^2}{K}.
$$

The expression relating the displacement components $u(x,z,t)$ and $w(x,z,t)$ to the scalar potential functions $\psi_1(x,z,t)$ and $\psi_2(x,z,t)$ in dimensionless form are given by

$$
u = \frac{\partial\psi_1}{\partial x} - \frac{\partial\psi_2}{\partial z}, \qquad w = \frac{\partial\psi_1}{\partial z} + \frac{\partial\psi_2}{\partial x}.
\tag{3.3}
$$

Applying the Laplace and Fourier transforms

$$\hat{f}(x,z,p)=\int_{0}^{\infty} f(x,z,t)\,e^{-pt}dt \quad \text{and} \quad \tilde{f}(\xi,z,p)=\int_{-\infty}^{\infty}\hat{f}(x,z,p)\,e^{i\xi x}dx, \tag{3.4}$$

to equations (2.1)–(2.3), after using Eqs. (3.1), (3.3) (suppressing the primes) and eliminating $\tilde{\psi}_1, \tilde{\phi}, \tilde{T}$ and $\tilde{\psi}_2$ from the resulting expressions, we obtain

$$\left(\frac{d^6}{dz^6}+Q\frac{d^4}{dz^4}+N\frac{d^2}{dz^2}+I\right)\left(\tilde{\psi}_1,\tilde{\phi},\tilde{T}\right)=0, \tag{3.5}$$

and

$$\left(\frac{d^2}{dz^2}-\lambda_4^2\right)\tilde{\psi}_2=0, \tag{3.6}$$

where Q, N, I are listed in Appendix A.

The roots of Eqs. (3.5) and (3.6) are $\pm\lambda_\ell$ ($\ell=1,2,3,4$). Assuming the regularity condition at $z=\infty$, the solution of Eqs. (3.5) and (3.6) may be written as

$$\tilde{\psi}_1=A_1\bar{e}^{\lambda_1 z}+A_2\bar{e}^{\lambda_2 z}+A_3\bar{e}^{\lambda_3 z}, \tag{3.7}$$

$$\tilde{\phi}=d_1A_1\bar{e}^{\lambda_1 z}+d_2A_2\bar{e}^{\lambda_2 z}+d_3A_3\bar{e}^{\lambda_3 z} \tag{3.8}$$

$$\tilde{T}=e_1A_1\bar{e}^{\lambda_1 z}+e_2A_2\bar{e}^{\lambda_2 z}+e_3A_3\bar{e}^{\lambda_3 z}, \tag{3.9}$$

$$\tilde{\psi}_2=A_4e^{-\lambda_4 z}, \tag{3.10}$$

with A_ℓ ($\ell=1,2,3,4$) being arbitrary constants and e_ℓ and d_ℓ are given in Appendix B.

4. Application

Consider a normal line load of intensity F_1, per unit length, acting in the positive z-direction on the plane boundary $z=0$ along the y-axis and a tangential line load F_2, per unit length, acting at the origin in the positive x-direction then the boundary conditions are

$$\begin{gathered} t_{zz}(x,z,t)=-F_1\psi(x)\,\delta(t), \qquad t_{zx}(x,z,t)=-F_2\zeta(x)\delta(t), \\ \frac{\partial\phi}{\partial z}=0, \qquad \frac{\partial T}{\partial z}+hT=0 \qquad \text{at} \quad z=0, \end{gathered} \tag{4.1}$$

where $\delta(\,)$ is the Dirac delta function, $\psi(x)$ and $\zeta(x)$ denote the vertical and horizontal load functions, respectively, distributed along the x-axis, h is the heat transfer coefficient, F_1 and F_2 are force intensities.

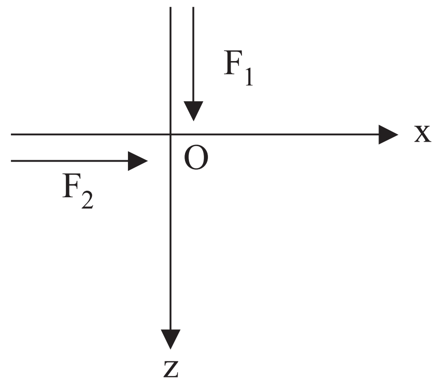

Fig. 2. Normal and tangential loadings.

Using Eqs. (3.1) and (3.2) along with $F_1' = \dfrac{F_1}{\beta T_0}$, $F_2' = \dfrac{F_2}{\beta T_0}$ in the boundary conditions (4.1) (suppressing the primes for convenience) and applying the Laplace and Fourier transforms defined by Eq. (3.4), we obtain the transformed boundary conditions as

$$\tilde{t}_{zz}(\xi,z,p) = -F_1\tilde{\psi}(\xi), \qquad \tilde{t}_{zx}(\xi,z,p) = -F_2\tilde{\zeta}(\xi),$$

$$\frac{d\tilde{\phi}}{dz} = 0, \qquad \frac{d\tilde{T}}{dz} + h\tilde{T} = 0 \tag{4.2}$$

where $\tilde{\psi}(\xi)$ and $\tilde{\zeta}(\xi)$ are the Fourier transforms of $\psi(x)$ and $\zeta(x)$ respectively.

Making use of Eqs. (2.4)–(3.3) (suppressing the primes for convenience) and applying the Laplace and Fourier transforms defined by (3.4) in the transformed boundary conditions (4.2) and substituting the values of $\tilde{\psi}_1, \tilde{\psi}_2, \tilde{T}, \tilde{\phi}$ from equations (3.7)–(3.10), we obtain the expressions for displacement components, stresses, temperature distribution and change in the volume fraction field which are presented in Appendix C.

Inclined line load

For an inclined line load F_0 per unit length, we have (see Fig. 1)

$$F_1 = F_0\cos\theta, \qquad F_2 = F_0\sin\theta. \tag{4.3}$$

Case 1. Concentrated force

In this case

$$\zeta(x) = \delta(x), \qquad \psi(x) = \delta(x),$$
$$\text{with} \tag{4.4}$$
$$\tilde{\zeta}(\xi) = 1, \qquad \tilde{\psi}(\xi) = 1.$$

CASE 2. Uniformly distributed force

The solution due to uniformly distributed force applied to the half-space surface is obtained by setting

$$\{\zeta(x), \psi(x)\} = \begin{cases} 1 & \text{if} \quad |x| \le a, \\ 0 & \text{if} \quad |x| > a, \end{cases}$$

in Eq. (4.1). The Laplace and Fourier transforms with respect to the pair (x, ξ) for the case of a uniform strip load of unit amplitude and width $2a$ applied at the origin of the coordinate system ($x = z = 0$) in dimensionless form, after suppressing the primes, becomes

$$\{\, \tilde{\zeta}(\xi),\ \tilde{\psi}(\xi)\} = \left[2\sin\left(\frac{\xi c_2 a}{\omega_1^*}\right) \Big/ \xi \right], \qquad \xi \neq 0\,. \tag{4.5}$$

CASE 3. Linearly distributed force

The solution due to linearly distributed force applied to the half-space surface is obtained by setting

$$\{\zeta(x), \psi(x)\} = \begin{cases} 1 - \dfrac{|x|}{a} & \text{if} \quad |x| \le a, \\ 0 & \text{if} \quad |x| > a, \end{cases}$$

in Eq. (4.1), where $2a$ is the width of the strip load. Using Eqs. (3.1)–(3.2) (suppressing the primes) and applying the transforms defined by Eq. (3.4), we get

$$\{\, \tilde{\zeta}(\xi),\ \tilde{\psi}(\xi)\} = \left[\frac{2\left\{1 - \cos\left(\dfrac{\xi c_2 a}{\omega_1^*}\right)\right\}}{\dfrac{\xi^2 c_2 a}{\omega_1^*}} \right]. \tag{4.6}$$

Using Eq. (4.3) in Eqs. (C.1) (Appendix C) and with the aid of Eqs. (4.4)–(4.6), we obtain the expressions for displacements, stresses, temperature distribution and change in the volume fraction field for different sources applied on the surface of the thermoelastic half-space with voids.

4.1. Particular case

If we neglect the voids effect, i.e. ($\alpha = b = \xi_1 = m = \chi = \omega_0 = 0$) in the Eqs. (C.1) along with Eqs. (4.3), we obtain the expressions for displacement components, stresses and temperature distribution in the thermoelastic half-space (see Appendix D).

The expressions for displacements, stresses and temperature distribution in the case of inclined line load can be obtained for concentrated, uniformly and linearly distributed force by substituting $\tilde{\zeta}(\xi)$, $\tilde{\psi}(\xi)$ from (4.4)–(4.6), respectively, in Eqs. (D.1).

SUB-CASE 1: If $h \to 0$, Eqs. (C.1) yield the expressions of displacements, stresses, temperature distribution and change in the volume fraction field for the insulated boundary.

SUB-CASE 2: If $h \to \infty$, Eqs. (C.1) yield the expressions of displacements, stresses, temperature distribution and change in the volume fraction field for the isothermal boundary.

SPECIAL CASE 1: By putting $k = 1$ and $\tau_1 = 0$ in Eqs. (C.1), we recover the displacements, stresses, temperature distribution and change in volume fraction field for L–S theory.

SPECIAL CASE 2: For the G–L theory, we obtain the corresponding expressions for displacements, stresses, temperature distribution and change in the volume fraction field by substituting $k = 2$ in Eqs. (C.1).

SPECIAL CASE 3: The expressions for displacements, stresses, temperature distribution and change in the volume fraction field for the theory of coupled thermoelasticity (CT) are obtained by putting $k = 3$, $\tau_0 = \tau_1 = 0$ in Eqs. (C.1).

5. Inversion of the transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms in equations (C.1) and (D.1), for the two theories, i.e., L–S and G–L. These expressions are functions of z, the parameters of Laplace and Fourier transforms p and ξ, respectively, and hence they are of the form $\tilde{f}(\xi, z, p)$. To get the function $f(x, z, t)$ in the physical domain, first we invert the Fourier transform using the formula:

$$\hat{f}(x,z,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x}\tilde{f}(\xi,z,p)d\xi = \frac{1}{\pi}\int_{0}^{\infty}(\cos(\xi x)f_e - i\sin(\xi x)f_0)d\xi, \tag{5.1}$$

where f_e and f_0 are, respectively, even and odd parts of the function $\hat{f}(\xi, z, p)$.

Thus, expression (5.1) gives us the Laplace transform $\hat{f}(x, z, p)$ of the function $f(x, z, t)$. Following HONIG and HIRDES [10], the Laplace transform $\hat{f}(x, z, p)$ can be inverted to $f(x, z, t)$.

The last step is to calculate the integral in Eq. (5.1). The method for evaluating this integral is described by PRESS *et al.* [13], which involves the application of Romberg's integration with adaptive step size. This uses also the results of successive approximations of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

6. Numerical results and discussion

Following Dhaliwal and Singh [8] we take the case of magnesium crystal-like material for numerical calculations. The physical constants used are

$$\lambda = 2.17 \times 10^{10}\ \mathrm{Nm}^{-2}, \quad \mu = 3.278 \times 10^{10}\ \mathrm{Nm}^{-2}, \quad \rho = 1.74 \times 10^{3}\ \mathrm{kgm}^{-3},$$
$$c_e = 1.04 \times 10^{3}\ \mathrm{J\ kg}^{-1}\mathrm{degree}^{-1}, \quad \omega_1^* = 3.58 \times 10^{11}\mathrm{s}^{-1}, \quad F_0 = 1, \quad T_0 = 298^\circ\ \mathrm{K},$$
$$\mathrm{K} = 1.7 \times 10^{2}\ \mathrm{Wm}^{-1}\mathrm{degree}^{-1}, \quad \beta = 2.68 \times 10^{6}\ \mathrm{Nm}^{-2}\mathrm{degree}^{-1},$$

and the void parameters are

$$\chi = 1.753 \times 10^{-15}\ \mathrm{m}^2, \quad \alpha = 3.688 \times 10^{-5}\ \mathrm{N}, \quad \xi_1 = 1.475 \times 10^{10}\ \mathrm{Nm}^{-2},$$
$$b = 1.13849 \times 10^{10}\ \mathrm{Nm}^{-2}, \quad \omega_0 = 0.0787 \times 10^{-3}\ \mathrm{Nm}^{-2}\ \mathrm{sec},$$
$$m = 2 \times 10^{6}\ \mathrm{Nm}^{-2}\mathrm{degree}^{-1}.$$

The comparison of the values of normal boundary displacement w, normal stress t_{zz}, boundary temperature field T and change in the volume fraction field ϕ with distance x for concentrated force (CF) are shown graphically in Figs. 3–6 for L–S and G–L theories for dimensionless relaxation times $\tau_0 = 0.02, \tau_1 = 0.05$ and for different values of $\theta = 0^\circ, 45^\circ, 90^\circ$. The computations are carried out for dimensionless time $t = 0.5$ at $z = 1.0$ in the range $0 \le x \le 10$. The solid, the small-dashed and the large-dashed lines without center symbol predicts the variations of L–S theory for different values of θ whereas the solid, the small-dashed and the large-dashed lines with center symbol predicts the variations of G–L theory for different values of θ. The results for distributed load are presented for dimensionless width $a = 1$.

Figure 3 depicts the variation of normal displacement w with distance x. Near the point of application of the source, the values of w increase as the angle of inclination increases for both the theories. The displacement for $\theta = 45^\circ$ lies between the corresponding displacements for a normal line load and tangential line load for both the theories in the range $0 \le x \le 10$. The behavior of variation of w for the L–S theory and G–L theory is the same, with difference in magnitude values in the whole range.

Figure 4 shows the variation of temperature distribution T with distance x. In the initial range, the values of T for $\theta = 0°$ are greater than for $\theta = 45°$ and $90°$ for both the theories. The values of T for $\theta = 0°$ exhibit small variations about zero in the whole range for both the theories, whereas for $\theta = 45°$ and $90°$ the values of T start with a small increase and then become oscillatory in the whole range for both the theories.

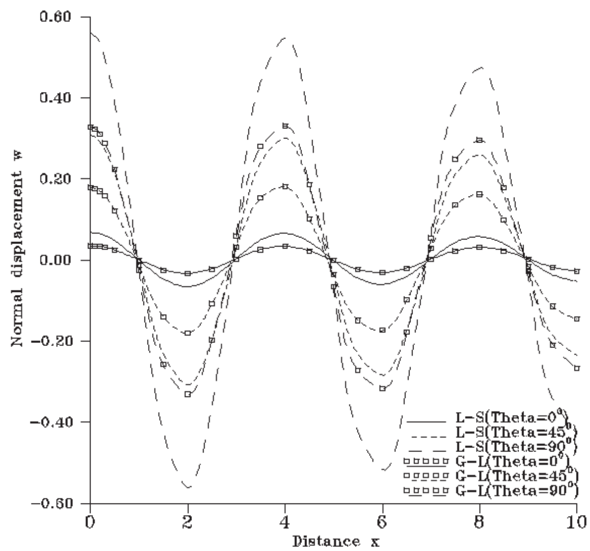

FIG. 3. Variation of normal displacement w with distance x.

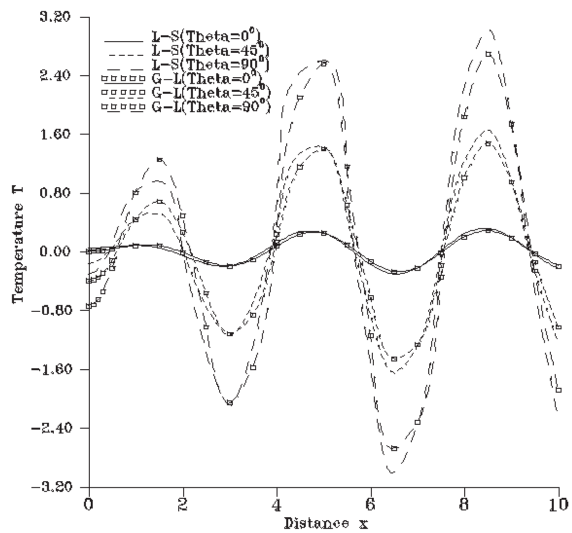

FIG. 4. Variation of temperature T with distance x.

Figure 5 depicts the variation of change in the volume fraction field ϕ with distance x. The nature of variation of ϕ is opposite to that of normal displacement w with difference in their magnitude as the angle of inclination increases from $\theta = 0°$ to $90°$.

Figure 6 shows the variation of normal stress t_{zz} with distance x. Initially the values of normal stress for the G–L theory are greater than the L–S theory and nature of variation of t_{zz} is opposite to that of T with difference in their magnitude as angle of inclination increases from $\theta = 0°$ to $90°$.

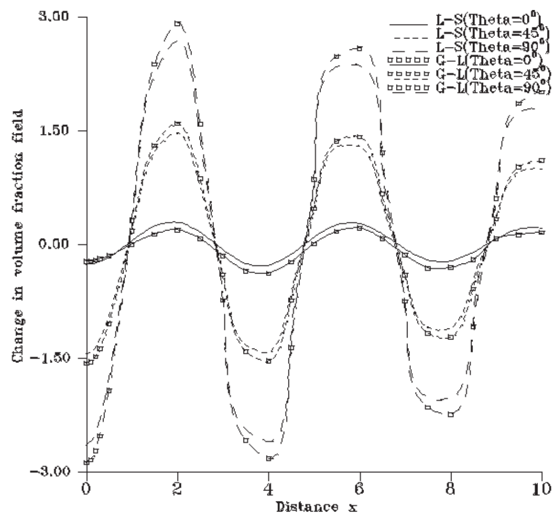

FIG. 5. Variation of change in the volume fraction field ϕ with distance x.

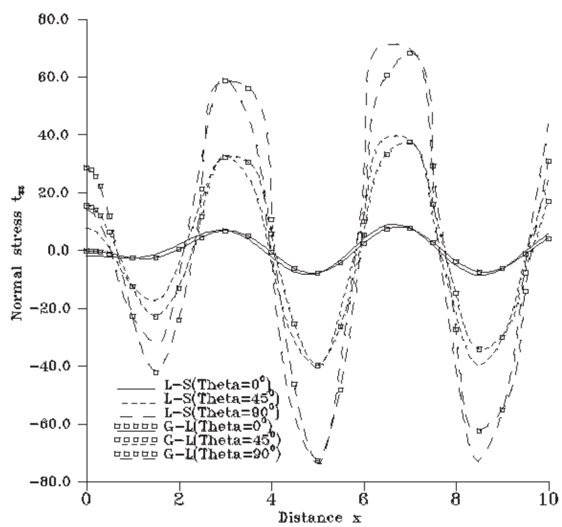

FIG. 6. Variation of normal stress t_{zz} with distance x.

7. Conclusions

1. It is evident from the figures that variations of normal displacement, stresses, temperature distribution and change in the volume fraction field for the L–S theory and the G–L theory are similar in nature for different angles of inclination when the load is applied.
2. At the point of application of the source, the values of normal displacement and normal stress increase as the angle of inclination increases, whereas for temperature distribution and change in volume fraction field the values show a reverse behaviour.
3. From the figures we conclude that the values of normal displacement, stresses, temperature distribution and change in the volume fraction field for $\theta = 45°$ lie between the values corresponding to $\theta = 0°$ and $\theta = 90°$.
4. The variations of normal displacement, normal stress, temperature distribution and change in volume fraction field for uniformly and linearly distributed force are the same as those of a concentrated force, with difference in their magnitude for both the theories.
5. The variations of normal displacement, stresses, temperature distribution and change in volume fraction field for CT theory are the same as those of the L–S theory and G–L theory, with a difference in the magnitude.

Appendix A

$$Q = \frac{1}{b_1}\{b_1(b_3 - b_5 - 3\xi^2) - p^2 + a_2a_4 - b_2b_4 \in_1\},$$

$$N = \frac{1}{b_1}\Big\{(a_6a_8p - b_3b_5 - 2b_3\xi^2 + 2b_5\xi^2 + 3\xi^4)b_1 - p^2\left(b_3 - b_5 - 2\xi^2\right)$$
$$+ a_2\left(a_4b_5 - 2a_4\xi^2 - a_6 \in_1 b_4\right) + b_2\left(\in_1 b_3b_4 - 2 \in_1 b_4\xi^2 - a_4a_8p\right)\Big\},$$

$$I = -\frac{1}{b_1}\Big\{b_1\xi^6 - \xi^4(p^2 + b_1b_3 - b_1b_5 + a_2a_4 + b_2b_4) + \xi^2(b_3p^2 + b_5p^2$$
$$+ b_1a_6a_8p - b_1b_3b_5 - a_2a_4b_5 - \in_1 b_4(a_2a_6 - b_2b_3)$$
$$+ p^2\left(a_6a_8p + b_3b_5\right)\Big\},$$

$$b_1 = (1 + a_1), \quad b_2 = a_3(1 + \tau_1\delta_{2k}\omega_1^* p), \quad b_3 = a_7p^2 + a_5 - \frac{\omega_0 c_2^2}{\omega_1^*\alpha},$$

$$b_4 = (p + p^2\tau_0\delta_{1k}\omega_1^*), \quad b_5 = (p + p^2\tau_0\omega_1^*), \quad \lambda_4^2 = \xi^2 + p^2,$$

$$a_1 = \frac{\lambda + \mu}{\mu}, \qquad a_2 = \frac{bc_2^2}{\chi\mu\omega_1^{*2}}, \qquad a_3 = \frac{\beta T_0}{\mu}, \qquad a_4 = \frac{b\chi}{\alpha},$$
$$a_5 = \frac{\xi_1 c_2^2}{\omega_1^{*2}\alpha}, \qquad a_6 = \frac{m\chi T_0}{\alpha}, \qquad a_7 = \frac{\rho\chi c_2^2}{\alpha}, \qquad a_8 = \frac{mc_2^4}{\omega_1^{*3}\chi K}$$

Appendix B

$$e_\ell = \frac{U^*\lambda_\ell^2 + V^*}{a_2\lambda_\ell^2 + T^*}, \qquad d_\ell = \frac{P^*\lambda_\ell^2 + Q^*}{R^*\lambda_\ell^2 + S^*}; \qquad (\ell = 1,\, 2,\, 3),$$
$$U^* = a_2 b_4 \in_1 + p b_1 a_8, \qquad V^* = -p^2 - \xi^2\left(b_1 + \in_1 a_2 b_4\right),$$
$$T^* = b_2 a_8 p - a_2\left(\xi^2 + b_5\right), \quad P^* = \frac{a_4}{a_6} - \frac{b_1}{b_2},$$
$$R^* = \frac{1}{a_6}, \qquad Q^* = \frac{1}{b_2}(\xi^2 b_1 + p^2) - \frac{a_4\xi^2}{a_6},$$
$$S^* = \frac{a_2}{b_2} + \left(\frac{b_3 - \xi^2}{a_6}\right),$$

Appendix C

The expressions for displacement components, stresses, temperature distribution and change in the volume fraction field are given as

$$\tilde{u} = -\frac{i\xi}{\Delta}\{F_1\tilde{\psi}(\xi)(\Delta_1\bar{e}^{\lambda_1 z} - \Delta_3\bar{e}^{\lambda_2 z} + \Delta_5\bar{e}^{\lambda_3 z} - \Delta_7\bar{e}^{\lambda_4 z})$$
$$+ F_2\tilde{\zeta}(\xi)(\Delta_2\bar{e}^{\lambda_1 z} - \Delta_4\bar{e}^{\lambda_2 z} + \Delta_6\bar{e}^{\lambda_3 z} - \Delta_8\bar{e}^{\lambda_4 z})\},$$

$$\tilde{w} = -\frac{1}{\Delta}\{F_1\tilde{\psi}(\xi)(\lambda_1\Delta_1\bar{e}^{\lambda_1 z} - \lambda_2\Delta_3\bar{e}^{\lambda_2 z} + \lambda_3\Delta_5\bar{e}^{\lambda_3 z} - i\xi\Delta_7\bar{e}^{\lambda_4 z})$$
$$+ F_2\tilde{\zeta}(\xi)(\lambda_1\Delta_2\bar{e}^{\lambda_1 z} - \lambda_2\Delta_4\bar{e}^{\lambda_2 z} + \lambda_3\Delta_6\bar{e}^{\lambda_3 z} - i\xi\Delta_8\bar{e}^{\lambda_4 z})\},$$

(C.1)

$$\tilde{t}_{zz} = \frac{1}{\Delta}\{F_1\tilde{\psi}(\xi)(s_1\Delta_1\bar{e}^{\lambda_1 z} - s_2\Delta_3\bar{e}^{\lambda_2 z} + s_3\Delta_5\bar{e}^{\lambda_3 z} - s_4\Delta_7\bar{e}^{\lambda_4 z})$$
$$+ F_2\tilde{\zeta}(\xi)(s_1\Delta_2\bar{e}^{\lambda_1 z} - s_2\Delta_4\bar{e}^{\lambda_2 z} + s_3\Delta_6\bar{e}^{\lambda_3 z} - s_4\Delta_8\bar{e}^{\lambda_4 z})\},$$

$$\tilde{t}_{zx} = \frac{1}{\Delta}\{F_1\tilde{\psi}(\xi)(\lambda_1\Delta_1\bar{e}^{\lambda_1 z} - \lambda_2\Delta_3\bar{e}^{\lambda_2 z} + \lambda_3\Delta_5\bar{e}^{\lambda_3 z} + n_1\Delta_7\bar{e}^{\lambda_4 z})$$
$$+ F_2\tilde{\zeta}(\xi)(\lambda_1\Delta_2\bar{e}^{\lambda_1 z} - \lambda_2\Delta_4\bar{e}^{\lambda_2 z} + \lambda_3\Delta_6\bar{e}^{\lambda_3 z} + n_1\Delta_8\bar{e}^{\lambda_4 z})\},$$

(C.1) [cont.]

$$\tilde{T} = \frac{1}{\Delta}\{F_1\tilde{\psi}(\xi)(e_1\Delta_1\bar{e}^{\lambda_1 z} - e_2\Delta_3\bar{e}^{\lambda_2 z} + e_3\Delta_5\bar{e}^{\lambda_3 z}) + F_2\tilde{\zeta}(\xi)(e_1\Delta_2\bar{e}^{\lambda_1 z} - e_2\Delta_4\bar{e}^{\lambda_2 z} + e_3\Delta_6\bar{e}^{\lambda_3 z})\},$$

$$\tilde{\phi} = \frac{1}{\Delta}\{F_1\tilde{\psi}(\xi)(d_1\Delta_1\bar{e}^{\lambda_1 z} - d_2\Delta_3\bar{e}^{\lambda_2 z} + d_3\Delta_5\bar{e}^{\lambda_3 z}) + F_2\tilde{\zeta}(\xi)(d_1\Delta_2\bar{e}^{\lambda_1 z} - d_2\Delta_4\bar{e}^{\lambda_2 z} + d_3\Delta_6\bar{e}^{\lambda_3 z})\},$$

where

$$\Delta = \Delta_1^* + h\Delta_2^*,$$

$$\Delta_1^* = \lambda_2\lambda_3\left(s_4\lambda_1 - n_1s_1\right)\left(d_3e_2 - d_2e_3\right) + \lambda_1\lambda_3\left(s_4\lambda_2 + n_1s_2\right)\left(d_1e_3 + e_1d_3\right) + \lambda_1\lambda_2\left(s_4\lambda_3 + n_1s_3\right)\left(d_1e_2 - e_1d_2\right),$$

$$\Delta_2^* = \left(s_4\lambda_1 - n_1s_1\right)\left(e_3\lambda_2d_2 - e_2d_3\lambda_3\right) + \left(e_3\lambda_1d_1 - e_1\lambda_3d_3\right)\left(s_4\lambda_2 + n_1s_2\right) - \left(s_4\lambda_3 + n_1s_3\right)\left(e_1\lambda_2d_2 - e_2\lambda_1d_1\right),$$

$$\Delta_{1,2} = (n_1, s_4)\Delta_{10}, \qquad \Delta_{3,4} = (-n_1, s_4)\Delta_{20}, \qquad \Delta_{5,6} = (-n_1, s_4)\Delta_{30},$$

$$\Delta_7 = \Big[\lambda_1\lambda_2\lambda_3\big\{e_3\left(d_1 - d_2\right) + e_2\left(d_3 - d_1\right) - e_1\left(d_3 - d_2\right)\big\} + h\big\{\lambda_1\left(e_3d_2\lambda_2 - e_2d_3\lambda_3\right) - \lambda_2\left(e_3\lambda_1d_1 - e_1\lambda_3d_3\right) + \lambda_3\left(\lambda_1d_1e_2 - e_1\lambda_2d_2\right)\big\}\Big],$$

$$\Delta_8 = \Big[\big\{s_2\lambda_1\lambda_3\left(d_1e_3 - e_1d_2\right) + s_1\lambda_2\lambda_3(e_2d_3 - e_3d_2) - s_3\lambda_2\lambda_1(e_1d_2 - e_2d_1)\big\} + h\big\{s_1\left(e_3d_2\lambda_2 - e_2d_3\lambda_3\right) - s_2\left(e_3\lambda_1d_1 - e_1\lambda_3d_3\right) + s_3\left(\lambda_1d_1e_2 - e_1\lambda_2d_2\right)\big\}\Big],$$

$$\Delta_{10} = \left(e_2d_3 - d_2e_3\right)\lambda_2\lambda_3 + h\left(e_3\lambda_2d_2 - \lambda_3d_3e_2\right),$$

$$\Delta_{20} = \left(e_1d_3 - d_1e_3\right)\lambda_1\lambda_3 + h\left(e_3\lambda_1d_1 + e_1\lambda_3d_3\right),$$

$$\Delta_{30} = \left(e_1d_2 - d_1e_2\right)\lambda_1\lambda_2 + h\left(e_2\lambda_1d_1 - \lambda_2d_2e_1\right),$$

$$s_\ell = -i\xi {s'}_{10} - \lambda_\ell^2 s'_{20} + d_\ell s'_{30} + e_\ell s'_{40}; \qquad (\ell = 1,\, 2,\, 3),$$

$$s_4 = -(s'_{10} + i\xi {s'}_{20})\lambda_4, \qquad s'_{10} = \frac{-i\xi\lambda}{\beta T_0},$$

$$s'_{20} = \frac{-\,(\lambda + 2\mu)}{\beta T_0}, \qquad s'_{30} = \frac{bc_2^2}{\beta T_0 \omega_1^{*2} \chi},$$

$$s'_{40} = (1 + p\tau_1\delta_{2k}), \qquad n_1 = \frac{\lambda_4^2 + \xi^2}{2i\xi}.$$

Appendix D

The expressions for displacement components, stresses and temperature distribution are are given by the formulae:

$$\tilde{u} = -\frac{F_0}{\Delta^{**}}\Big[\tilde{\psi}(\xi)\cos\theta\big\{-i\xi(\Delta_3^{**}\bar{e}^{\lambda_1^* z} + \Delta_4^{**}\bar{e}^{\lambda_2^* z}) + \Delta_5^{**}\lambda_4\bar{e}^{\lambda_4 z}\big\} + \tilde{\zeta}(\xi)\sin\theta\big\{i\xi(\Delta_6^{**}\bar{e}^{\lambda_1^* z} + \Delta_7^{**}\bar{e}^{\lambda_2^* z}) - \Delta_8^{**}\lambda_4\bar{e}^{\lambda_4 z}\big\}\Big],$$

$$\tilde{w} = \frac{F_0}{\Delta^{**}}\Big[\tilde{\psi}(\xi)\cos\theta\big\{\lambda_1^*\Delta_3^{**}\bar{e}^{\lambda_1^* z} + \lambda_2^*\Delta_4^{**}\bar{e}^{\lambda_2^* z} + i\xi\Delta_5^{**}\bar{e}^{\lambda_4 z}\big\} - \tilde{\zeta}(\xi)\sin\theta\big\{\lambda_1^*\Delta_6^{**}\bar{e}^{\lambda_1^* z} + \lambda_2^*\Delta_7^{**}\bar{e}^{\lambda_2^* z} + i\xi\Delta_8^{**}\bar{e}^{\lambda_4 z}\big\}\Big],$$

(D.1)
$$\tilde{t}_{zz} = \frac{F_0}{\Delta^{**}}\Big[\tilde{\psi}(\xi)\cos\theta\big\{s_1^*\Delta_3^{**}\bar{e}^{\lambda_1^* z} + s_2^*\Delta_4^{**}\bar{e}^{\lambda_2^* z} + s_3^*\Delta_5^{**}\bar{e}^{\lambda_4 z}\big\} - \tilde{\zeta}(\xi)\sin\theta\big\{s_1^*\Delta_6^{**}\bar{e}^{\lambda_1^* z} + s_2^*\Delta_7^{**}\bar{e}^{\lambda_2^* z} + s_3^*\Delta_8^{**}\bar{e}^{\lambda_4 z}\big\}\Big],$$

$$\tilde{t}_{zx} = -\frac{F_0}{\Delta^{**}}\Big[\tilde{\psi}(\xi)\cos\theta\big\{\lambda_1^*\Delta_3^{**}\bar{e}^{\lambda_1^* z} + \lambda_2^*\Delta_4^{**}\bar{e}^{\lambda_2^* z} - n_1\Delta_5^{**}\bar{e}^{\lambda_4 z}\big\} - \tilde{\zeta}(\xi)\sin\theta\big\{\lambda_1^*\Delta_6^{**}\bar{e}^{\lambda_1^* z} + \lambda_2^*\Delta_7^{**}\bar{e}^{\lambda_2^* z} - n_1\Delta_8^{**}\bar{e}^{\lambda_4 z}\big\}\Big],$$

$$\tilde{T} = -\frac{F_0}{\Delta^{**}}\Big[\tilde{\psi}(\xi)\cos\theta\big\{e_1^*\Delta_3^{**}\bar{e}^{\lambda_1^* z} + e_2^*\Delta_4^{**}\bar{e}^{\lambda_2^* z}\big\} - \tilde{\zeta}(\xi)\sin\theta\big\{e_1^*\Delta_6^{**}\bar{e}^{\lambda_1^* z} + e_2^*\Delta_7^{**}\bar{e}^{\lambda_2^* z}\big\}\Big],$$

where

$$\Delta^{**} = \Delta_1^{**} + h\Delta_2^{**},$$
$$\Delta_1^{**} = -\left(s_1^* e_2^* \lambda_2^* - s_2^* e_1^* \lambda_1^*\right) n_1 - \left(s_3^* e_2^* - e_1^*\right) \lambda_1^* \lambda_2^*,$$
$$\Delta_2^{**} = \left(s_1^* e_2^* - s_2^* e_1^*\right) n_1 + s_3^* e_2^* \lambda_1^* - e_1^* \lambda_2^*,$$
$$\Delta_3^{**} = -(\lambda_2^* - h) e_2^* n_1,$$
$$\Delta_4^{**} = (\lambda_1^* + h) e_1^* n_1,$$
$$\Delta_5^{**} = (e_1^* - e_2^*)\lambda_1^* \lambda_2^* + h(e_2^* \lambda_1^* - \lambda_2^* e_1^*),$$
$$\Delta_6^{**} = (\lambda_2^* - h) e_2^* s_3^*,$$
$$\Delta_7^{**} = (\lambda_1^* + h) e_1^* s_3^*,$$
$$\Delta_8^{**} = -(e_1^* s_2^* \lambda_1^* - e_2^* s_1^* \lambda_2^*) + h(e_2^* s_1^* - s_2^* e_1^*),$$
$$\lambda_\ell^{*2} = \frac{-A + (-1)^{\ell+1}\sqrt{A^2 - 4B}}{2}; \qquad \ell = 1, 2,$$
$$A = -\frac{p^2 + (2\xi^2 + b_5)b_1 + \in_1 b_2 b_4}{b_1},$$
$$B = \frac{(p^2 + \xi^2 b_1)(\xi^2 + b_5) + \in_1 b_2 b_4 \xi^2}{b_1},$$
$$s_\ell^* = -i\xi {s'}_{10} - \lambda_\ell^{*2} s'_{20} - e_\ell^*; \qquad \ell = 1, 2,$$
$$s_3^* = ({s'}_{10} - i\xi {s'}_{20})\lambda_4,$$
$$e_\ell^* = \frac{b_1 \lambda_\ell^{*2} - p^2 - b_1 \xi^2}{b_2}; \qquad \ell = 1, 2.$$

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