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Maysel's formula in the generalized linear micropolar thermoviscoelasticity

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Generalizations of Maysel's formula to generalized linear micropolar thermoviscoelasticity is given. Fundamental solutions in the Laplace transform domain are obtained. The results are applicable to the following generalized thermoelasticity theories: Lord–Shulman theory with one relaxation time, Green–Lindsay theory with two relaxation times, Green–Naghdi theory of type III, and the Chandrasekharaiah and Tzou theory with dual-phase lag, as well as to the dynamic coupled theory. The cases of generalized linear micropolar thermoviscoelasticity of the Kelvin–Voigt model, and the generalized linear micropolar thermoelasticity can be obtained from the given results.

Key words: Maysel's formula, generalized micropolar thermoviscoelasticity, viscoelasticity, generalized thermoelasticity, green's functions, fundamental solutions.

Notations

u_i	components of displacement vector,
σ_{ij}	components of force stress tensor,
m_{ij}	components of couple stress tensor,
e_{ij}	components of strain tensor,
ε_{ij}	components of micro-strain tensor,
r_i	components of rotation vector,
ω_i	components of micro-rotation vector,
$e = e_{kk} = \varepsilon_{kk}$	dilatation,
M_i	mass couple vector,
F_i	mass force vector,
ρ	density,
J	micro-inertia coefficient,
δ_{ij}	Kronecker delta,
$\in_{ijk}$	permutation tensor,
a	coefficient of linear thermal expansion,
t	time,
$\lambda, \mu, k, \alpha, \beta, \gamma$	elastic coefficients,
$\hat{\gamma}$	$= (3\lambda + 2\mu + k)a$,
$R_\xi(t), (\,\xi = \lambda, \mu, k, \alpha, \beta, \gamma)$	relaxation functions,

ω $= \omega_{ii} = \omega_{i,i}$,
τ_q phase-lag of heat flux,
τ_θ phase-lag of temperature gradient,
T absolute temperature,
T_0 reference temperature chosen so that $\dfrac{|T - T_0|}{T_0} \ll 1$,
Θ $= T - T_0$,
$\overset{\wedge}{\Theta}$ $= \Theta + \nu\,\dot{\Theta}$,
K thermal conductivity,
C_E specific heat at constant strain,
Q intensity of applied heat source per unit mass;
$n^*, n_1, n_0, t_1, t_2, \nu, \tau_0$ constants,
λ_v retardation period of the Kelvin–Voigt model.

1. Introduction

The general theory of linear and nonlinear micropolar continuum mechanics was given by ERINGEN and SUHUBI [1, 2], ERINGEN [3, 4]. It was extended to include thermal effects by NOWACKI [5], ERINGEN [6, 7], TAUCHERT *et al.* [8], TAUCHERT [9] and NOWACKI and OLSZAK [10]. One can refer to DHALIWAL and SINGH [11] for a review on the micropolar thermoelasticity and a historical survey of the subject, as well as to the "continuum physics" series by ERINGEN and KAFADAR [12] in which the general theory of micromorphic media has been summed up. The micropolar viscoelasticty theory was investigated by many authors (e.g. ERINGEN [13]).

BIOT [14] formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for both theories are of parabolic type, predicting infinite speeds of propagation for heat waves contrary to physical observations. HETNARSKI and IGNACZAK in their survey article [15] examined five generalizations to the coupled theory and obtained a number of important analytical results. The first generalization is due to LORD and SHULMAN [16] (L–S theory). The second generalization to the coupled theory is known as the generalized theory with two relaxation times (G–L theory) [17]. One can refer to IGNACZAK [18] for a review, presentation of the two theories and some important results obtained in this field. The third generalization to the coupled theory is known as the thermoelasticity without energy dissipation, proposed by GREEN and NAGHDI [19] (G–N theory *of type II*). The so-called Green–Naghdi theory *of type III*, can be derived from GREEN and NAGHDI [20, 21]. The fourth Generalization is the low temperature thermoelastic model due to HETNARSKI and IGNACZAK (the H–I theory), which is characterized by a system of nonlinear field equations. The fifth generalization to the coupled theory is known as the dual-phase-lag thermoelasticity, proposed by CHANDRASEKHARAIAH and TZOU [22] (C–T theory), which can be considered as an extension of the L–S theory [15].

The technique frequently used in isothermal elasticity [23], known as Betti's method, has been extended to thermoelasticity by V.M. MAYSEL [24], who deduced on the basis of reciprocity theorem a method of integration of the boundary value problems of thermoelasticity. The Maysel formula to determine the displacements $u_j(\overline{x})$ in a body D, due to the action of a steady temperature field T, has the form $u_j(\mathbf{x}) = a \int\limits_D T(\mathbf{y})\sigma_{kk}^{(j)}(\mathbf{y},\mathbf{x})dV(\mathbf{y})$, where $\sigma_{kk}^{(j)}$ is the sum of normal stresses, at the point $\overline{y}$ of the elastic body in the isothermal state $(T = 0)$, due to the action of a concentrated unit force located at the point $\mathbf{x}$ in the direction of the x_j-axis.

Maysel's formula (published in Russian) became known to a wider audience through Nowacki's famous monograph [25], where it was used also for the quasi-static problems. Extensions of Maysel's method to quasi-static problems for viscoelastic bodies were given by NOWACKI [26]. Nowacki generalized Maysel's formula to the dynamic coupled thermoelasticity [27], to uncoupled micropolar thermoelasticity [28] and obtained the Green functions for the micropolar thermoelasticity [29]. Maysel's method is used extensively in the theory of plates and shells. One can refer to Franz ZIEGLER and Hans IRSCHIK [30] for the methods of solutions in thermoelasticity, based on Maysel's formula and its implementation in the direct boundary integral equation methods.

In the present work, the mathematical model of generalized linear micropolar thermoviscoelasticity is given. Generalizations of Maysel's formula to the given model are established. Fundamental solutions in Laplace transform domain are obtained.

2. Mathematical model

Assume a linear micropolar thermoviscoelastic material occupies a regular region D with a smooth boundary surface B in the three-dimensional Euclidian space. The material is assumed to be microisotropic and isotropic. In this paper, a rectangular coordinate system (x_1, x_2, x_3) is employed. $\mathbf{x}$ is the position vector and t the time. All the functions are considered to be functions of $(\mathbf{x}, t)$, defined on $\overline{D}(= D \cup B) \times [0, \infty)$. A superposed dot denotes differentiation with respect to time, while a comma denotes partial differentiation with respect to the space variables x_i. The summation notation is used. The system of governing equations of a linear micropolar thermoviscoelastic solid [5, 10, 13, 31] consists of:

- Equations of motion (on $D \times (0, \infty)$)

$$\sigma_{ji,j} + \rho F_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \qquad \in_{ijp} \ \sigma_{jp} + m_{ji,j} + \rho M_i = J\rho \frac{\partial^2 \omega_i}{\partial t^2}. \tag{2.1}$$

- Kinematic relations (on $D \times (0,\infty)$)

$$\varepsilon_{ij} = e_{ij} - \in_{ijp} (r_p - \omega_p), \qquad e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad r_i = \frac{1}{2} \in_{ipq} u_{q,p}. \tag{2.2}$$

- Constitutive laws (on $\overline{D} \times (0,\infty)$)

$$\sigma_{ij} = \lambda \breve{R}_\lambda(e)\delta_{ij} + (2\mu\breve{R}_\mu + k\breve{R}_k)(e_{ij}) + k\breve{R}_k(\in_{ijp} (r_p - \omega_p)) - (3\lambda\breve{R}_\lambda + 2\mu\breve{R}_\mu + k\breve{R}_k)(a\hat{\Theta})\sigma_{ij}, \tag{2.3}$$

$$m_{ij} = \alpha\breve{R}_\alpha(\omega)\delta_{ij} + \beta\breve{R}_\beta(\omega_{i,j}) + \gamma\breve{R}_\gamma(\omega_{j,i}), \tag{2.4}$$

where the operator $\breve{R}_\xi(f)$, $(\xi = \lambda, \mu, k, \alpha, \beta, \gamma)$ is defined for any function $f(\mathbf{x}, t)$ of class C^1, as

$$\breve{R}_\xi(f) = \breve{R}_\xi(f(\mathbf{x}, t)) = \int_0^t R_\xi(t-\tau)\, \frac{\partial f(\mathbf{x}, t)}{\partial \tau}\, d\tau \tag{2.5}$$

and where $R_\xi(t)$ are six relaxation functions.

Using the kinematic relations, Eq. (2.3) takes the form

$$\sigma_{ji} = \lambda\breve{R}_\lambda(u_{p,p})\delta_{ij} + (\mu\breve{R}_\mu + k\breve{R}_k)(u_{i,j}) + \mu\breve{R}_\mu(u_{j,i}) + k\breve{R}_k(\in_{ijp} \omega_p) - (3\lambda\breve{R}_\lambda + 2\mu\breve{R}_\mu + k\breve{R}_k)(a\hat{\Theta})\delta_{ij}. \tag{2.6}$$

From Eqs. (2.1)–(2.4) we get

$$(\lambda\breve{R}_\lambda + \mu\breve{R}_\mu)(u_{j,ji}) + (\mu\breve{R}_\mu + k\breve{R}_k)(u_{i,jj}) + k\breve{R}_k(\in_{ijp} \omega_{p,j}) - (3\lambda\breve{R}_\lambda + 2\mu\breve{R}_\mu + k\breve{R}_k)(a\hat{\Theta}_{,i}) = \rho(\ddot{u}_i - F_i), \tag{2.7}$$

$$(\alpha\breve{R}_\alpha + \beta\breve{R}_\beta)(\omega_{j,ji}) + \gamma\breve{R}_\gamma(\omega_{i,jj}) + k\breve{R}_k(\in_{ijp} u_{p,j}) - 2k\breve{R}_k(\omega_i) = \rho(J\ddot{\omega}_i - M_i). \tag{2.8}$$

The heat equation (on $D \times (0,\infty)$)

$$K\left(n^* + t_1\frac{\partial}{\partial t}\right)\Theta_{,ii} = \rho C_E(n_1\dot{\Theta} + \tau_0\ddot{\Theta} + t_2^2\dddot{\Theta}) + T_0 a(3\lambda\breve{R}_\lambda + 2\mu\breve{R}_\mu + k\breve{R}_k)(n_1\dot{e} + n_0\tau_0\,\ddot{e} + t_2^2\dddot{e}) - (n_1 Q + n_0\tau_0\dot{Q} + t_2^2\ddot{Q}). \tag{2.9}$$

Equations (2.7)–(2.9) are the *field equations (on $D\times(0,\infty)$) of the generalized* linear micropolar thermoviscoelasticity, *applicable to the coupled theory in four generalizations, and to several special cases as follows:*

1. The equations of the coupled linear micropolar thermoviscoelasticity, when

$$n^* = n_1 = 1, \qquad t_1 = t_2 = \tau_0 = \nu = 0, \qquad n_0\tau_0 = 0. \tag{2.10}$$

2. The equations of the generalized linear micropolar thermoviscoelasticity with one relaxation time (L–S theory), when

$$n^* = n_1 = 1, \qquad n_0 = 1, \qquad t_1 = t_2 = \nu = 0, \qquad \tau_0 > 0, \tag{2.11}$$

where τ_0 is relaxation time.

3. The equations of the generalized linear micropolar thermoviscoelasticity with two relaxation times (G–L theory), when

$$n^* = n_1 = 1, \qquad n_0 = 0, \qquad t_1 = t_2 = 0, \qquad \nu \geq \tau_0 > 0, \tag{2.12}$$

where ν and τ_0 are two relaxation times.

4. The equations of the generalized linear micropolar thermoviscoelasticity in case of the linearized G–N theory *of type III*, when

$$n^* > 0, \qquad n_1 = 0, \qquad n_0 = 1, \qquad t_1 = 1, \qquad t_2 = \nu = 0, \qquad \tau_0 = 1. \tag{2.13}$$

Here $n^* = \text{const}$ has the dimension of [1/sec], and ($n^*K = K^*$) is a characteristic constant of this theory. It is worth noting that the linearized G–N theory *of type I*, reduces to the parabolic heat equation, and only the theory of *type II* involves no energy dissipation [19].

5. The equations of the generalized linear micropolar thermoviscoelasticity with dual phase-lag (C–T theory), when

$$\begin{gathered} n^* = n_1 = n_0 = 1, \qquad t_1 = \tau_\theta > 0, \qquad \tau_0 = \tau_q > 0, \\ t_2^2 = \frac{1}{2}\tau_q^2, \qquad \nu = 0, \qquad \tau_q > 0, \qquad \tau_\theta > 0. \end{gathered} \tag{2.14}$$

6. The equations of the generalized linear micropolar thermoviscoelasticity of Kelvin–Voigt model can be obtained from the above equations by replacing the operator $\breve{R}_\xi(f)$ with

$$R_\xi^{(v)}(f(\mathbf{x},t)) = \left(1 + \lambda_v \frac{\partial}{\partial t}\right) f(\mathbf{x},t), \tag{2.15}$$

where $\lambda_v > 0$ is the retardation period of the Kelvin–Voigt model [32].

7. The equations of the generalized linear micropolar thermoelasticity can be obtained from the equations (2.3), (2.4), (2.6)–(2.14) by replacing the operator $\breve{R}_\xi(f)$ with the function $f(\mathbf{x},t)$. The heat equation (2.9) in this case takes the form

$$K\left(n^* + t_1\frac{\partial}{\partial t}\right)\Theta_{,ii} = \rho C_E\left(n_1\dot{\Theta} + \tau_0\ddot{\Theta} + t_2^2\dddot{\Theta}\right) + T_0\hat{\gamma}\left(n_1\dot{e} + n_0\tau_0\,\ddot{e} + t_2^2\dddot{e}\right) - \left(n_1 Q + n_0\tau_0\dot{Q} + t_2^2\ddot{Q}\right). \tag{2.16}$$

For example, the heat equation of the generalized linear micropolar thermoelasticity without energy dissipation (the linearized G–N theory *of type II*), can be obtained from Eq. (2.16) when

$$n^* > 0, \qquad n_1 = 0, \qquad n_0 = 1, \qquad t_1 = t_2 = \nu = 0, \qquad \tau_0 = 1. \tag{2.17}$$

8. The corresponding equations of the generalized linear thermoviscoelasticity can be obtained from the above system by setting $k = 0$, $\omega_i = 0$, $M_i = 0$.

9. The corresponding equations of the generalized linear thermoviscoelasticity of the Kelvin–Voigt model can be obtained from Eqs. (2.3), (2.4), (2.6)–(2.14) by setting $k = 0$, $\omega_i = 0$, $M_i = 0$ and replacing the operator (2.5) by the operator (2.15).

10. The corresponding equations of the generalized linear thermoelasticity can be obtained from the equations (2.3), (2.4), (2.6)–(2.14) by replacing $\breve{R}_\xi(f)$ by f, and setting $k = 0$, $\omega_i = 0$, $M_i = 0$.

The system of equations (2.7)–(2.9) is completed by the initial and boundary conditions.

The Initial conditions will be assumed homogeneous

$$u_i(\mathbf{x},t) = 0, \qquad \omega_i(\mathbf{x},t) = 0, \qquad \Theta(\mathbf{x},t) = 0, \qquad \mathbf{x} \in \overline{D}, \qquad t \le 0, \tag{2.18}$$

$$\frac{\partial^n u_i(\mathbf{x},t)}{\partial t^n} = 0, \qquad \frac{\partial^n \omega_i(\mathbf{x},t)}{\partial t^n} = 0, \qquad \frac{\partial^n \Theta(\mathbf{x},t)}{\partial t^n} = 0, \qquad \mathbf{x} \in \overline{D}, \quad t \le 0, \quad (n \ge 1). \tag{2.19}$$

The boundary conditions

$$\begin{aligned} \sigma_{ji}n_j &= f_i(\mathbf{x}_\sigma, t) \quad \text{on} \quad B_\sigma \times (0,\infty); \\ u_i &= g_i(\mathbf{x}_{B_u}, t) \quad \text{on} \quad B_u \times (0,\infty), \end{aligned} \tag{2.20}$$

$$\begin{aligned} m_{ji}n_j &= \Gamma_i(\mathbf{x}_{B_m}, t) \quad \text{on} \quad B_m \times (0,\infty); \\ \omega_i &= \Xi_i(\mathbf{x}_{B_\omega}, t) \quad \text{on} \quad B_\omega \times (0,\infty), \end{aligned} \tag{2.21}$$

$$\Theta = \Phi(\mathbf{x}_{B_1}, t) \quad \text{on} \quad B_1 \times (0,\infty); \quad \Theta_{,n} = G(\mathbf{x}_{B_2}, t) \quad \text{on} \quad B_2 \times (0,\infty), \tag{2.22}$$

where the functions f_i, g_i, Γ_i, Ξ_i, Φ and G are given functions, equal to zero when $t \le 0$. (B_u, B_σ), (B_ω, B_m) and (B_1, B_2) are three partitions of the boundary surface B such that $B = B_u \cup B_\sigma = B_\omega \cup B_m = B_1 \cup B_2$, $B_u \cap B_\sigma = B_\omega \cap B_m = B_1 \cap B_2 = \phi$, and $n_i = n_i(\mathbf{x}_B)$ are the components of the outer normal vector to the surface at $\mathbf{x}_B$.

3. The formulation of the problem in the Laplace transform domain

Performing the Laplace transform defined as $\overline{f}(\mathbf{x}, s) = \int\limits_0^\infty e^{-st} f(\mathbf{x}, t)dt$ over Eqs. (2.1), (2.4) and (2.6) with homogeneous initial conditions and omitting the bars, we obtain

$$\sigma_{ji,j} = \rho(s^2 u_i - F_i); \quad \in_{ijl} \sigma_{jl} + m_{ji,j} = \rho(J s^2 \omega_i - M_i),$$

$$\sigma_{ji} = \lambda_1 u_{l,l}\delta_{ij} + (\mu_1 + k_1)u_{i,j} + \mu_1 u_{j,i} + k_1 \in_{ijl} \omega_l - \hat{\gamma}_1 \eta_2 \Theta \delta_{ij}, \tag{3.1}$$

$$m_{ji} = \alpha_1 \omega \delta_{ij} + \beta_1 \omega_{j,i} + \gamma_1 \omega_{i,j}. \tag{3.2}$$

The field equations (2.7)–(2.9) in Laplace transform domain take the form

$$(\lambda_1 + \mu_1)\, u_{j,ji} + (\mu_1 + k_1)\, u_{i,jj} + k_1 \in_{ijl} \omega_{l,j} - \hat{\gamma}_1 \eta_2\, \Theta_{,i} = \rho(s^2 u_i - F_i), \tag{3.3}$$

$$(\alpha_1 + \beta_1)\omega_{j,ji} + \gamma_1 \omega_{i,jj} + k_1 \in_{ijl} u_{l,j} - 2k_1 \omega_i = \rho(J s^2 \omega_i - M_i), \tag{3.4}$$

$$K \eta_3 \Theta_{,ii} = \rho C_E s \eta_1 \Theta + T_0 s \eta\, \hat{\gamma}_1\, u_{i,i} - \eta\, Q, \tag{3.5}$$

where

$$\xi_1 = s\,\xi\, R_\xi(s); \qquad (\xi = \lambda, \mu, k, \alpha, \beta, \gamma); \qquad \hat{\gamma}_1 = (3\lambda_1 + 2\mu_1 + k_1)a; \tag{3.6}$$

$$\begin{aligned} \eta &= n_1 + n_0 \tau_0 s + t_2^2 s^2, & \eta_1 &= n_1 + \tau_0 s + t_2^2 s^2, \\ \eta_2 &= 1 + \nu s, & \eta_3 &= n^* + t_1 s. \end{aligned} \tag{3.7}$$

$R_\xi(s)$ is the Laplace transform of the relaxation functions $R_\xi(t)$. ($\xi_1 = \xi(1+\lambda_v s)$ for the Kelvin–Voigt model and $\xi_1 = \xi$ for the generalized linear micropolar thermoelasticity). The boundary conditions (2.20)–(2.22) in the Laplace transform domain are

$$\begin{aligned} &\sigma_{ji}(\mathbf{x}_{B_\sigma}, s) n_j = f_i, && u_i(\mathbf{x}_{B_u}, s) = g_i, && m_{ji}(\mathbf{x}_{B_m}, s) n_j = \Gamma_i, \\ &\omega_i(\mathbf{x}_{B_\omega}, s) = \Xi_i, && \Theta(\mathbf{x}_{B_1}, s) = \Phi, && \Theta_{,i}(\mathbf{x}_{B_2}, s) n_i = G. \end{aligned}$$

The reciprocity relation in the Laplace transform domain for the generalized micropolar thermoviscoelasticity theory is [31]:

$$
\begin{aligned}
(3.8)\qquad & T_0 s\eta\rho \left[\int_D F_i^{(1)} u_i^{(2)}\, dV + \int_D M_i^{(1)} \omega_i^{(2)}\, dV \right] - \eta\eta_2 \int_D Q^{(1)} \Theta^{(2)}\, dV \\
& + T_0 s\eta \left[\int_{B_u} \sigma_{ji}^{(1)} n_j g_i^{(2)}\, dA + \int_{B_\sigma} f_i^{(1)} u_i^{(2)}\, dA + \int_{B_\omega} m_{ji}^{(1)} n_j \Xi_i^{(2)}\, dA \right. \\
& \left. + \int_{B_m} \Gamma_i^{(1)} \omega_i^{(2)}\, dA \right] - K\eta_3\, \eta_2 \left[\int_{B_1} \Theta_{,n}^{(1)} \Phi^{(2)}\, dA + \int_{B_2} G^{(1)} \Theta^{(2)}\, dA \right] = S_{21}^{12}.
\end{aligned}
$$

Here S_{21}^{12} indicates the same expression as that on the left-hand side, except that superscripts (1) and (2) are interchanged.

4. Generalizations of Maysel's formula

The problem to be solved will consist in determination of $u_i(\mathbf{x},t)$, $\omega_i(\mathbf{x},t)$ and $\Theta(\mathbf{x},t)$, $\mathbf{x} \in D, t > 0$, i.e. the solution of the system of equations (2.7)–(2.9), subjected to the homogeneous initial conditions (2.18) and (2.19), and the boundary conditions:

$$
(4.1)\qquad \begin{aligned}
u_i(\mathbf{x}_B,t) &= g_i(\mathbf{x}_B,t), & \omega_i(\mathbf{x}_B,t) &= \Xi_i(\mathbf{x}_B,t), \\
\Theta_{,n}(\mathbf{x}_B,t) &= G(\mathbf{x}_B,t), & \mathbf{x}_B &\in B_2 = B_u = B_\omega,
\end{aligned}
$$

$$
(4.2)\qquad \begin{aligned}
\sigma_{ij}(\mathbf{x}_B,t)n_j(\mathbf{x}_B) &= f_i(\mathbf{x}_B,t), & m_{ij}(\mathbf{x}_B,t)n_j(\mathbf{x}_B) &= \Gamma_i(\mathbf{x}_B,t), \\
\Theta(\mathbf{x}_B,t) &= \Phi(\mathbf{x}_B,t), & \mathbf{x}_B &\in B_1 = B_\sigma = B_m,
\end{aligned}
$$

where $g_i(\mathbf{x}_B,t)$, $\Xi_i(\mathbf{x}_B,t)$, $\Phi(\mathbf{x}_B,t)$, $f_i(\mathbf{x}_B,t)$, $\Gamma_i(\mathbf{x}_B,t)$ and $G(\mathbf{x}_B,t)$ are given functions.

Consider now the three cases:

CASE 1. We assume that $F_i = 0$, $M_i = 0$ and that an instantaneous source of heat located at $x_i = y_i$ where $\mathbf{y} \in (D \cup B)$, is acting upon a linear micropolar viscoelastic body, i.e. we assume $Q = Q_0\delta(R)\,\delta(t)$, $F_i = 0$, $M_i = 0$, where $Q_0 > 0$ is constant, $R = \sqrt{(x_i - y_i)\,(x_i - y_i)}$ and $\delta(...)$ is a Dirac delta function. Thus in the Laplace transform domain (omitting the bars) we have

$$
(4.3)\qquad Q = Q_0\delta(R), \qquad F_i = 0, \qquad M_i = 0.
$$

The corresponding fundamental solutions of the system of Eqs. (3.4)–(3.6) are

$$u_i^{(1)},\ \omega_i^{(1)},\ \Theta^{(1)}. \tag{4.4}$$

CASE 2. We assume now that $Q = 0$, $M_i = 0$ and an instantaneous concentrated body force $F_i = F_i^{(j)} = F_0\delta(\mathbf{x}-\mathbf{y})\,\delta(t)\,\delta_{ij}$ is acting at the point $x_i = y_i$, where $\mathbf{y} \in (D \cup B)$, in the direction of x_j-axis, where $F_0 > 0$ is constant. Taking the Laplace transform of F_i and omitting the bars, we have

$$F_i = F_i^{(j)} = \delta_{ij}F_0\delta(R), \qquad Q = 0, \qquad M_i = 0. \tag{4.5}$$

The corresponding fundamental solutions (Green's functions) are

$$u_i^{(j)},\ \omega_i^{(j)},\ \Theta^{(j)}. \tag{4.6}$$

CASE 3. We assume now that $Q = 0$, $F_i = 0$ and an instantaneous concentrated body couple force $M_i = M_i^{(q)} = M_0\delta(\mathbf{x}-\mathbf{y})\,\delta(t)\,\delta_{iq}$ is acting at the point $x_i = y_i$, where $\mathbf{y} \in (D \cup B)$, in the direction of x_q-axis, where $M_0 > 0$ is constant. The Laplace transform of M_i is

$$M_i^{(q)} = \delta_{iq}\,M_0\delta(R), \qquad Q = 0, \qquad F_i = 0. \tag{4.7}$$

The corresponding fundamental solutions are

$$u_i^{(q)},\ \omega_i^{(q)},\ \Theta^{(q)}. \tag{4.8}$$

Assuming the boundary conditions to be satisfied by the fundamental solutions (4.4), (4.6) and (4.8) in the form:

$$g_i^{(l)}(\mathbf{x}_B, s) = \Xi_i^{(l)}(\mathbf{x}_B, s) = G^{(l)}(\mathbf{x}_B, s) = 0, \qquad \mathbf{x}_B \in B_2 = B_u = B_\omega, \tag{4.9}$$

$$f_i^{(l)}(\mathbf{x}_B, s) = \Gamma_i^{(l)}(\mathbf{x}_B, s) = \Phi^{(l)}(\mathbf{x}_B, s) = 0, \qquad \mathbf{x}_B \in B_1 = B_\sigma = B_m; \tag{4.10}$$

where $l = 1, j, q$, and substituting from Eqs. (4.1)–(4.10) into the reciprocity relation (3.8), one obtains the generalizations of Maysel's formula, in the Laplace transform domain, to the generalized micropolar thermoviscoelasticity theory in the form:

$$\begin{aligned}\eta\eta_2 Q_0 \Theta(\mathbf{x}, s) &= \eta\eta_2 \int_D Q\Theta^{(1)} dV - T_0\rho s\eta \int_D F_i\, u_i^{(1)} dV \\ &\quad + K\eta_3\eta_2 \left[\int_{B_2} G\Theta^{(1)} dA - \int_{B_1} \Phi\Theta_{,n}^{(1)} dA \right] \\ &\quad + T_0 s\eta \left[\int_{B_2} g_i \sigma_{ji}^{(1)} n_j dA - \int_{B_1} f_i u_i^{(1)} dA \right],\end{aligned} \tag{4.11}$$

$$F_0 T_0 \rho s \eta u_j(\mathbf{x}, s) = -\eta\eta_2 \int_D Q\Theta^{(j)} dV + T_0 \rho s \eta \left[\int_D F_i u_i^{(j)} dV + \int_D M_i \omega_i^{(j)} dV \right] + T_0 s \eta \left[\int_{B_1} f_i u_i^{(j)} dA - \int_{B_2} g_i \sigma_{ki}^{(j)} n_k dA + \int_{B_1} \Gamma_i \omega_i^{(j)} dA - \int_{B_2} \Xi_i m_{ki}^{(j)} n_k dA \right] + K\eta_3\eta_2 \left[\int_{B_1} \Phi \Theta_{,n}^{(j)} dA - \int_{B_2} G\Theta^{(j)} dA \right], \tag{4.12}$$

$$\rho M_0 \omega_q(\mathbf{x}, s) = \rho \left[\int_D F_i u_i^{(q)} dV + \int_D M_i \omega_i^{(q)} dV \right] + \int_{B_1} f_i u_i^{(q)} dA - \int_{B_2} g_i \sigma_{ki}^{(q)} n_k dA + \int_{B_1} \Gamma_i \omega_i^{(q)} dA - \int_{B_2} \Xi_i m_{ki}^{(q)} n_k dA. \tag{4.13}$$

For all the considered generalized theories we have in view Eqs. (2.9)–(2.14), (2.16), (2.17) and (3.7): $\nu t_1 = \nu t_2 = \nu n_0 t_0 = 0$ and therefore

$$\begin{aligned} \eta\eta_2 &= (n_1 + \nu_2 s + t_2^2 s^2), & \eta_2\eta_3 &= (n^* + \nu_1 s), \\ \nu_1 &= (\nu n^* + t_1), & \nu_2 &= (\nu n_1 + n_0 \tau_0). \end{aligned} \tag{4.14}$$

Inverting Eqs. (4.1)–(4.13) we obtain the generalizations of Maysel's formula in the form

$$\begin{aligned} L_1(\Theta(\mathbf{x}, t)) &= W_1^M(\mathbf{x}, t); & L_2(u_j(\mathbf{x}, t)) &= W_2^M(\mathbf{x}, t), \\ \omega_q(\mathbf{x}, t) &= W_3^M(\mathbf{x}, t); & \mathbf{x} &\in D \end{aligned} \tag{4.15}$$

where $W_1^M(\mathbf{x}, t)$, $W_2^M(\mathbf{x}, t)$, and $W_3^M(\mathbf{x}, t)$ are listed in the Appendix.

$$L_1(f(\mathbf{x}, t)) = \left(n_1 + \nu_2 \frac{\partial}{\partial t} + t_2^2 \frac{\partial^2}{\partial t^2} \right) f(\mathbf{x}, t),$$

$$L_2(f(\mathbf{x}, t) = \left(n_1 + n_0 \tau_0 \frac{\partial}{\partial t} + t_2^2 \frac{\partial^2}{\partial t^2} \right) f(\mathbf{x}, t).$$

From Eqs. (4.15) we obtain the following *generalizations of Maysel's formula:*

(i) For the dynamic coupled theory:

$$\begin{aligned}\Theta(\mathbf{x},t) &= W_1^{MC}(\mathbf{x},t),\\ u_j(\mathbf{x},t) &= W_2^{MC}(\mathbf{x},t), \qquad \omega_q(\mathbf{x},t) = W_3^{MC}(\mathbf{x},t).\end{aligned}$$

(ii) For the L–S theory:

$$\begin{aligned}\Theta(\mathbf{x},t) &= \frac{1}{\tau_0}\,e^{-t/\tau_0}\int_0^t e^{\tau/\tau_0}W_1^{MLS}(\mathbf{x},\tau)d\tau,\\ u_j(\mathbf{x},t) &= \frac{1}{\tau_0}\,e^{-t/\tau_0}\int_0^t e^{\tau/\tau_0}W_2^{MLS}(\mathbf{x},\tau)d\tau,\\ \omega_q(\mathbf{x},t) &= W_3^{MLS}(\mathbf{x},t).\end{aligned}$$

(iii) For the G–L theory:

$$\begin{aligned}\Theta(\mathbf{x},t) &= \frac{1}{\nu}e^{-t/\nu}\int_0^t e^{\tau/\nu}W_1^{MGL}(\mathbf{x},\tau)d\tau,\\ u_j(\mathbf{x},t) &= W_2^{MGL}(\mathbf{x},t), \qquad \omega_q(\mathbf{x},t) = W_3^{MGL}(\mathbf{x},t).\end{aligned}$$

(iv) For the G–N theory of Type III:

$$\begin{aligned}\Theta(\mathbf{x},t) &= \int_0^t W_1^{MGN_3}(\mathbf{x},\tau)d\tau,\\ u_j(\mathbf{x},t) &= \int_0^t W_2^{MGN_3}(\mathbf{x},\tau)d\tau, \qquad \omega_q(\mathbf{x},t) = W_3^{MGN_3}(\mathbf{x},t).\end{aligned}$$

(v) For the C–T theory:

$$\begin{aligned}\Theta(\mathbf{x},t) &= \frac{2}{\tau_q}e^{-t/\tau_q}\Big[\Theta_1\sin(t/\tau_q) - \Theta_2\cos(t/\tau_q)\Big],\\ u_j(\mathbf{x},t) &= \frac{2}{\tau_q}e^{-t/\tau_q}\Big[u_1\sin(t/\tau_q) - u_2\cos(t/\tau_q)\Big],\\ \omega_q(\mathbf{x},t) &= W_3^{MCT}(\mathbf{x},t).\end{aligned}$$

Here

$$\Theta_1 = \int_0^t e^{\tau/\tau_q} \cos(\tau/\tau_q)\, W_1^{MCT}(\mathbf{x},\tau) d\tau,$$

$$\Theta_2 = \int_0^t e^{\tau/\tau_q} \sin(\tau/\tau_q) W_1^{MCT}(\mathbf{x},\tau)\, d\tau,$$

$$u_1 = \int_0^t e^{\tau/\tau_q} \cos(\tau/\tau_q) W_2^{MCT}(\mathbf{x},\tau)\, d\tau,$$

$$u_2 = \int_0^t e^{\tau/\tau_q} \sin(\tau/\tau_q) W_2^{MCT}(\mathbf{x},\tau)\, d\tau.$$

(vi) For the G–N theory of Type II (for the micropolar thermoelasticity theory):

$$\Theta(\mathbf{x},t) = \int_0^t W_1^{MGN_2}(\mathbf{x},\tau) d\tau,$$

$$u_j(\mathbf{x},t) = \int_0^t W_2^{MGN_2}(\mathbf{x},\tau) d\tau, \qquad \omega_q(\mathbf{x},t) = W_3^{MGN_2}(\mathbf{x},t).$$

5. The fundamental solutions

According to the Helmholtz theorem [25], the displacement and the body forces can be expressed in the form:

(5.1) $$u_i = \phi_{,i} + \in_{ijk} \Psi_{k,j}, \quad \Psi_{i,i} = 0; \quad F_i = X_{,i} + \in_{ijk} Y_{k,j}, \quad Y_{i,i} = 0,$$

(5.2) $$\omega_i = \Omega_{,i} + \chi_i, \quad \chi_{i,i} = 0; \quad M_i = J(Z_{,i} + N_i), \quad N_{i,i} = 0,$$

where φ, X, Ω, Z are the scalar potentials and Ψ_k, Y_k, χ_k, N_k are the vector potentials of the vector fields u_i, F_i, ω_i and M_i respectively. Equations (5.1) and (5.2) with Eqs. (3.3)–(3.5) lead to

(5.3) $$\begin{aligned} (\nabla^2 - P_1^2)\varphi - b_1\Theta &= -\frac{X}{C_1^2}; & (\nabla^2 - P_2^2)\Psi_i + b_2\chi_i &= -\frac{Y_i}{C_2^2}; \\ (\nabla^2 - a_3^2)\Omega &= -\frac{Z}{C_3^2}, & (\nabla^2 - a_4^2)\chi_i - b_4\nabla^2\Psi_i &= -\frac{N_i}{C_4^2}; \\ (\nabla^2 - P^2)\Theta - b\nabla^2\varphi &= -b_0 Q, \end{aligned}$$

where, taking into consideration Eqs. (3.6) and (3.7), $a_3^2 = P_3^2 + b_3$, $a_4^2 = P_4^2 + 2b_4$ and

$$C_1^2 = \frac{(\lambda_1 + 2\mu_1 + k_1)}{\rho}, \quad C_2^2 = \frac{\mu_1 + k_1}{\rho},$$
$$C_3^2 = \frac{\alpha_1 + \beta_1 + \gamma_1}{\rho J}, \qquad C_4^2 = \frac{\gamma_1}{\rho J},$$
$$P_n = \frac{s}{C_n}, \qquad (n = 1, 2, 3, 4);$$
$$P^2 = \frac{\rho C_E s\,\eta_1}{K\eta_3}, \qquad b_0 = \frac{\eta}{K\eta_3},$$
$$b = \frac{\hat{\gamma}_1 T_0 s\,\eta}{K\eta_3}, \qquad b_1 = \frac{\hat{\gamma}_1 \eta_2}{\rho C_1^2},$$
$$b_2 = \frac{k_1}{\mu_1 + k_1}, \qquad b_3 = \frac{2k_1}{\alpha_1 + \beta_1 + \gamma_1}, \qquad b_4 = \frac{k_1}{\gamma_1}.$$

To obtain $u_i^{(1)}$, $\omega_i^{(1)}$, $\Theta^{(1)}$ in the Laplace transform domain, we substitute the relation (4.3) into the system of the governing equations (5.3), using the Helmholtz equation [33]

$$\frac{1}{\nabla^2 - m_n^2}[\delta(R)] = -\frac{1}{4\pi R}\, e^{-m_n R} \tag{5.4}$$

and introducing the notations

$$E_n = (-1)^{n-1} e^{-m_n R}, \qquad \xi_n = (-1)^{n-1}\left(\frac{1}{R} + m_n\right) e^{-m_n R},$$
$$V_n = 3\xi_n + m_n^2 R E_n, \qquad A_1 = \frac{Q_0 b_0 b_1}{4\pi(m_1^2 - m_2^2)}$$

we obtain for an infinite region, with the homogeneous initial conditions [27] the result:

$$\Omega^{(1)} = 0, \qquad \Psi_i^{(1)} = 0, \qquad \chi_i^{(1)} = 0, \qquad \omega_i^{(1)} = 0, r_i^{(1)} = 0, \qquad m_{ij} = m_{ji} = 0$$

$$\varphi^{(1)} = \frac{A_1}{R}\sum_1^2 E_n, \qquad u_i^{(1)} = -\frac{A_1 R_{,i}}{R}\sum_1^2 \xi_n, \qquad \Theta^{(1)} = \frac{A_1}{b_1 R}\sum_1^2 (m_n^2 - P_1^2) E_n,$$

where m_1^2, m_2^2 are the roots of the characteristic equation:

$$m^4 - (P_1^2 + b_1 b + P^2) m^2 + P_1^2\, P^2 = 0.$$

The fundamental solutions $u_i^{(j)}$, $\omega_i^{(j)}$, $\Theta^{(j)}$ are obtained by substituting from Eqs. (4.5) into Eqs. (5.3). Taking into consideration that $\in_{ilk}\ Y_{k,li}^{(j)} = 0$ and

$\in_{iqp} X^{(j)}_{,iq} = 0$, using Eq. (5.4) with $m_n = 0$ and Eq. $(5.1)_2$ with Eqs. (4.5), we obtain $X^{(j)} = -\dfrac{F_0}{4\pi}\left(\dfrac{\delta_{ij}}{R}\right)_{,i}$ and $Y^{(j)}_k = \dfrac{F_0}{4\pi} \in_{iqk} \left(\dfrac{\delta_{qj}}{R}\right)_{,i}$. The governing Eqs. (5.3) now lead to:

$$\varphi^{(j)} = \frac{B_0\,\delta_{ij}\,R_{,i}}{R^2} + \frac{\delta_{ij}\,R_{,i}}{R}\sum_1^2 \varsigma_n\xi_n, \qquad \Theta^{(j)} = -\frac{B_1\delta_{ij}R_{,i}}{R}\sum_1^2 \xi_n,$$

where

$$\varsigma_n = \frac{(m_n^2 - P^2)F_0}{4\pi C_1^2 m_n^2 (m_2^2 - m_1^2)} \quad \text{and} \quad B_0 = \frac{F_0}{4\pi s^2}, \qquad B_1 = \frac{bF_0}{4\pi C_1^2 (m_1^2 - m_2^2)},$$

$$\chi^{(j)}_i = \in_{jik}\left(\frac{A_2 b_4 R_{,k}}{R}\sum_3^4 \xi_n\right), \qquad \Psi^{(j)}_i = \in_{ijk}\left(\frac{R_{,k}}{R}\right)\left[\frac{B_0}{R} - \sum_3^4 \varsigma^*_n\xi_n\right],$$

$$u^{(j)}_i(\overline{x},\overline{y},s) = \frac{U_1\delta_{ij}}{R^2} - \frac{U_2 R_{,i}R_{,j}}{R^2},$$

$$\omega^{(j)}_i = \chi^{(j)}_i = \in_{jil}\left(\frac{A_2 b_4 R_{,l}}{R}\sum_3^4 \xi_n\right), \qquad \omega^{(j)}_{i,\,i} = 0,$$

where m_3^2 and m_4^2 are the roots of the following second characteristic equation:

$$m^4 - \left(P_2^2 + a_4^2 - b_2 b_4\right) m^2 + a_4^2 P_2^2 = 0,$$

$$A_2 = \frac{F_0}{4\pi C_2^2 (m_3^2 - m_4^2)}, \qquad \varsigma^*_n = \frac{A_2\left(m_n^2 - a_4^2\right)}{m_n^2},$$

$$U_1 = \sum_1^2 \varsigma_n\xi_n + \sum_3^4 \varsigma^*_n(\xi_n + R m_n^2 E_n), \qquad U_2 = \sum_1^2 \varsigma_n V_n + \sum_3^4 \varsigma^*_n V_n.$$

To determine the *Green functions* $u^{(q)}_i, \omega^{(q)}_i, \Theta^{(q)}$ we substitute from Eqs. (4.7) into the governing equations (5.3), and we obtain, for an infinite region, taking into consideration the homogeneous initial conditions $\varphi^{(q)} = 0$, $e^{(q)} = 0$ and $\Theta^{(q)} = 0$.

From Eqs. (4.7) and (5.2) we get

$$Z^{(q)} = \frac{M_0\delta_{iq}R_{,i}}{4\pi J R^2}, \qquad N^{(q)}_i = \frac{M_0}{J}\left[\delta_{iq}\delta(R) + \frac{1}{4\pi}\left(\frac{1}{R}\right)_{,iq}\right]$$

from which and Eq. $(5.3)_3$ we obtain $\Omega^{(q)} = \dfrac{A_3 R_{,i}\delta_{iq}}{R^2}\left[1-(1+a_3R)e^{-a_3R}\right]$, therefore

$$
\begin{aligned}
\Omega^{(q)}_{,i} &= -\frac{A_3(3R_{,i}R_{,q}-\delta_{iq})}{R^3}\left[1-(1+a_3R)e^{-a_3R}\right]+\frac{A_3a_3^2R_{,i}R_{,q}\,e^{-a_3R}}{R},\\
\Psi^{(q)}_i &= \frac{U_3(3R_{,i}R_{,q}-\delta_{iq})}{R^3}-\frac{2A_4\delta_{iq}}{3R}\sum_3^4 E_n,\\
\chi^{(q)}_i &= -\frac{U_4(3R_{,i}R_{,q}-\delta_{iq})}{b_2R^3}+\frac{2\delta_{iq}A_4}{3b_2R}\sum_3^4\left(m_n^2-P_2^2\right)E_n,\\
u^{(q)}_i &= \in_{ilq}\left(\frac{A_4R_{,l}}{R}\sum_3^4\xi_n\right), \qquad \omega^{(q)}_i=\Omega^{(q)}_{,i}+\chi^{(q)}_i.
\end{aligned}
$$

Here

$$
U_3=B_3+\frac{A_4R}{3}\sum_3^4\frac{V_n}{m_n^2}, \qquad U_4=-P_2^2B_3+\frac{A_4R}{3}\sum_3^4\left(\frac{m_n^2-P_2^2}{m_n^2}\right)V_n,
$$

$$
A_3=\frac{M_0}{4\pi JC_3^2a_3^2}, \qquad A_4=\frac{M_0b_2}{4\pi JC_4^2(m_3^2-m_4^2)},
$$

and

$$
B_3=\frac{M_0b_2}{4\pi JC_4^2m_3^2m_4^2}.
$$

6. Conclusions

1. For the linear micropolar thermoviscoelasticity and the generalizations of Maysel's formula to the dynamic coupled theory, four generalized theories are obtained. The corresponding generalizations to the linear micropolar thermoviscoelasticity of Kelvin–Voigt model and to the linear micropolar thermoelasticity can be obtained from the given results as special cases.

2. The Green functions for an infinite region are obtained in Laplace transform domain. Appropriate numerical methods for evaluating the corresponding expressions should be applied for the implementations of the generalizations of Maysel's formula.

Appendix

$$
\begin{aligned}
W_1^M(\mathbf{x},t) = & \frac{1}{Q_0}\int_0^t\int_D Q(\mathbf{y},t-\tau)L_1(\Theta^{(1)}(\mathbf{y},\mathbf{x},\tau))\,dV(\mathbf{y})\,d\tau \\
& -\frac{T_0\rho}{Q_0}\int_0^t\int_D F_i(\mathbf{y},t-\tau)\,\frac{\partial L_2\,(u_i^{(1)}(\mathbf{y},\mathbf{x},\tau))}{\partial\tau}dV(\mathbf{y})\,d\tau \\
& +\frac{T_0}{Q_0}\left[\int_0^t\int_{B_2} g_i(\mathbf{y},t-\tau)\frac{\partial L_2(\sigma_{ji}^{(1)}(\mathbf{y},\mathbf{x},\tau)n_j)}{\partial\tau}\,dA(\mathbf{y})\,d\tau\right. \\
& \left.-\int_0^t\int_{B_1} f_i(\mathbf{y},t-\tau)\,\frac{\partial L_2(u_i^{(1)}(\mathbf{y},\mathbf{x},\tau))}{\partial\tau}\,dA(\mathbf{y})\,d\tau\right] \\
& +\frac{K}{Q_0}\left[\int_0^t\int_{B_2} G(\mathbf{y},t-\tau)\,L_3\,(\Theta^{(1)}(\mathbf{y},\mathbf{x},\tau))\,dA(\mathbf{y})\,d\tau\right. \\
& \left.-\int_0^t\int_{B_1}\Phi(\mathbf{y},t-\tau)\,L_3(\,\Theta_{,n}^{(1)}(\mathbf{y},\mathbf{x},\tau))\,dA(\mathbf{y})\,d\tau\right];
\end{aligned}
$$

$$
\begin{aligned}
W_2^M(\mathbf{x},t) = & \frac{1}{F_0}\int_0^t\int_D F_i(\mathbf{y},t-\tau)\,L_2\,(u_i^{(j)}(\mathbf{y},\mathbf{x},\tau))\,dV(\mathbf{y})d\tau \\
& +\frac{1}{F_0}\int_0^t\int_D M_i(\mathbf{y},t-\tau)\,L_2\,(\omega_i^{(j)}(\mathbf{y},\mathbf{x},\tau))\,dV(\mathbf{y})d\tau \\
& -\frac{1}{F_0T_0\rho}\int_0^t\int_D Q(\mathbf{y},t-\tau)\,L_1^*(\Theta^{(j)}(\mathbf{y},\mathbf{x},\tau))\,dV(\mathbf{y})\,d\,\tau \\
& +\frac{K}{F_0T_0\rho}\left[\int_0^t\int_{B_1}\Phi(\mathbf{y},t-\tau)\,L_3^*(\Theta_{,n}^{(j)}(\mathbf{y},\mathbf{x},\tau))\,dA(\mathbf{y})\,d\tau\right. \\
& \left.-\int_0^t\int_{B_2} G(\mathbf{y},t-\tau)\,L_3^*(\Theta^{(j)}(\mathbf{y},\mathbf{x},\tau))\,dA(\mathbf{y})\,d\tau\right]
\end{aligned}
$$

[cont.]

$$+ \frac{1}{F_0\rho}\left[\int_0^t\int_{B_1} f_i(\mathbf{y},t-\tau)\; L_2\,(u_i^{(j)}(\mathbf{y},\mathbf{x},\tau))dA(\mathbf{y})\; d\tau - \int_0^t\int_{B_2} g_i(\mathbf{y},t-\tau)L_2(\sigma_{ki}^{(j)}(\mathbf{y},\mathbf{x},\tau)n_k)\; dA(\mathbf{y})\; d\tau\right]$$

$$+ \frac{1}{F_0\rho}\left[\int_0^t\int_{B_1} \varGamma_i(\mathbf{y},t-\tau)\; L_2\,(\omega_i^{(j)}(\mathbf{y},\mathbf{x},\tau))\,dA(\mathbf{y})\; d\tau - \int_0^t\int_{B_2} \varXi_i(\mathbf{y},t-\tau)L_2(m_{ki}^{(j)}(\mathbf{y},\mathbf{x},\tau)n_k)\; dA(\mathbf{y})\; d\tau\right];$$

$$W_3^M(\mathbf{x},t) = \frac{1}{M_0}\left[\int_0^t\int_D F_i(\mathbf{y},t-\tau)\,u_i^{(q)}(\mathbf{y},\mathbf{x},\tau)\,dV(\mathbf{y})d\tau + \int_0^t\int_D M_i(\mathbf{y},t-\tau)\,\omega_i^{(q)}(\mathbf{y},\mathbf{x},\tau)dV(\mathbf{y})d\tau\right]$$

$$+ \frac{1}{M_0\rho}\left[\int_0^t\int_{B_1} f_i(\mathbf{y},t-\tau)u_i^{(q)}(\mathbf{y},\mathbf{x},\tau)dA(\mathbf{y})\; d\tau - \int_0^t\int_{B_2} g\,(\mathbf{y},t-\tau)\sigma_{ji}^{(q)}(\mathbf{y},\mathbf{x},\tau)n_j dA(\mathbf{y})\; d\tau\right]$$

$$+ \frac{1}{M_0\rho}\left[\int_0^t\int_{B_1} \varGamma_i(\mathbf{y},t-\tau)\omega_i^{(q)}(\mathbf{y},\mathbf{x},\tau)dA(\mathbf{y})\; d\tau - \int_0^t\int_{B_2} \varXi_i(\mathbf{y},t-\tau)m_{ji}^{(q)}(\mathbf{y},\mathbf{x},\tau)n_j dA(\mathbf{y})\; d\tau\right],$$

where

$$L_3(f(\mathbf{x},t)) = \left(n^* + \nu_1\frac{\partial}{\partial t}\right) f(\mathbf{x},t),$$

$$L_1^*(f) = \int_0^\tau L_1(f(\mathbf{y},\mathbf{x},\varsigma))d\varsigma,$$

$$L_3^*(f) = \int_0^\tau L_3(f(\mathbf{y},\mathbf{x},\varsigma))d\varsigma,$$

$$L_1(f) = L_2(f) = L_3(f) = f \quad \text{(for the DCT)},$$

$$L_1(f) = L_2(f) = \left(1 + \tau_0 \frac{\partial}{\partial t}\right) f, \qquad L_3(f) = f \quad \text{(for L–S theory)},$$

$$L_1(f) = L_3(f) = \left(1 + \nu \frac{\partial}{\partial t}\right) f, \qquad L_2(f) = f \quad \text{(for G–L theory)},$$

$$\left.\begin{aligned} L_1(f) &= L_2(f) = \frac{\partial f}{\partial t}, \\ L_3(f) &= \left(n^* + \frac{\partial}{\partial t}\right) f \end{aligned}\right. \quad \text{(for G–N theory of Type III)}$$

$$\left.\begin{aligned} L_1(f) &= L_2(f) = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right) f, \\ L_3(f) &= \left(1 + \tau_\Theta \frac{\partial}{\partial t}\right) f, \end{aligned}\right. \quad \text{(for C–T theory), and}$$

$$L_1(f) = L_2(f) = \frac{\partial f}{\partial t}, \qquad L_3(f) = n^* f \quad \text{(for G–N theory of Type II).}$$

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