

Transmission conditions for a soft elasto-plastic interphase between two elastic materials. Plane strain state

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A THIN INTERPHASE between two different elastic media is under consideration. It is assumed that the intermediate layer consists of a soft elasto-plastic material whose Young's modulus is small enough in comparison with those of the bounding materials. Using an asymptotic technique, nonlinear transmission conditions for the bimaterial structure are evaluated. As a numerical example, a FEM analysis of a bimaterial structure with an interface is performed to investigate the accuracy of the derived transmission conditions.

Key words: elastic-plastic layer, imperfect interface, nonclassical transmission conditions.

1. Introduction

THIN INTERPHASES appearing in dissimilar bodies such as composite structures with adhesively bonded materials may influence significantly the whole spectrum of structural parameters: strength, dynamics, fracture, long lifetime and so on. Recently, significant efforts have been done to clarify various phenomena connected with the so-called imperfect interface approach. It consists in replacing the real thin interphase between two different materials by an infinitesimal layer of zero thickness. This layer is then modeled by special transmission conditions which incorporate information about geometrical and mechanical properties of the thin interphase. At first, such proposed conditions were based on phenomenological arguments and have been sufficiently exploited (see [3, 6, 7] among others and the respective references). Later, various imperfect transmission conditions have been evaluated by asymptotic methods in [2, 4, 8, 13] for different types of interfaces and materials. Accurate asymptotic behaviour of solutions of interface crack problems in the imperfect interface formulation have been done in [1, 14, 15] where it has been shown that the behaviour may be very compli-

cated and essentially depends on the material and geometrical properties of the imperfect interfaces. Possible error estimates and ranges of the edge zone effects connected with utilisation of the imperfect interface models have been discussed in [16, 17] by the FEM analysis. This short overview shows that *elastic* imperfect interfaces have been intensively investigated in different directions.

However, thin elasto-plastic interfaces appear very often in real applications and the respective plastic properties may even have a greater influence than the elastic ones [11]. On the other hand, the numerical FEM simulation of the thin elasto-plastic interphase is more complicated than a pure elastic simulation. Unfortunately, results which have been obtained up to now have been absolutely insufficient and are mainly concentrated on problems of thin plastic interphases between rigid adherends [10, 12].

In the present work, imperfect transmission conditions for a soft elasto-plastic interphase are evaluated by asymptotic methods. The interface is described by the simple Hencky's deformation theory model. Only the main terms, i.e. zero-order expressions, of the asymptotic analysis are considered. Respective transmission conditions are naturally nonlinear. Higher-order expressions can be much easier to construct continuing the asymptotic procedure from the respective linear boundary problems. A numerical example based on an accurate finite element simulation shows a high efficiency of the approach, in spite of the fact that the deformation theory has its strong restrictions.

2. Basic interphase equations

In this section, only the interphase is considered. It is assumed that the material behaviour can be modeled by the elasto-plastic Hencky law [5, 12]:

$$(2.1) \quad \varepsilon = \frac{1-2\nu}{E}\sigma, \quad \mathbf{D}_\varepsilon = \left(\phi + \frac{1}{2\mu} \right) \mathbf{D}_\sigma,$$

where ν is Poisson's ratio, μ and E are the shear and Young's moduli of the material in the elastic regime ($E = 2\mu(1 + \nu)$). Here, \mathbf{D}_ε and \mathbf{D}_σ denote the deviatoric parts of the strain and stress tensors:

$$(2.2) \quad \mathbf{D}_\varepsilon = \boldsymbol{\varepsilon} - \frac{1}{3}\boldsymbol{\varepsilon}\mathbf{I}, \quad \mathbf{D}_\sigma = \boldsymbol{\sigma} - \frac{1}{3}\boldsymbol{\sigma}\mathbf{I},$$

while

$$(2.3) \quad \varepsilon = I_1(\boldsymbol{\varepsilon}) = \sum_{i=1}^3 \varepsilon_{ii}, \quad \sigma = I_1(\boldsymbol{\sigma}) = \sum_{i=1}^3 \sigma_{ii}$$

are the first invariants of the tensors.

Function ϕ is assumed to be known in Eq. (2.1) and depends only on the second invariant of the strain deviator [12]:

$$(2.4) \quad \phi = \phi(J_2(\boldsymbol{\varepsilon})), \quad \phi(0) = 0.$$

Here, as usually,

$$(2.5) \quad J_2(\boldsymbol{\varepsilon}) = I_2(\mathbf{D}_\varepsilon) = \frac{1}{2} \sum_{i,j=1}^3 e_{ij}e_{ij}$$

and e_{ij} are components of the deviator \mathbf{D}_ε . It is well known that such a model describes appropriately only monotonic or near-monotonic loading and, in fact, comprises one of the nonlinear elasticity models [9, 12].

After some standard transformations, Eq. (2.1) can be rewritten in a form of nonlinear elasticity as:

$$(2.6) \quad \sigma_{ij} = 2\tilde{\mu}\varepsilon_{ij} + \tilde{\lambda}\varepsilon\delta_{ij}, \quad i, j = 1, 2, 3,$$

where the generalised Lamé's coefficients have been introduced:

$$(2.7) \quad \begin{aligned} \tilde{\mu}(\phi) &= \frac{1}{2} \left(\phi + \frac{1+\nu}{E} \right)^{-1}, \\ \tilde{\lambda}(\phi) &= \frac{1}{3} \left(\phi + \frac{1+\nu}{E} \right)^{-1} \left(\frac{3\nu}{1-2\nu} + \phi \frac{E}{1-2\nu} \right). \end{aligned}$$

It should be noted here that these new coefficients coincide in the pure elastic regime ($\phi = 0$) with the elastic Lamé's parameters:

$$(2.8) \quad \tilde{\mu}(0) = \mu = \frac{E}{2(1+\nu)}, \quad \tilde{\lambda}(0) = \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}.$$

Also the generalised Poisson's ratio can be introduced in the model:

$$(2.9) \quad \tilde{\nu}(\phi) = \frac{\tilde{\lambda}(\phi)}{2(\tilde{\lambda}(\phi) + \tilde{\mu}(\phi))} = \frac{3\nu + \phi E}{3 + 2\phi E}.$$

It is easy to show from Eqs. (2.7) and (2.9) that for $0 < \nu < 0.5$

$$(2.10) \quad 0 < \tilde{\mu}(\phi) \leq \mu, \quad \nu \leq \tilde{\nu}(\phi) < \frac{1}{2}, \quad \lambda \leq \tilde{\lambda}(\phi) < \frac{1+\nu}{3\nu}\lambda,$$

where the function $\tilde{\mu}(\phi)$ monotonically decreases, while functions $\tilde{\nu}(\phi)$ and $\tilde{\lambda}(\phi)$ monotonically increase.

REMARK 1. If the initial elastic parameters of the intermediate layer are essentially smaller than those corresponding to the bonded materials, i.e.

$$(2.11) \quad \tilde{\mu}(0) \ll \mu_{\pm}, \quad \tilde{\lambda}(0) \ll \lambda_{\pm},$$

then one can immediately conclude from (2.10) the same properties of the generalised parameters for any $\phi \geq 0$ provided ν is not too close to zero:

$$(2.12) \quad \tilde{\mu}(\phi) \ll \mu_{\pm}, \quad \tilde{\lambda}(\phi) \ll \lambda_{\pm}.$$

REMARK 2. It follows from (2.9) and (2.10) that $\tilde{\nu}(\phi) \rightarrow 1/2$ as $\phi \rightarrow \infty$.

REMARK 3. Function $\phi = \phi(J_2(\boldsymbol{\varepsilon}))$ can not behave arbitrarily. In fact, it should be determined from the yield criterion [12]. On the other hand, one can deduce from the monotonicity of the true stress-strain curve behaviour that the function $\tilde{\mu}(\phi(t))\sqrt{t}$ has to be non-decreasing. Moreover, in the case of hardening materials without saturation, $J_2(\boldsymbol{\sigma}) \rightarrow \infty$ as $J_2(\boldsymbol{\varepsilon}) \rightarrow \infty$, or

$$(2.13) \quad \tilde{\mu}(\phi(J_2(\boldsymbol{\varepsilon})))\sqrt{J_2(\boldsymbol{\varepsilon})} \rightarrow \infty, \quad \text{as} \quad J_2(\boldsymbol{\varepsilon}) \rightarrow \infty.$$

Taking into account Eq. (2.7), it is clear that condition (2.13) holds always in the case when $\phi(t)$ is a bounded function. In the opposite case, if $\phi(t) \rightarrow \infty$ as $t \rightarrow \infty$, the following estimate for the function ϕ has to be satisfied:

$$(2.14) \quad \sqrt{t}/\phi(t) \rightarrow \infty, \quad \text{as} \quad t \rightarrow \infty.$$

If one additionally assumes that there exists some parameter $\alpha > 0$ such that

$$(2.15) \quad \phi(t) = O(t^\alpha), \quad \text{as} \quad t \rightarrow \infty,$$

then it is easy to see that estimate (2.14) is satisfied only under the condition $0 < \alpha < 1/2$.

On the other hand, in the case of ideal plastic materials or plastic hardening laws with saturation:

$$(2.16) \quad \tilde{\mu}(\phi(J_2(\boldsymbol{\varepsilon})))\sqrt{J_2(\boldsymbol{\varepsilon})} \rightarrow \text{const}, \quad \text{as} \quad J_2(\boldsymbol{\varepsilon}) \rightarrow \infty.$$

This leads to:

$$(2.17) \quad \sqrt{t}/\phi(t) \rightarrow \text{const}, \quad \text{as} \quad t \rightarrow \infty,$$

or, under assumption (3.2), it is equivalent to $\alpha = 1/2$.

To finish the preliminary part of the paper, equations for the plane strain state are presented below. Thus, if $u_x = u_x(x, y)$, $u_y = u_y(x, y)$, $u_z = 0$ then

$$(2.18) \quad \varepsilon_z = \varepsilon_{xz} = \varepsilon_{yz} = 0, \quad \sigma_{xz} = \sigma_{yz} = 0, \quad \sigma_{zz} = \tilde{\lambda}(\phi)\varepsilon,$$

and for the remaining displacement and strain components, the following 2D relationships hold:

$$(2.19) \quad \varepsilon_x = \frac{\partial u_x}{\partial x}, \quad \varepsilon_y = \frac{\partial u_y}{\partial y}, \quad \varepsilon_{xy} = \frac{1}{2} \left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right)$$

and the stress components are defined by generalised Hooke's law (2.6):

$$(2.20) \quad \sigma_{ij} = 2\tilde{\mu}\varepsilon_{ij} + \tilde{\lambda}\varepsilon\delta_{ij}, \quad i, j = 1, 2, \quad \varepsilon = \varepsilon_x + \varepsilon_y.$$

Finally, the second invariant of the strain deviator can be calculated in this case as:

$$(2.21) \quad J_2(\boldsymbol{\varepsilon}) = \varepsilon_{xy}^2 + \frac{1}{3} (\varepsilon_x^2 + \varepsilon_y^2 - \varepsilon_x \varepsilon_y).$$

3. Problem formulation and its asymptotic analysis

A bi-material domain with a thin elasto-plastic layer between two different elastic materials with Lamé's parameters μ_{\pm}, λ_{\pm} , respectively, is considered in the following (Fig. 1). It is assumed that conditions (2.11) and, hence, (2.12) are satisfied. The intermediate layer is thin and soft so that simultaneously the conditions

$$(3.1) \quad 2h = 2\epsilon h_0, \quad \tilde{\mu} = \epsilon \tilde{\mu}_0, \quad \tilde{\lambda} = \epsilon \tilde{\lambda}_0,$$

hold where $\epsilon \ll 1$ is a small parameter, and

$$(3.2) \quad h_0 \sim L, \quad \tilde{\mu}_0 \sim \mu_{\pm}, \quad \tilde{\lambda}_0 \sim \lambda_{\pm},$$

while L is a characteristic size of the body.

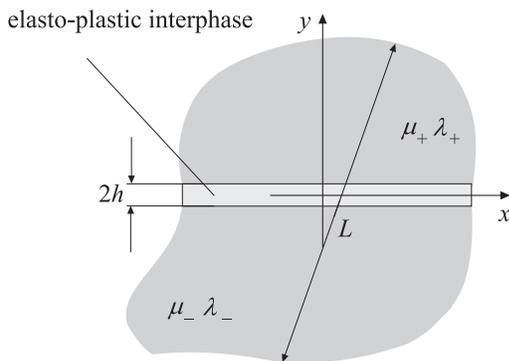


FIG. 1. Bimaterial structure under consideration.

The stresses satisfy within the interface, together with Eqs. (2.19) and (2.20), the equilibrium conditions:

$$(3.3) \quad \frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0, \quad \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0, \quad y \in (-\epsilon h_0, \epsilon h_0).$$

Along two bimaterial interfaces where $y = \pm \epsilon h_0$ holds, the perfect transmission conditions are assumed to be true:

$$(3.4) \quad u_x(x, \pm \epsilon h_0) = u_x^\pm(x, \pm \epsilon h_0), \quad u_y(x, \pm \epsilon h_0) = u_y^\pm(x, \pm \epsilon h_0),$$

$$(3.5) \quad \sigma_{xy}(x, \pm \epsilon h_0) = \sigma_{xy}^\pm(x, \pm \epsilon h_0), \quad \sigma_y(x, \pm \epsilon h_0) = \sigma_y^\pm(x, \pm \epsilon h_0).$$

Let us intentionally assume that the solution of the problem is known. Then, the nonlinear material parameters $\tilde{\mu}$ and $\tilde{\lambda}$ depend, generally speaking, on the geometrical position of the point under consideration:

$$(3.6) \quad \tilde{\mu} = \tilde{\mu}(x, y), \quad \tilde{\lambda} = \tilde{\lambda}(x, y),$$

via the known strain $\varepsilon_{ij}(x, y)$ and, hence, the second invariant $J_2(\boldsymbol{\varepsilon}) = J_2(x, y)$ of the strain deviator. During the evaluation of the transmission conditions it was assumed that functions (3.6) are known. As a result, this interphase can be analysed as an inhomogeneous elastic interphase [16] and only in the last stage, an additional equation to determine the second invariant of the strain deviator will be extracted.

Here, the standard asymptotic procedure, explained in detail in [19], is applied. Namely, rescaling one of the space variables by the formula:

$$(3.7) \quad y = \epsilon \xi, \quad \xi \in (-h_0, h_0),$$

and seeking for the solution of the problem in the form of asymptotic series:

$$(3.8) \quad \mathbf{u}(x, y) = \sum_{j=0}^{\infty} \epsilon^j \mathbf{u}_j(x, \xi), \quad \boldsymbol{\sigma}(x, \xi) = \sum_{j=0}^{\infty} \epsilon^j \boldsymbol{\sigma}_j(x, \xi),$$

one should collect the terms of the same order with respect to the small parameter ϵ in Eqs. (2.20), (2.21) and (3.3) and in the transmission conditions (3.4)–(3.5) and then to solve step by step the corresponding boundary value problems. Thus, repeating the line of reasoning applied in [16], one can find the solution for the zero-order approximation within the interface in the following form as [16]:

$$(3.9) \quad \sigma_{xy}(x, y) = \sigma_{xy}^\pm(x, 0), \quad \sigma_y(x, y) = \sigma_y^\pm(x, 0),$$

$$(3.10) \quad u_x(x, y) = u_x^-(x, 0) + \sigma_{xy}(x, 0) \int_{-h}^y \frac{dt}{\tilde{\mu}(x, t)},$$

$$u_y(x, y) = u_y^-(x, 0) + \sigma_y(x, 0) \int_{-h}^y \frac{dt}{\tilde{\lambda}(x, t) + 2\tilde{\mu}(x, t)},$$

whereas the imperfect transmission conditions along the soft inhomogeneous elastic interface with known distribution of the *elastic* parameters $\tilde{\lambda}(x, y)$ and $\tilde{\mu}(x, y)$ are:

$$(3.11) \quad [\sigma_{xy}]_{y=0} = 0, \quad [\sigma_y]_{y=0} = 0,$$

$$(3.12) \quad [u_x]_{y=0} = \sigma_{xy}(x) \int_{-h}^h \frac{dt}{\tilde{\mu}(x, t)}, \quad [u_y]_{y=0} = \sigma_y(x) \int_{-h}^h \frac{dt}{\tilde{\lambda}(x, t) + 2\tilde{\mu}(x, t)}.$$

Here, the symbol $[f]_\Gamma$ denotes as usually the jump of the function f across the surface Γ . Let us underline here that integrals in (3.12) are estimated like $O(1)$ in view of the assumptions (3.1). On the other hand, one can conclude from (3.10) and (2.19) that:

$$(3.13) \quad \varepsilon_y, \varepsilon_{xy} = O(\epsilon^{-1}), \quad \varepsilon_x = O(1), \quad \epsilon \rightarrow 0.$$

Taking these estimates into account, one can rewrite Eq. (2.21) in the following manner:

$$(3.14) \quad J_2(\boldsymbol{\varepsilon}) = \left(\varepsilon_{xy}^2 + \frac{1}{3} \varepsilon_y^2 \right) (1 + O(\epsilon)), \quad \epsilon \rightarrow 0,$$

and utilising the generalised Hooke's law (2.20) and neglecting the terms of higher orders, one can deduce that the second invariant of the strain deviator can be calculated in the following manner:

$$(3.15) \quad J_2(\boldsymbol{\varepsilon}) = \frac{\sigma_{xy}^2}{4\tilde{\mu}^2} + \frac{\sigma_y^2}{3(2\tilde{\mu} + \tilde{\lambda})^2}.$$

This equation should be considered as an additional relationship to the transmission conditions (3.11), (3.12) connecting stress and strain quantities within the thin soft elasto-plastic layer.

Note that the stress components σ_y and σ_{xy} do not depend on the variable y (compare it with (3.9)). As a result, it is natural to assume that

$$(3.16) \quad J_2(\boldsymbol{\varepsilon}) = J_2(x) \quad \text{and} \quad \phi = \phi(x).$$

Taking this fact into account, one can simplify the transmission conditions (3.12) to obtain:

$$(3.17) \quad [u_x] = \frac{2h}{\tilde{\mu}(x)} \sigma_{xy}(x), \quad [u_y] = \frac{2h}{\tilde{\lambda}(x) + 2\tilde{\mu}(x)} \sigma_y(x).$$

The system of five equations (3.11), (3.17) and (3.15) establish the sought for transmission conditions for the soft elasto-plastic interface. Three of the equations in the transmission conditions are nonlinear (cf. (3.17) and (3.15)). Fortunately, it is possible to reduce the number of equations. Namely, to stay only with two nonlinear transmission conditions, equations (3.17) are substituted into equation (3.15) to obtain:

$$(3.18) \quad J_2(\boldsymbol{\varepsilon}) = \frac{[u_x]^2}{16h^2} + \frac{[u_y]^2}{12h^2}.$$

As a result, one can receive two nonlinear equations

$$(3.19) \quad \frac{1}{2h} \tilde{\mu}(\phi(J_2(\boldsymbol{\varepsilon}))) \cdot [u_x] = \sigma_{xy}, \quad \frac{1}{2h} (\tilde{\lambda} + 2\tilde{\mu})(\phi(J_2(\boldsymbol{\varepsilon}))) \cdot [u_y] = \sigma_y,$$

which constitute together with (3.11) the complete set of the transmission conditions. Here, $J_2(\boldsymbol{\varepsilon})$ is calculated only basing on the displacement jumps $[u_x]$ and $[u_y]$ in (3.18). It should be noted that the transmission conditions (3.19) can be written in abstract form as:

$$(3.20) \quad F_x([u_x], [u_y]) = \sigma_{xy}, \quad F_y([u_x], [u_y]) = \sigma_y,$$

where functions $F_x(t, \cdot)$ and $F_y(\cdot, t)$ monotonically increase with respect to the variable t (cf. equations (2.4), (2.7) and (3.18)). Moreover, one can conclude from (2.7) and (3.1) that the left-hand sides of equations (3.20) are of the order $O(1)$.

Equations (3.11) and (3.20) substitute the complete system of nonlinear transmission conditions for the soft elasto-plastic interface in the bimaterial structure under consideration. Another peculiarity of the conditions obtained in comparison with the imperfect elastic interface [16] is that the displacement jumps in different directions are not separated for the soft elasto-plastic interface, but both participate in each transmission condition from (3.20). However, in a particular case, when only the elastic regime appears in the elasto-plastic layer, conditions (3.20) degenerate to the imperfect elastic interface [16]:

$$(3.21) \quad \frac{\tilde{\mu}(0)}{2h} \cdot [u_x] = \sigma_{xy}, \quad \frac{\tilde{\lambda}(0) + 2\tilde{\mu}(0)}{2h} \cdot [u_y] = \sigma_y.$$

Another possibility to separate the displacement jumps from each other, even under plastic regime, can appear for some special loading conditions (e.g. simple

tensile or simple shear load), where one of the nonlinear transmission conditions (3.20) is satisfied identically whereas the other contains at the left-hand side the remaining non-zero jump (generally speaking in a nonlinear form).

REMARK 4. Transmission conditions (3.11) and (3.20) are valid, generally speaking, at some distance at the interaction of the interface with the external boundary. The range of the distance cannot be exactly predicted but can be estimated numerically, what will be done in the next section.

4. Numerical example and discussions

First of all, it is important to note that only terms of zero order have been evaluated by means of the asymptotic procedure. However, next terms can be found also in the same manner. Moreover, the boundary value problems which appear for the next terms will be linear, in contradiction to the zero-order term. However, as it has been demonstrated earlier in the case of the purely elastic interface [16], it will be shown for the elasto-plastic case that it is also sufficient to restrict the analysis to the zero-order approximation.

To show this, a numerical simulation of a bimaterial interface problem has been done. The geometry of the sample and respective loading conditions are shown in Fig. 2. The elastic materials which are glued by the interphase are assumed to be identical with Young's moduli $E_{\pm} = 72700$ MPa and Poisson's ratio $\nu_{\pm} = 0.34$. The geometrical dimensions are $L = 10$ mm, $H = 1$ mm and $2h = 0.01$ mm. As a result, the value of $\epsilon = 2h/H = 0.01$ can be considered as the small parameter. The elasto-plastic interface is represented by a linear hardening model whose parameters are described in Fig. 3. Namely, the elastic parameters: $E = 813$ MPa, $\nu = 0.3$. In the plastic region which is appearing after reaching the Huber–Mises stress of value $k_{t,0} = 50$ MPa, the constant hardening modulus $E_p = 81.3$ MPa is prescribed. Let us underline that all commercial

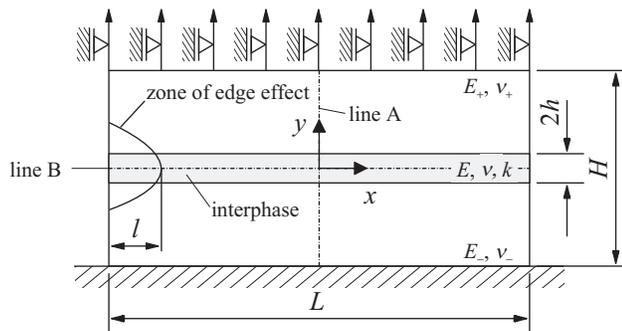


FIG. 2. Geometry and loading conditions of the bimaterial sample with a thin soft elasto-plastic interface for FEM simulation.

FEM codes are based on the more general theory of plastic flow [5, 9, 12]. As it has been mentioned above, the results with these models, i.e. deformation and plastic flow theories, coincide only under monotonic or nearly monotonic loading. Because of this, only monotonic external loading is applied (Dirichlet's boundary condition at the top of the sample).

The function ϕ from the deformation theory equations (2.1)–(2.4) was calculated by the given interphase properties of the flow theory [5, 12] and is shown in Fig 3b). Furthermore, it has been assumed that the material is obeying the Huber–Mises yield criterion.

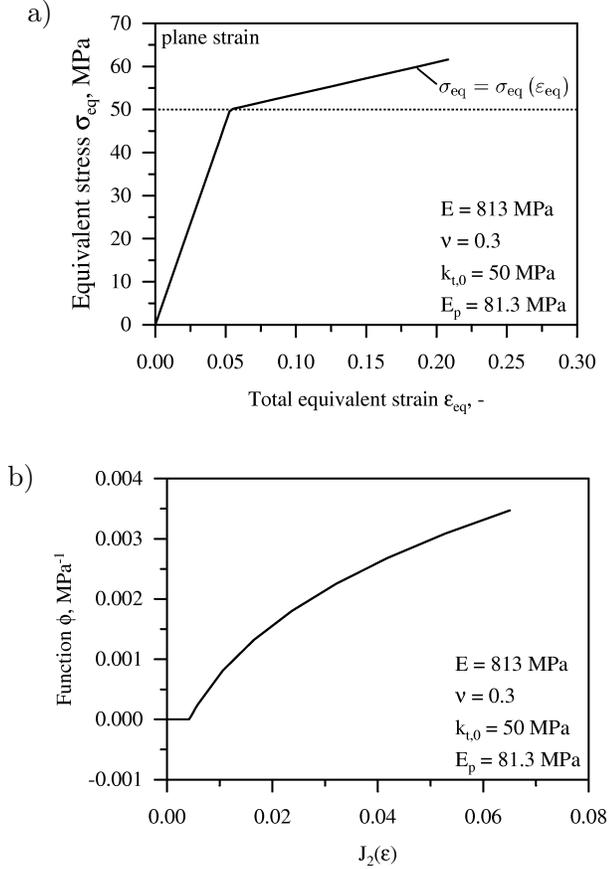


FIG. 3. Evaluation of the function ϕ from plastic flow parameters.

A simple tensile monotonic loading ($u_x(x, H/2) = 0, u_y(x, H/2) = v_y$) is applied at the top of the bimaterial sample in the range from 0% to 0.6% of v_y/H in 100 incremental steps. Due to the symmetry of the loading and the sample geometry, two of the transmission conditions, i.e. $[\sigma_{xy}] = 0$ and $F_x([u_x], [u_y]) = \sigma_{xy}$,

are satisfied identically because of $[u_x] = 0$ and $\sigma_{xy} = 0$ holds in this case. The two remaining conditions $[u_y] = 0$ and $F_y(0, [u_y]) = \sigma_y$ have to be verified. The first one is the same as in the case of the pure elastic imperfect interface [16] and is of less interest in comparison with the second one.

In Fig. 4a), a comparison of the left and right-hand side of the condition $F_y(0, [u_y]) = \sigma_y$ is presented. The traction is represented by the solid line while the values of the left-hand side function are depicted by circles in several points. The visible plastic zone appears in the middle of the interface after 30 increments. The accuracy of the evaluated transmission condition is in the same range as it has been checked for the pure elastic interface [17]. Moreover, the region where the transmission conditions are valid does not change practically, regardless whether the interphase material is in the elastic or plastic region. To illustrate this fact, a magnification of the same functions as in Fig. 4a) is presented in Fig. 4b). The 1% accuracy criterion has been chosen to indicate the validity regions. It is also important to note that the plastic zones which appear

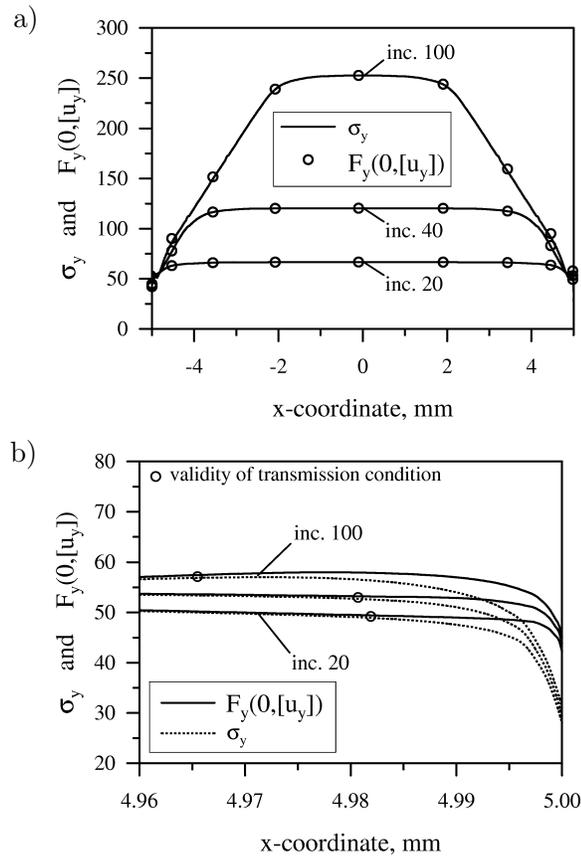


FIG. 4. Validity of the transmission conditions for thin elasto-plastic interface.

near the free edges are very small and therefore invisible in the scale of Fig. 4a). The range of the plastic zone coincides more or less with the range of singularity dominated domains for the elastic interface [17] and starts to be smaller during the plastic deformation.

Additionally to the presented analysis, investigations of possible singularity of the solution for a bimaterial body with a soft imperfect elasto-plastic interface model near the interface crack tip or near free edges should be done. Respective results concerning the pure elastic imperfect interface have been obtained in [1, 14, 15].

One of the crucial points to underline is the fact that the stress-strain state of the 2D bimaterial structure under consideration is not purely monotonic due to definition in [12]. Thus, it would be natural to expect a more significant difference between the numerically and analytically predicted interfacial conditions than it was clarified for the pure elastic interface in [16]. However, as it follows from the results presented in Fig. 4, the accuracy of the transmission conditions for the elasto-plastic interface is much better than one could expect due to the limitations of the deformation theory.

Another important fact which should be mentioned here concerns Remark 2. It may happen for very large plastic deformations that the generalised Poisson's ratio will approach its maximal value of 0.5 and, as a result, the transmission conditions evaluated here should be used with a reservation, as it follows from the results obtained in [18] for the soft, weakly compressible elastic interface. Nevertheless, if Poisson's ratio of the elasto-plastic interphase is sufficiently smaller than 0.5 in the elastic regime, then the transmission conditions evaluated in the paper can be applied in the range of usual plastic deformations. For example, the maximum value of the generalised Poisson's ratio takes in the numerical simulation the value of $\tilde{\nu} = 0.42$ after 100 increments, while $\tilde{\nu}(0) = 0.3$.

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