Arch. Mech., **56**, 2, pp. 87–101, Warszawa 2004

Heat transfer due to permeable stretching. Wall in presence of transverse magnetic field

B. S. DANDAPAT, S. N. SINGH, R. P. SINGH

Physics and Applied Mathematics Unit
Indian Statistical Institute,
03 B. T. Road, Calcutta–700 108, India
e-mail: dandapat@isical.ac.in

Exact similarity solution for the viscous flow due to stretching surface in the presence of magnetic field is derived. Further exact solutions for the temperature distributions in terms of Kummer's functions are obtained. Two cases of heat transfer are considered: the sheet (i) with prescribed surface temperature and (ii) – the prescribed wall heat flux. Both the cases are further extended to study the heat transfer due to suction and injection. A simple relation for the two cases of heat transfer is obtained.

1. Introduction

Flow problem with obvious relevance to polymer extrusion is an interesting area of present-day research. In a melt-spinning process, the extrudate from the die is generally drawn and simultaneously stretched into a filament or sheet, which is thereafter solidified through rapid quenching or gradual cooling by direct contact with water or chilled metal rolls. In fact, stretching imports a unidirectional orientation to the extrudate, thereby improving its mechanical properties and the quality of the final product greatly depends on the rate of cooling. Crane [1] studied the two-dimensional boundary-layer flow caused by the stretching of the sheet which moves in its own plane at a velocity that varies linearly with the distance from the slit. This problem was extended to heat and mass transfer with suction or blowing by Gupta and Gupta [2] who studied the temperature and concentration distributions for isothermal case. Dutta, Roy and Gupta [3] analyzed the temperature distribution in the flow over a stretching sheet with uniform heat flux. Grubka and Bobba [4] studied the heat transfer characteristics of a continuous stretching surface with variable temperature. Further study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid due to the stretching of the sheet is of considerable interest in modern metallurgical and metal-working process. To be more specific, it may be mentioned that many metallurgical processes involve cooling of the continuous strips or filaments by drawing them through a quiescent fluid and that in process of the drawing, these strips are sometimes stretched. Mention may be made of drawing, annealing and thinning of copper wires. In all these cases the properties of the final product depend to a great extent on the rate of cooling. By drawing such strips in an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final products of the desired characteristics might be achieved. Pavlov [5] presented an exact similarity solution of the boundary-layer equation for the steady two-dimensional flow of an electrically conducting incompressible fluid due to stretching of a plane elastic surface in a uniform transverse magnetic field. In this analysis he has neglected the induced magnetic field under the assumption of small magnetic Reynolds number. Andersson [6] has demonstrated that the similarity solution derived by Pavlov [5] is not only a boundary layer equation but also represents an exact solution of the complete Navier–Stokes equation. Chakrabarti and Gupta [7] extended the above analysis of Pavlov to study the temperature distribution for isothermal boundary when a uniform suction is applied at the surface. It would be of interest to study the effects of power-law variations of temperature and heat flux distribution on the heat transfer characteristics of stretching sheet in presence of a uniform transverse magnetic field subject to suction or blowing.

The analysis is carried out in a general form allowing us, in a unified approach, to describe the heat transfer on a stretching sheet in presence of a transverse magnetic field with suction or blowing for different types of thermal boundary conditions on the surface.

2. Formulation of the problem

Consider the flow of an incompressible electrically conducting fluid (with electrical conductivity σ and thermal diffusivity α) due to the stretching of a permeable flat sheet. It is assumed that the speed of a point on the sheet is proportional to its distance from the slit at $x = 0,\ y = 0$. Further we assume that the sheet lies in the $x-z$ plane and is stretched along the $x-$ axis. A uniform transverse magnetic field B_0 acts parallel to the y axis and the conducting fluid occupies the half space $y > 0$. We have also assumed that the sheet is subjected to either (a) prescribed surface temperature or (b) prescribed wall heat flux. The steady velocity field $[u(x,y), v(x,y), 0]$ that developed due to the stretching of the sheet with velocity Ax satisfies the boundary-layer equation of mass, momentum and thermal energy.

$$\partial_x u + \partial_y v = 0, \tag{2.1}$$

$$u\partial_x u + v\partial_y u = \nu\partial_{yy} u - (\sigma B_0^2/\rho)u, \tag{2.2}$$

$$u\partial_x T + v\partial_y T = \alpha\partial_{yy} T, \tag{2.3}$$

where the induced magnetic field is neglected by assuming the flow for small magnetic Reynolds number, as justified by SHERCLIFF [8]. It is also assumed that the external electrical field due to polarization of charges is negligible. Further we assume that ν, the kinematic viscosity of the ambient fluid, is constant and the gravity force gives rise to a hydrostatic pressure variation in the liquid. In order to justify the boundary layer approximation, length scale in the primary flow direction should be significantly larger than the length scale in the cross-stream direction. In fact, the flow takes place within a thin layer of thickness $(\nu/A)^{1/2}$ due to the stretching of the sheet, the scale ratio $x/(\nu/A)^{1/2} \gg 1$. Further it is possible to define a local Reynolds number $\mathrm{Re}_x = Ux/\nu = Ax^2/\nu$ which initially equals the square of the above scale ratio. Thus, just as in aerodynamic boundary layer theory, cross-stream diffusion of momentum and thermal energy can only be neglected at a high Reynolds number. The corresponding boundary conditions are:

$$u(x,0) = Ax, \qquad v(x,0) = V_w(x), \qquad u(x,\infty) = 0, \tag{2.4}$$

$$\text{either} \qquad T_w(x,0) = T_1(x), \qquad T(x,\infty) = T_\infty, \tag{2.5}$$

$$\text{or,} \qquad -k\partial_y T = q_w(x) \qquad \text{for} \quad y = 0, \qquad T(x,\infty) = T_\infty, \tag{2.6}$$

where V_w denotes the lateral mass flux of velocity which occurs due to suction or injection. T_w, T_∞ and q_w denote the temperature at the wall, temperature at a large distance from the wall and heat flux at the wall, respectively. Assuming the functional structure of the similarity solutions in the form :

$$u(x,y) = Axf'(\eta), \tag{2.7}$$

$$v(x,y) = -\left(\nu A\right)^{1/2} f(\eta), \tag{2.8}$$

$$\eta = (A/\nu)^{1/2} y \tag{2.9}$$

and substituting (2.7)–(2.9) in the system of Eqs. (2.1) to (2.3), it can be shown that similarity solution of the above set of boundary layer equations exists and reduces to:

$$f''' + ff'' - (f')^2 - Hf' = 0 \tag{2.10}$$

subject to the boundary conditions

$$f(0) = f_w, \qquad f'(0) = 1, \qquad f'(\infty) = 0, \tag{2.11}$$

where $H(\equiv \sigma B_0^2/(A\rho))$ and $f_w(\equiv -V_w/(\nu A)^{1/2})$ denote the magnetic parameter, suction/injection parameter, respectively and prime ($'$) denotes derivative with respect to the similarity variable η. In Eq. (2.11), $f_w = 0$ corresponds to an impermeable wall, $f_w > 0$ and $f_w < 0$ denote respectively the suction and injection of the fluid through the permeable wall.

3. Exact analytic solutions for $f_w \neq 0$

System of Eqs. (2.10) and (2.11) gives an exact analytical form for the stream function f as:

$$f(\eta) = f_w + (1/\beta)\left[1 - \exp^{(-\beta\eta)}\right], \tag{3.1}$$

where

$$\beta = 1/2\left[f_w + \sqrt{f_w^2 + 4(H+1)}\right]. \tag{3.2}$$

The above form of solution was first obtained by GUPTA and GUPTA [2] for $H = 0$. In absence of the suction/injection velocity f_w ($f_w = 0$), the solution (3.1) exactly agrees with ANDERSSON [6]. The skin friction acting on the stretching surface in contact with the ambient fluid of constant density (ρ) is given by

$$\tau_w = \rho\nu\frac{\partial u}{\partial y}(x,0) = \rho\sqrt{A^3\nu}\;\; x f''(0).$$

Substituting the value of $f''(0)$ deduced from (3.1) and (3.2) we get

$$\tau_w = -\frac{1}{2}\rho\sqrt{A^3\nu}\;\; x\left[f_w + \left\{f_w^2 + 4(H+1)\right\}^{1/2}\right]. \tag{3.3}$$

It is clear from Eq. (3.3) that the skin friction τ_w is always negative irrespective of the sign of f_w. This implies that the stretching wall experiences a drag for any finite value of f_w. Then the drag increases as H increases.

4. Heat transfer

In this section we are interested to study the heat transfer for two different heating processes.

4.1. Prescribed surface temperature

We assign a general functional structure in Eq. (2.5) to prescribe the temperature at the boundary as

$$T_w = T_1(x) = T_\infty + Cx^r \qquad \text{for} \qquad y = 0 \tag{4.1}$$

and

$$T = T_\infty \qquad \text{as} \qquad y \to \infty, \tag{4.2}$$

where r is the temperature parameter and T_∞ denotes temperature at a large distance from the wall and it is constant. For $r = 0$, the thermal boundary will be isothermal. Introducing

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{4.3}$$

in Eq. (2.3) and using (2.9), we get

$$\theta'' + \mathrm{P}_r[f\theta' - rf'\theta] = 0 \tag{4.4}$$

where $\mathrm{P}_r = \nu/\alpha$ is the Prandtl number. The corresponding boundary conditions (2.5) reduce to

$$\theta(0) = 1, \qquad \theta(\infty) = 0. \tag{4.5}$$

Substituting the values of $f(\eta)$ and $f'(\eta)$ from Eqs. (3.1) in (4.4) and using a new variable ζ as

$$\zeta = -(\mathrm{P}_r/\beta^2)\mathrm{exp}^{-\beta\eta}, \tag{4.6}$$

we arrive at

$$\zeta\frac{d^2\theta}{d\zeta^2} + (1 - P - \zeta)\frac{d\theta}{d\zeta} + r\theta = 0, \tag{4.7}$$

where

$$P = \mathrm{P}_r(f_w\beta + 1)/\beta^2. \tag{4.8}$$

The corresponding boundary conditions (4.5) reduce to

$$\theta\left(\frac{-\mathrm{P}_r}{\beta^2}\right) = 1, \qquad \theta(0) = 0. \tag{4.9}$$

The solution of (4.7) satisfying (4.9) can be obtained in terms of Kummer's function (ABRAMOWITZ and STEGUN [9])

$$\theta(\zeta) = [-\beta^2\zeta/\mathrm{P}_r]^P \frac{M(P - r, P + 1, \zeta)}{M(P - r, P + 1, -\mathrm{P}_r/\beta^2)}, \tag{4.10}$$

where $M(a, b, x) = 1 + \sum_{n=1}^{\infty} \frac{a_n x^n}{b_n n!}$, $a_n = a(a + 1)(a + 2)(a + 3)...(a + n - 1)$ and $b_n = b(b+1)(b+2)(b+3)...(b+n-1)$. Equation (4.10) can be expressed in terms of η as

$$\theta(\eta) = [\mathrm{exp}^{-\beta P\eta}]\frac{M(P - r, P + 1, -(\mathrm{P}_r/\beta^2)\,\mathrm{exp}^{(-\beta\eta)})}{M(P - r, P + 1, -\mathrm{P}_r/(\beta^2))}. \tag{4.11}$$

The wall temperature gradient becomes

$$\theta'(0) = -\beta P + \frac{P-r}{P+1}\left(\frac{\mathrm{P_r}}{\beta}\right)\frac{M(P-r+1, P+2, -\mathrm{P}_r/\beta^2)}{M(P-r, P+1, -\mathrm{P}_r/\beta^2)}, \tag{4.12}$$

and the local wall heat flux can be expressed as

$$q_w = -K(\partial_y T)_w = -kC\sqrt{\left(\frac{A}{\nu}\right)}\; x^r \theta'(0). \tag{4.13}$$

The dimensionless surface temperature gradient $\theta'(0)$ is related to the local Nusselt and Reynolds numbers:

$$\mathrm{Nu}_x = \frac{q_w(x)x}{K(T_w(x) - T_\infty)}$$

and

$$\mathrm{Re}_x = \frac{Ux}{\nu} = \frac{Ax^2}{\nu},$$

as

$$\frac{\mathrm{Nu}_x}{\sqrt{\mathrm{Re}_x}} = -\theta'(0). \tag{4.14}$$

A few special cases may be deduced depending on the relative values of r and P.

4.1.1. Case of $r = P$. We have

$$\theta(\eta) = \exp^{-\beta P \eta} \tag{4.15}$$

and

$$\theta'(0) = -\beta P. \tag{4.16}$$

$\theta'(0) < 0$ implies that the heat flows from the stretching surface to the ambient fluid. We like to add a few comments regarding the physical validity of the temperature fields. It is clear from Eq. (4.10) that the temperature field may become positive or negative, depending on the values of the Kummer's function at different ranges of r and P. It is therefore obvious for a physically consistent result that the parameter range should be such that the corresponding θ is everywhere finite and non-negative, i.e.

$$0 \le \theta(\eta; r, P) < \infty \qquad \text{for any} \qquad 0 \le \eta \le \infty. \tag{4.17}$$

A negative θ is unrealistic since according to Eq. (4.3) we have

$$T = T_\infty + Cx^r\theta. \tag{4.18}$$

Now for θ to be negative for any region of η, the steady temperature $T(x, y)$ of the ambient fluid would become smaller than T_∞ if $C > 0$ and greater than T_∞ if $C < 0$; in spite of the fact that, in accordance with Eq. (4.1), the wall temperature is everywhere higher for $C > 0$ and everywhere lower than T_∞ for $C < 0$. Such situations are physically impossible. Therefore for a physically consistent result one must have

$$P \geq r. \tag{4.19}$$

4.1.2. Case of $r = 0$ and $P \neq 0$. We have

$$\theta(\eta) = \gamma(P, (\mathrm{P}_r e^{-\beta\eta}/\beta^2))/\gamma(P, \mathrm{P}_r/\beta^2)$$

and

$$\theta'(0) = -\beta(\mathrm{P}_r/\beta^2)^P e^{(-\mathrm{P}_r/\beta^2)}/\gamma(P, \mathrm{P}_r/\beta^2),$$

where γ is incomplete gamma function. This result coincides with GUPTA and GUPTA [2] for isothermal case.

4.1.3. Case of $r = -1$ and $P > 0$. We have

$$\theta(\eta) = \exp[\mathrm{P}_r(1 - [f_w\beta + 1]\beta\eta - e^{(-\beta\eta)}/\beta^2] \quad \text{and} \quad \theta'(0) = -\mathrm{P}_r f_w < 0.$$

4.2. Prescribed wall heat flux

In this case the thermal boundary conditions will be

$$-k\partial_y T = q_w = Dx^s \qquad \text{for} \qquad y = 0 \tag{4.20}$$

and

$$T = T_\infty \qquad \text{as} \qquad y \to \infty, \tag{4.21}$$

where s is the heat flux parameter. For $s = 0$ the stretching sheet is under uniform heat flux. We assume the similar solution as

$$T = T_\infty + \frac{Dx^s}{K}\sqrt{\nu/A}\;\; g(\eta) \tag{4.22}$$

where $\eta = \sqrt{A/\nu}\;\; y$. Using Eqs. (3.1) and (4.22) in (2.3), we get

$$g'' + \mathrm{P}_r\left[f_w + (1 - \exp^{-\beta\eta})/(\beta)\right]g' - \mathrm{P}_r \exp^{-\beta\eta} sg = 0. \tag{4.23}$$

The corresponding boundary conditions (4.20) and (4.21) will reduce to

$$g'(0) = -1 \qquad \text{and} \qquad g(\infty) = 0. \tag{4.24}$$

Introducing (4.6) in (4.23) and (2.3), we have

$$\zeta \frac{d^2 g}{d\zeta^2} + (1 - P - \zeta)\frac{dg}{d\zeta} + sg = 0, \tag{4.25}$$

$$g'(-\mathrm{P}_r/\beta^2) = -\beta/\mathrm{P}_r, \qquad g(0) = 0, \tag{4.26}$$

where P is defined in Eq. (4.8). Similarity solution for (4.25) and (4.26) can also be expressed in terms of Kummer's function

$$g(\eta) = \left(\frac{1}{\beta P}\right) \exp^{-\beta P \eta} \frac{M(P - s, P + 1, (-\mathrm{P}_r/\beta^2)\exp^{-\beta\eta})}{M(P - s, P, -\mathrm{P}_r/\beta^2)}. \tag{4.27}$$

The wall temperature T_w can be obtained as

$$T_w - T_\infty = \frac{Dx^s}{k}\sqrt{(\nu/A)}\;\; g(0). \tag{4.28}$$

For a consistent and physically realistic temperature field one must have

$$P \geq s. \tag{4.29}$$

Again several closed-form solutions can be obtained from (41) for specific values of s and P and these solutions are presented below:

- for $s = P \neq 0$,

$$g(\eta) = \exp[-\beta P \eta]/(\beta P)$$

and

- for $s = 0$, $P \neq 0$

$$g(\eta) = \beta^{-1}\left(\frac{\beta^2}{\mathrm{P}_r}\right)^P \exp\left(\frac{\mathrm{P}_r}{\beta^2}\right)\gamma(P, \mathrm{P}_r e^{-\beta\eta}/\beta^2).$$

5. Results and discussion

To get insight into the problem, a few figures are drawn by evaluating Eqs. (3.1), (4.11), (4.12) and (4.27) for different values of the parameters. Figure 1 represents the variation of $f'(\eta)$ with respect to η for different values of suction, injection and magnetic parameters. It is clear that as suction parameter increasing (curves b and e), $f'(\eta)$ decreases while for increasing of injection parameters (curves c and f), $f'(\eta)$ increases. Further it is clear that when magnetic parameter increases either for suction (curves a and b) or for injection (curves c and d), $f'(\eta)$ decreases for all values of η. This is due to the fact that the increase of H implies the increase of Lorentz force which puts greater resistance to the flow, and as a result the flow decelerates. For prescribed surface temperature case, Eqs. (4.11) and (4.12) are evaluated for different values of P_r, H, f_w and r. The results are presented in Figs. 2, 3, 4 and 5. Figure 2 represents the variation of temperature distribution with η for different values of P_r, H and r in the case of suction. Figures 3 and 4 show the temperature variation with η for injection when P_r, H and r vary. It is clear from Fig. 2 (in case of suction) that the temperature θ at a given point decreases if the Prandt number (P_r), suction (f_w) and r the temperature parameter increase while θ increases with the magnetic parameter H. But for case of injection, as depicted in Fig. 3, it shows that the temperature θ at a given point decreases with the increase of P_r and r for all $r > 0$, where as θ increases with the increase of magnetic parameter H, injection parameter $(-f_w)$ and r for all $r > 0$. Figure 4 shows that, for $r > 0$, heat flows from the stretching surface to the ambient fluid. The magnitude of the temperature gradient increases as r increases (curve c and d). Further for $r = -2$, the wall temperature gradient is positive and heat flows into the stretching surface from the ambient fluid. It is to be mentioned here that heat always flows from

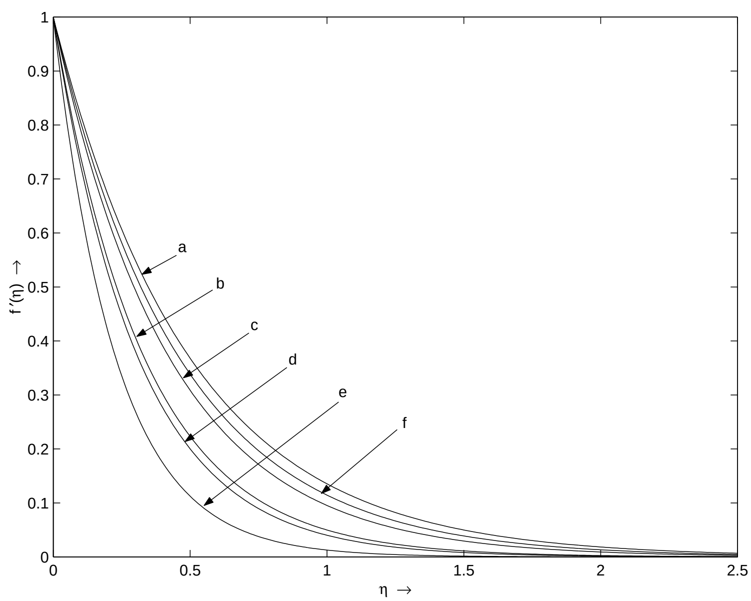

FIG. 1. Variation of $f'(\eta)$ with η for different values of suction, injection and magnetic parameters, (a) $f_w = 1$, $H = 1$; (b) $f_w = 1$, $H = 5$; (c) $f_w = -.2$, $H = 5$; (d) $f_w = -.2$, $H = 10$; (e) $f_w = 2$, $H = 5$; (f) $f_w = -.8$, $H = 5$.

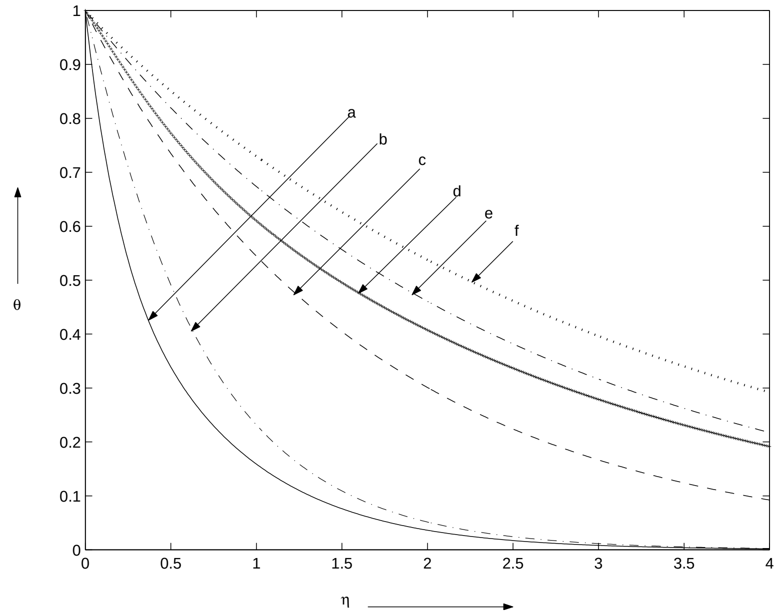

Fig. 2. Variation of θ with η for different values of f_w, H, P_r and r (a) $f_w = 1$, $H = 1$, $P_r = 1$, $r = -1$; (b) $f_w = 1$, $H = 1$, $P_r = 1$, $r = 1$; (c) $f_w = 2$, $H = 1$, $P_r = .25$, $r = 1$; (d) $f_w = 1$, $H = 1$, $P_r = .25$, $r = 5$; (e) $f_w = 1$, $H = 1$, $P_r = .25$, $r = 1$; (f) $f_w = 1$, $H = 15$, $P_r = .25$, $r = 1$.

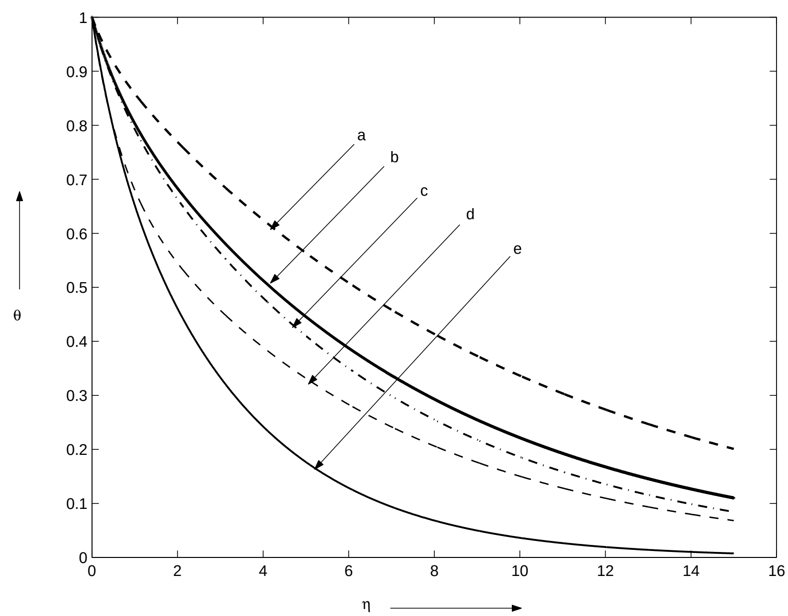

Fig. 3. Variation of θ with η for different values of f_w, H, P_r and r (a) $f_w = -.1$, $H = 3$, $P_r = .25$, $r = 1$; (b) $f_w = -.2$, $H = 1$, $P_r = .25$, $r = 1$; (c) $f_w = -.1$, $H = 1$, $P_r = .25$, $r = 1$; (d) $f_w = -.1$, $H = 1$, $P_r = .25$, $r = 3$; (e) $f_w = -.1$, $H = 1$, $P_r = .5$, $r = 1$.

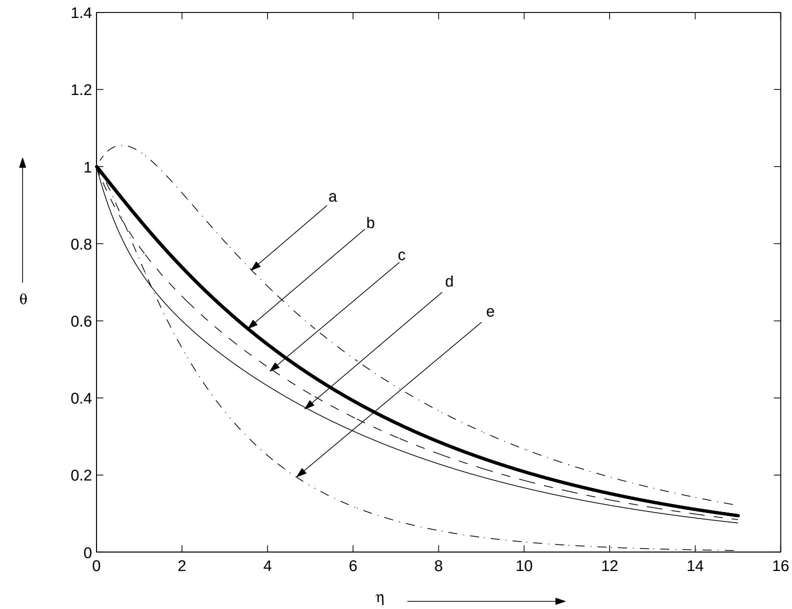

FIG. 4. Variation of θ with η for different values of f_w, H, P_r and r (a) $f_w = -.1$, $H = 1$, $\mathrm{P}_r = .25$, $r = -2$; (b) $f_w = -.1$, $H = 1$, $\mathrm{P}_r = .25$, $r = 0$; (c) $f_w = -.1$, $H = 1$, $\mathrm{P}_r = .25$, $r = 1$; (d) $f_w = -.1$, $H = 1$, $\mathrm{P}_r = .25$, $r = 2$; (e) $f_w = 1$, $H = 1$, $\mathrm{P}_r = .25$, $r = -2$.

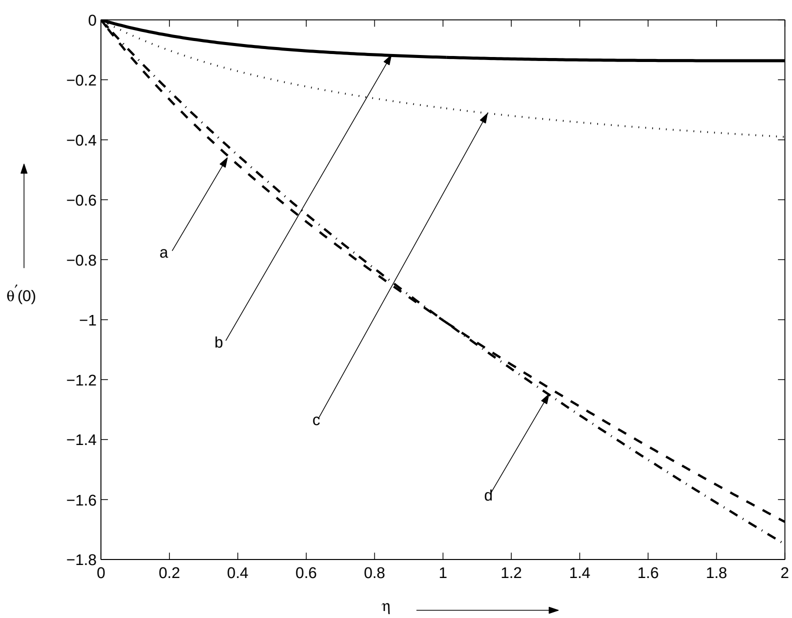

FIG. 5. Variation of $\theta'(0)$ with η for different values of f_w, H and r (a) $f_w = 1$, $H = 1$, $r = 1$; (b) $f_w = -.5$, $H = 1$, $r = 1$; (c) $f_w = -.1$, $H = 1$, $r = 1$; (d) $f_w = 1$, $H = 15$, $r = 1$.

the stretching surface to the ambient fluid in case of suction. One can also see from Fig. 4 that zero temperature gradient occurs (curve a) implying heat flows into the thermal boundary layer from both the stretching sheet and the ambient fluid. The wall temperature gradient $\theta'(0)$ is plotted as a function of P_r for selected values of suction/blowing parameter, magnetic parameter H and r in Fig. 5. It is clear from Fig. 5 that $\theta'(0)$ becomes more negative for suction than injection. Further one can see that the wall temperature gradient $\theta'(0)$ is negative for $r > 0$. For the prescribed wall heat flux case, the temperature distribution obtained in (41) is plotted in Figs. 6 and 7 for two different cases of suction and blowing respectively, when other parameters P_r, H and s change. It is clear from Fig. 6 that g at a point decreases as s, f_w or P_r increases, while increase of H produces increase of g. Figure 7 shows the temperature variation g with η for blowing. It is evident that g increases with the increase of the blowing parameter f_w or H, while g decreases when P_r or s increases. It is to be pointed out here that the dimensionless temperature distribution $\theta = (T - T_\infty)/(T_w - T_\infty)$ is equal to the ratio of $g(\eta)$ and $g(0)$. This shows that

$$\theta(\eta) = \frac{g(\eta)}{g(0)} \tag{5.1}$$

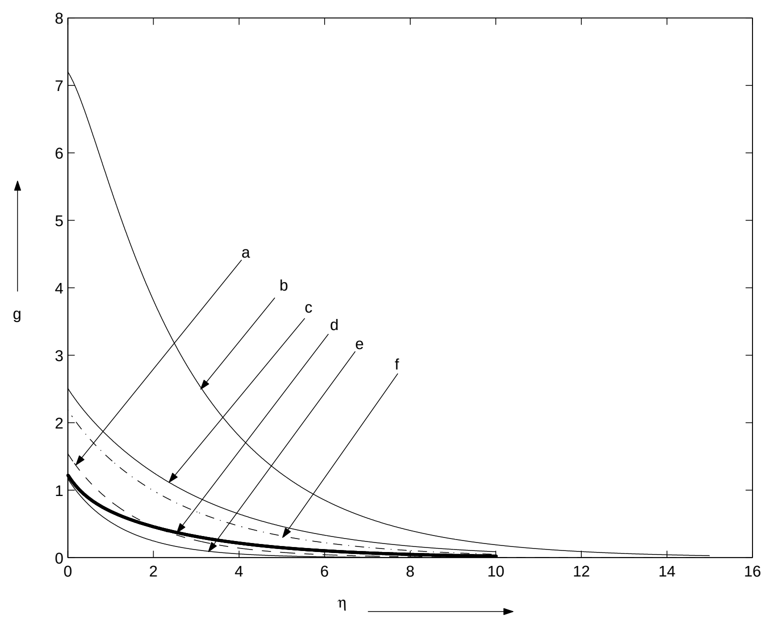

FIG. 6. Variation of g with η for different values of f_w, H, P_r and s (a) $f_w = 2$, $H = 1$, $\mathrm{P}_r = .25$, $s = 1$; (b) $f_w = 1$, $H = 1$, $\mathrm{P}_r = .25$, $s = -2$; (c) $f_w = 1$, $H = 5$, $\mathrm{P}_r = .25$, $s = 1$; (d) $f_w = 1$, $H = 1$, $\mathrm{P}_r = .25$, $s = 5$; (e) $f_w = 1$, $H = 1$, $\mathrm{P}_r = .5$, $s = 1$; (f) $f_w = 1$, $H = 1$, $\mathrm{P}_r = .25$, $s = 1$.

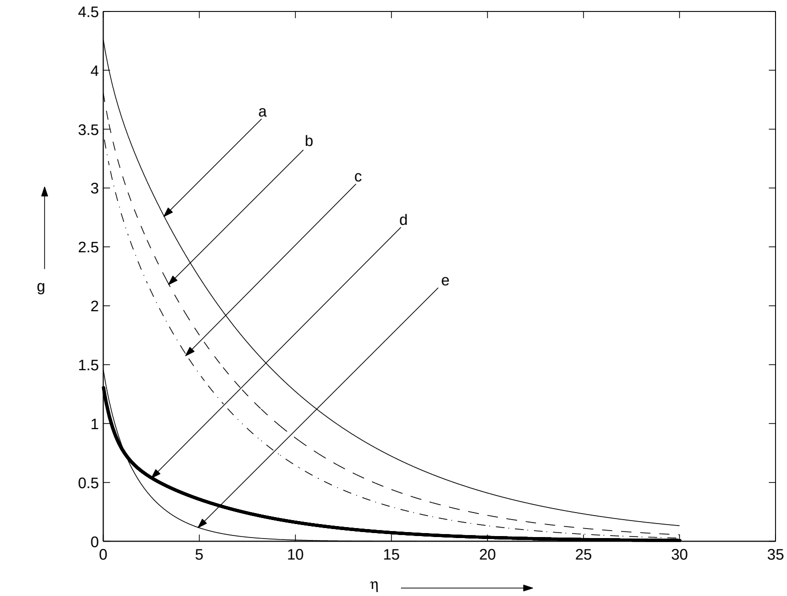

FIG. 7. Variation of g with η for different values of f_w, H, P_r and s (a) $f_w = -.15$, $H = 2$, $\mathrm{P}_r = .25$, $s = 1$; (b) $f_w = -.1$, $H = 2$, $\mathrm{P}_r = .25$, $s = 1$; (c) $f_w = -.1$, $H = 1$, $\mathrm{P}_r = .25$, $s = 1$; (d) $f_w = -.1$, $H = 1$, $\mathrm{P}_r = .25$, $s = 5$; (e) $f_w = -.1$, $H = 1$, $\mathrm{P}_r = .75$, $s = 1$.

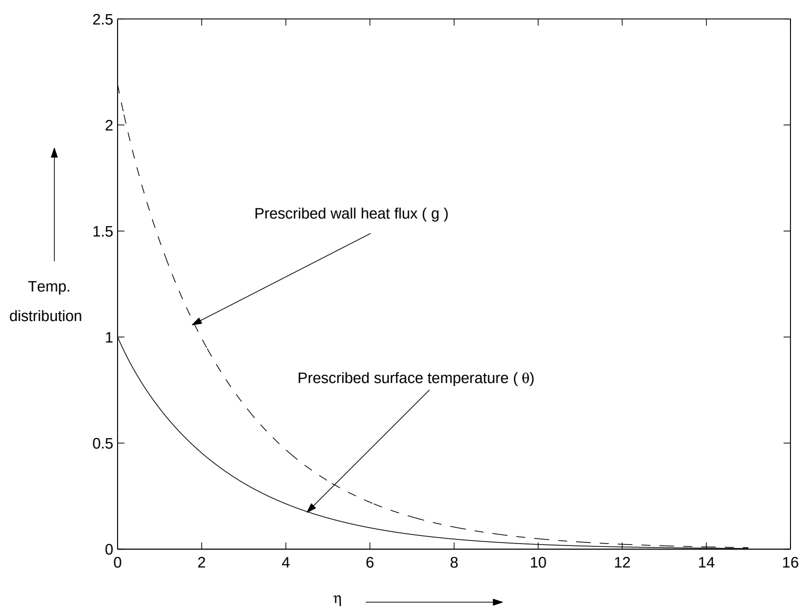

FIG. 8. Variation of temperature distribution with η for prescribed surface temperature and wall heat flux (suction) (a) $f_w = 1$, $H = 1$, $\mathrm{P}_r = .25$, $r = 1$; (b) $f_w = 1$, $H = 1$, $\mathrm{P}_r = .25$, $s = 1$.

if only r is replaced by s. Through analogy with Fig. 4, one can say that the wall temperature gradient for prescribed wall heat flux condition will be negative for $s = 0, 1$ and 2. This implies that heat flows from the stretching surface to the ambient fluid. For $s = -2$, the sign of temperature gradient changes but the value of $g(0)$ will be negative indicating the heat flux at the sheet flows into the fluid. Further we conclude from (4.21) that the nature of the temperature distribution in both the cases will be same except for a shift by a constant $g(0)$. This result can be seen in Figs. 8 and 9 representing the comparison of temperature distribution for both the suction and blowing when the prescribed surface temperature θ and prescribed wall heat flux g are considered.

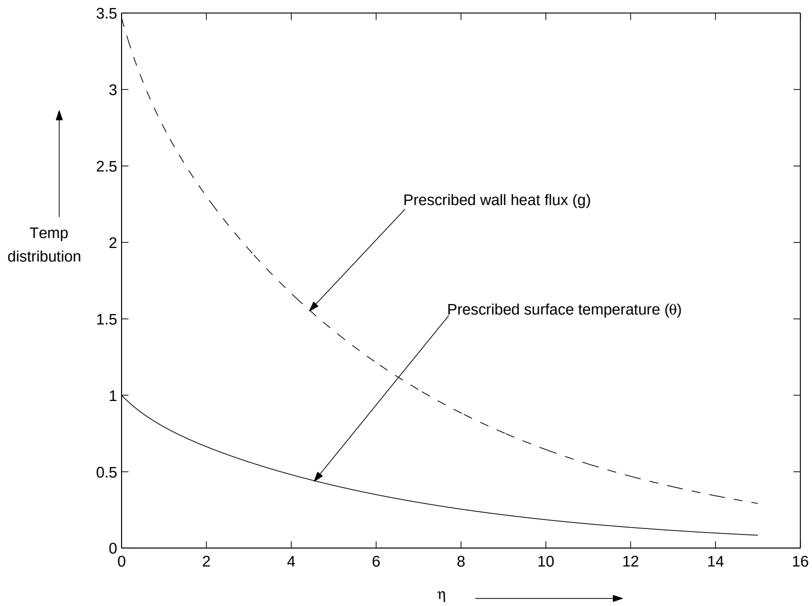

FIG. 9. Variation of temperature distribution with η for prescribed surface temperature and wall heat flux (injection) (a) $f_w = -.1$, $H = 1$, $P_r = .25$, $r = 1$; (b) $f_w = -.1$, $H = 1$, $P_r = .25$, $s = 1$.

Acknowledgments

The authors are thankful to the referee's constructive suggestions to improve the quality of the present paper.

References

1. L. E. CRANE, *Flow past a stretching plane*, Z. A. M. P., **21**, 645–647, 1970.
2. P. S. GUPTA and A. S. GUPTA, *Heat and mass transfer on a stretching sheet with suction or blowing*, Can. J. Chem. Eng., **55**, 744–746, 1977.

3. B. K. Dutta, P. Roy and A. S. Gupta, *Temperature field in flow over a stretching sheet with uniform heat flux*, Int. commun., Heat Mass Transfer, **12**, 89–94, 1985.

4. L. J. Grubka and K. M. Bobba, *Heat transfer characteristics of a continuous stretching surface with variable temperature*, J. Heat Trans., -T ASME, **107**, 248–250, 1985.

5. K. B. Pavlov, *Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a plane surface*, Magnitnaya Gidrodinamica, **4**, 146–147, 1974.

6. H. I. Andersson, *An exact solution of the Navier–Stokes equation for magnetohydrodynamic flow*, Acta Mechanica, **113**, 241–244, 1995.

7. A. Chakrabarti and A. S. Gupta, *Hydromagnetic flow and heat transfer over a stretched sheet*, Quart. Appl. Math., **37**, 73–78, 1979.

8. J. A. Shercliff, *A textbook of magnetohydrodynamics*, Pergamon Press, 1965.

9. M. Abramowitz and I. A. Stegun, *Handbook of mathematics functions*, Dover, New York 1965.

Received April 9, 2003; revised version December 23, 2003.