

Viscoelastic boundary layer MHD flow through a porous medium over a porous quadratic stretching sheet

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BOUNDARY LAYER MHD flow of a viscoelastic fluid in a porous medium over a porous stretching sheet has been presented in this article. A typical choice of quadratic stretching of the boundary, having a quadratic part in velocity parallel to the boundary sheet and a linear mass flux in the velocity normal to the stretching sheet, constitutes the boundary conditions of the problem. The effect of various values of non-dimensional physical parameters on streamline patterns and skin friction coefficient are discussed. Some of the important findings of the article are: (a) the flow is enhanced by the positive values of linear mass flux parameter and suppressed by the negative values of linear mass flux parameter; (b) the effect of permeability parameter is not significant when linear mass flux parameter takes zero or negative values; (c) the combined effect of reduction of the values of permeability parameter, Hartmann number and linear mass flux parameter is expected to reduce largely the values of skin friction coefficient.

Key words: viscoelastic fluid, quadratic stretching, stream function, skin friction.

Notations

- k_0 – dimensional viscoelastic parameter,
- k', k_2 – dimensional and dimensionless permeability parameters,
- B_0 – magnetic strength parameter,
- γ – kinematic coefficient of viscosity,
- u, v – axial and transverse velocities,
- σ – electrical conductivity,
- ρ – density of the viscoelastic fluid,
- x, y – axial and transverse coordinates,
- b – linear stretching rate,
- α – quadratic stretching coefficient,
- v_w – dimensional constant mass flux,
- δ – dimensional linear mass flux coefficient,
- ψ – stream function,
- η – dimensionless transverse coordinate, $y\sqrt{\frac{b}{\gamma}}$,
- k_1^* – dimensionless viscoelastic parameter, $\frac{k_0 b}{\gamma}$,
- M_n – Hartmann number, $\sqrt{\frac{\sigma}{\rho b}} B_0$,

- v_w^* – dimensionless constant mass flux, $\frac{v_w}{\sqrt{b\gamma}}$,
 s – real positive root of Eq. (3.14),
 C_f – dimensionless skin friction coefficient,
 b_0 – modified linear mass flux parameter,
 Re_x – local Reynolds number, $\frac{\sqrt{b\gamma}}{\gamma} x$,
 ψ^* – dimensionless stream function,
 ξ – dimensionless axial coordinate.

1. Introduction

SAKIADIS [1] was the first to initiate pioneering work on the boundary layer flow of incompressible fluid over continuous solid surface, which finds its application in the problem of polymer sheet extruded from a dye. In the recent years, ever increasing application of viscoelastic fluids (e.g. dilute polymer solution of 0.83% ammonium alginate in water and 5.4% polyisobutylene in cetane at 30°C, MARKOVITZ and COLEMAN [2]) in polymer processing industries has led to a renewed interest among researchers to investigate the viscoelastic boundary layer flow over a stretching plastic sheet (RAJAGOPAL *et al.* [3, 4], DANDAPAT and GUPTA [5], ROLLINS and VAJRARELU [6], ANDERSSON [7], LAWERENCE and RAO [8], CHAR [9] and RAO [10]). A significant effort has been directed to study the boundary layer viscous fluid flow over a porous stretching sheet where the flow is influenced by suction/blowing of liquid through the porous sheet (VAJRARELU and NAYFEH [11], VAJRARELU [12], AHMAD and MUBEEN [13], CHIAM [14], YIH [15], ACHARYA *et al.* [16]). A new dimension has been added in this study by investigating such situation for viscoelastic fluid flow in our recent works (PRASAD *et al.* [17] and SONTI *et al.* [18]). Stability analysis of viscoelastic fluid flow over a stretching sheet with and without magnetic field has been carried out by DANDAPAT *et al.* [19, 20].

Exhaustive literature is available including the papers cited above on two-dimensional viscoelastic boundary layer flow over a stretching surface, where the velocity of the stretched surface is assumed to be linearly proportional to the distance from a fixed origin. However, GUPTA and GUPTA [21] have pointed out that in reality, stretching of the sheet might not necessarily be linear. Also, there might be a situation of flow of linear mass flux addition or annihilation in addition to constant mass flux through the pores of the boundary sheet. This situation was dealt with by KUMARAN and RAMANAIHAH [22] in their work on boundary layer flow over a quadratic stretching sheet. However, their work is confined to the viscous fluid flow over a stretching sheet in absence of a magnetic field. Estimation of skin friction which is very important from the industrial application point of view is also not presented in their analysis. Skin friction and streamline pattern might vary to a certain extent if the fluid flows through porous media.

In view of this, a study was carried out by GUPTA and SRIDHAR [23] on a viscoelastic fluid flow through a tube having periodically varying diameter which is often used to represent a porous medium. The applied magnetic field, which also might play a decisive role in skin friction, is excluded from this analysis.

Therefore, in the present article the authors make an attempt to investigate the viscoelastic boundary layer fluid flow through porous media over a quadratic stretching of a sheet. We know that magnetic field stabilizes such a flow, a fact pointed out by DANDAPAT *et al.* [20]. Hence, we consider an electrically conducting fluid region which is exposed to a uniform transverse magnetic field. In this work we aim at investigating the effect of permeability of the porous medium, viscoelastic parameter, Hartmann number, and mass flux parameters which appear in linear and quadratic stretching part, on the stream function characteristics and the skin friction coefficient. Results of ANDERSSON [7] concerning the linear stretching problem may be deduced from this study as a limiting case when there would be no linear mass flux, no quadratic stretching and no porous medium.

2. Mathematical formulation

Two-dimensional viscoelastic boundary layer flow through a porous medium over a porous stretching sheet in a semi-infinite region $y > 0$ is considered for investigation (Fig. 1). The flow is assumed to be generated solely due to stretching of the adjacent flat boundary sheet so that no free stream velocity exists within the boundary layer. The sheet extends along the xz -plane and is stretched along the x -axis. The stretching is being done by applying two equal and opposite forces whilst keeping the origin fixed. The sheet is stretched in such a way that the velocity of the sheet is a quadratic polynomial of the distance

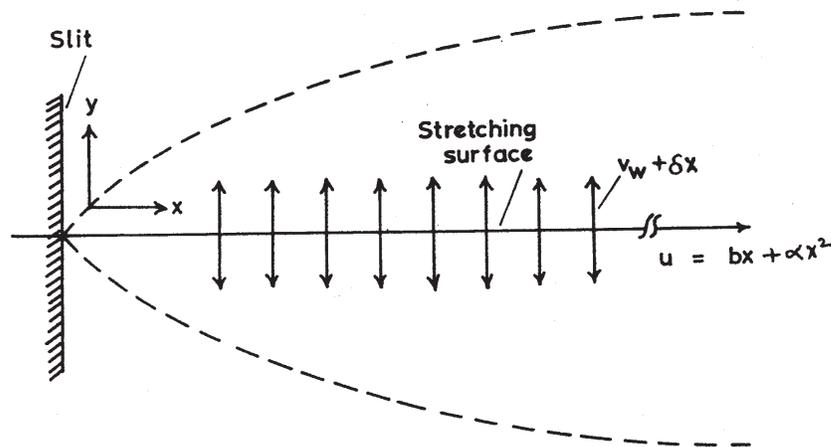


FIG. 1. Boundary layer over a porous stretching surface.

from the origin and there is a linear mass flux in addition to constant mass flux to the boundary layer region through the porous boundary sheet. The electrically conducting boundary layer fluid flow is assumed to be exposed to the influence of a transverse uniform magnetic field of strength B_0 . The transverse magnetic field acts parallel to the y -axis. The magnetic Reynolds number is considered to be small so that the induced magnetic field is negligible. Also, we assume that the external electric field is zero and that the electric field as a result of polarization of charges is negligible.

The basic governing equations in the present flow situation are the modified version of the boundary layer equations of BEARD and WALTERS [24] and of ANDERSSON [7], and they are as follows.

$$(2.1) \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$(2.2) \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} - k_0 \left\{ u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} \right\} - \frac{\sigma B_0^2}{\rho} u - \frac{\gamma}{k'} u.$$

Here k_0 is the elastic parameter, γ is the kinematic viscosity of the fluid, k' is the permeability parameter of porous medium, B_0 is the magnetic strength parameter and σ is the electrical conductivity of viscoelastic fluid. The other quantities have their usual meanings. The above Eq. (2.2) has been derived under the assumption that the normal stress is of the same order of magnitude as that of the shear stress, in addition to the usual boundary layer approximations. The model equation (2.2) is valid for small values of elastic parameter k_0 since it has been derived to the first order in elasticity representing the short memory fluid with smaller relaxation time.

2.1. Boundary conditions

The porous boundary sheet is assumed to be stretched quadratically along the x -direction in such a way that constant and linear mass fluxes through the pores of the boundary influence the boundary layer flow. The appropriate boundary conditions on velocity are:

$$(2.3) \quad \begin{array}{lll} u = bx + \alpha x^2 & v = v_w + \delta x & \text{at } y = 0 \\ u = 0 & u_y = 0 & \text{as } y \rightarrow \infty. \end{array}$$

The subscript y represents differentiation with respect to y ; b , α , v_w and δ are constants, where b is known as linear stretching rate, α is the quadratic stretching

rate, v_w represents constant mass flux and δx represents linear mass flux to the boundary layer region through the porous boundary sheet. The boundary conditions as $y \rightarrow \infty$ in Eq. (2.3) are due to the stress-free conditions at the outer boundary and free stream velocity being taken zero there. It is of interest to note that the quadratic stretching part αx^2 in the x -directional velocity must be accompanied by the linear mass flux δx in y -directional velocity in order to satisfy the equation of continuity.

3. Mathematical analysis

We seek a self-similar solution of Eq. (2.2) by defining the stream function as

$$(3.1) \quad \psi = \sqrt{b\gamma} x f(\eta) - \frac{\delta}{2} x^2 f_\eta(\eta), \quad \eta = y \sqrt{\frac{b}{\gamma}}.$$

This yields

$$(3.2) \quad u = \psi_y = b x f_\eta(\eta) - \frac{\delta}{2} \sqrt{\frac{b}{\gamma}} x^2 f_{\eta\eta}(\eta),$$

$$(3.3) \quad v = -\psi_x = -\sqrt{b\gamma} f(\eta) + \delta x f_\eta(\eta).$$

Substituting the Eqs. (3.1)–(3.3) in the Eq. (2.2) and then equating the coefficients of x , x^2 , x^3 of the resultant equations to zero, we obtain the following nonlinear differential equations:

$$(3.4) \quad f_\eta^2 - f f_{\eta\eta} = f_{\eta\eta\eta} - k_1^* \{ 2f_\eta f_{\eta\eta\eta} - f f_{\eta\eta\eta\eta} - f_{\eta\eta}^2 \} - M_n^2 f_\eta - k_2 f_\eta,$$

$$(3.5) \quad f_\eta f_{\eta\eta} - f f_{\eta\eta\eta} = f_{\eta\eta\eta\eta} - k_1^* \{ f_\eta f_{\eta\eta\eta\eta} - f f_{\eta\eta\eta\eta} \} - M_n^2 f_{\eta\eta} - k_2 f_{\eta\eta},$$

$$(3.6) \quad f_{\eta\eta}^2 - f_\eta f_{\eta\eta\eta} = k_1^* \{ f_\eta f_{\eta\eta\eta\eta} - 2f_{\eta\eta} f_{\eta\eta\eta\eta} + f_{\eta\eta\eta}^2 \}.$$

Here $M_n = \sqrt{\frac{\sigma}{\rho b}} B_0$ is the Hartmann number, $k_1^* = \frac{k_0 b}{\gamma}$ is the viscoelastic parameter and $k_2 = \frac{\gamma}{k' b}$ is the permeability parameter of the medium. In non-dimensional form the corresponding boundary conditions are derived as

$$(3.7) \quad \begin{aligned} f = -v_w^*, \quad f_\eta = 1 \quad f_{\eta\eta} = -\frac{2\alpha}{\delta} \sqrt{\frac{\gamma}{b}} & \quad \text{at } \eta = 0 \\ f_\eta = 0, \quad f_{\eta\eta} = 0, & \quad \text{as } \eta \rightarrow \infty. \end{aligned}$$

Here, $v_w^* = \frac{v_w}{\sqrt{b\gamma}}$. From the mathematical point of view, the Eq. (3.5) doesn't have any special significance since it may be obtained from the Eq. (3.4) by simple

differentiation. Using the first two boundary conditions with $v_w^* = 0$ at $\eta = 0$ and the first boundary condition at $\eta \rightarrow \infty$, RAJAGOPAL *et al.* [3] obtained the corresponding solution of the Eq. (3.4). Subsequently TROY *et al.* [25] obtained a unique solution of the Eq. (3.4), for $0 < k_1^* < 1$, in the form

$$(3.8) \quad f(\eta) = \sqrt{1 - k_1^*} \left(1 - e^{-\eta/\sqrt{1-k_1^*}} \right).$$

Later on CHANG [26] showed that the solution of the Eq. (3.4) with those boundary conditions was not unique. Taking $k_1^* = 1/2$, CHANG [26] derived other solution of the form

$$(3.9) \quad f(\eta) = \sqrt{2} \left(1 - e^{-\eta/\sqrt{2}} \cos(\sqrt{3/2} \eta) \right)$$

Recently RAO [10] derived another closed-form solution

$$(3.10) \quad f(\eta) = A \left[1 - e^{-A\eta/2} \cos(\sqrt{3} A \eta) + \frac{1 + 2k_1^*}{\sqrt{3}} \sin(\sqrt{3} A \eta/2) \right],$$

$$A = \frac{1}{\sqrt{-k_1^*}}.$$

This form of solution exists only when $k_1^* \in (-1, 0)$.

Among all the above solutions, the solution (3.8) is realistic as we can recover the Navier–Stokes' solution only in its limiting case $k_1^* \rightarrow 0$. Also, for a viscoelastic fluid of the Walters type liquid B where k_1^* should be small real positive (DANDAPAT and GUPTA [5]), the solution of the form (3.8) is the only solution of the problem with those three boundary conditions. Therefore, in view the nature of the boundary conditions and the results known from the literature, we seek the solution of the Eq. (3.4) with the prescribed boundary conditions (3.7) in the form

$$(3.11) \quad f(\eta) = A_1 + B_1 e^{-s\eta}.$$

The assumption of solution of the Eq. (3.4) in the above form is most appropriate as it also satisfies the Eq. (3.6). Now, we proceed further to determine the constants A_1 , B_1 and s . Substituting the Eq. (3.11) in the Eq. (3.4) and making use of all the boundary conditions of Eq. (3.7), we obtain

$$(3.12) \quad A_1 = \frac{1}{s} - v_w^*, \quad B_1 = -\frac{1}{s}.$$

Hence we obtain

$$(3.13) \quad f(\eta) = \frac{1 - e^{-s\eta}}{s} - v_w^*.$$

Here, s is the real positive root of the following cubic algebraic equation

$$(3.14) \quad s^3 + \frac{1 - k_1^*}{v_w^* k_1^*} s^2 + \frac{1}{k_1^*} s - \frac{1 + M_n^2 + k_2}{v_w^* k_1^*} = 0.$$

We must note that in the range $0 \leq k_1^* < 1$ for which the model Eq. (2.2) is valid and for all real values of v_w^* , there exists always a real positive root of the Eq. (2.2). It is of some interest to note that the quadratic stretching parameter α and the linear mass flux parameter δ must be related by the equation

$$(3.15) \quad s = \frac{2\alpha}{\delta} \sqrt{\frac{\gamma}{b}}.$$

Therefore, when α and δ are simultaneously absent, $k_2 = 0$ and $v_w^* = 0$, we would obtain the solution of ANDERSSON [7] as the limiting case of our result. The quadratic equation for s , in such a case, may be deduced from the Eq. (3.14) in the limit $v_w^* \rightarrow 0$ and $k_2 = 0$. The solution represented by Eqs. (3.13)–(3.15) is the only realistic solution satisfying all the boundary conditions of Eq. (3.7).

The dimensionless skin friction coefficient C_f is expressed as

$$(3.16) \quad C_f = - \frac{\left[\gamma \left(\frac{\partial u}{\partial y} \right) - k_0 \left\{ u \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^2 u}{\partial y^2} - 2 \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} \right\} \right]_{y=0}}{(bx)^2}$$

$$= \frac{s}{\sqrt{\text{Re}_x}} (1 - 3k^*) + b_0 s^3 (1 - 7k_1^*) + b_0 s^3 k_1^* (v_w^* - 4\text{Re}_x b_0) + \frac{v_w^* s^2 k_1^*}{\sqrt{\text{Re}_x}}.$$

Here, $\text{Re}_x = \frac{\sqrt{b} \gamma x}{\gamma}$ is the local Reynolds number, which depends on the rate of linear stretching b . The effect of mass flux parameters v_w^* and δ is demonstrated by the relations (3.14)–(3.15).

It will be more convenient to analyse the flow characteristics if we further introduce the dimensionless quantities in the following way

$$(3.17) \quad \psi^* = \frac{\psi}{\gamma}, \quad \xi = x \sqrt{\frac{b}{\gamma}} \quad \text{and} \quad b_0 = \frac{\delta}{2b}.$$

Hence the Eq. (3.1) takes the form

$$(3.18) \quad \psi^* = \xi f(\eta) - b_0 \xi^2 f_\eta(\eta).$$

The streamline equation is derived as

$$(3.19) \quad \eta = \frac{1}{s} \left\{ \log \left(b_0 \xi^2 + \frac{\xi}{s} \right) - \log \left[\frac{s^2 - M_n^2 - k_2}{s(1 + k_1^* s^2)} \xi - C \right] \right\}.$$

Here, $\psi^* = C$, a constant along a particular streamline. The Eq. (3.19) yields the expression of KUMARAN and RAMANAIAH [22] as a limiting case when $k_1^* = 0$, $M_n = 0$ and $k_2 = 0$.

4. Results and discussion

We have considered the viscoelastic boundary layer flow in a porous medium over a porous stretching sheet. In contrast to the linear stretching of the boundary sheet, the flow is considered to be generated solely by general quadratic stretching of the boundary and influenced by a uniform transverse magnetic field. We assume that horizontal stretching velocity consists of two parts, namely (i) a linear part which is linear in x and (ii) a quadratic part which is quadratic in x . The cross directional velocity, which is normal to the stretching boundary, has also two parts, namely (i) a constant mass flux due to suction/blowing and (ii) a linear mass flux. Analytical expressions for dimensionless stream function and dimensionless skin friction coefficient C_f have been derived. To acquire the knowledge on the effect of viscoelastic parameter k_1^* , the permeability parameter k_2 , linear mass flux parameter b_0 , the constant mass flux parameter v_w^* and the Hartmann number M_n on the streamline pattern, several graphs are plotted in the Figs. 2–6. The numerical values of the skin friction coefficient C_f for various flow-controlling physical parameters are recorded in a Table.

Table 1. Values of skin friction parameter C_f for various values of the physical parameters.

k_1^*	v_w^*	b_0	Re_x	M_n	C_f ($k_2 = 0.0$)	C_f ($k_2 = 0.5$)
10^{-10} 0.2	-0.07	-0.01	0.5	0.0	1.454 0.643	1.766 0.778
10^{-10} 0.2				0.5	1.767 0.778	2.029 0.890
10^{-10} 0.2		0.0		0.0	1.465 0.637	1.782 0.769
10^{-10} 0.2				0.5	1.782 0.769	2.050 0.879
10^{-10} 0.2	0.07	-0.01		0.0	1.356 0.629	1.668 0.782
10^{-10} 0.2				0.5	1.668 0.782	1.931 0.913
10^{-10} 0.2		0.0		0.0	1.365 0.625	1.682 0.776
10^{-10} 0.2		-0.01	2.5		0.601 0.283	0.738 0.353

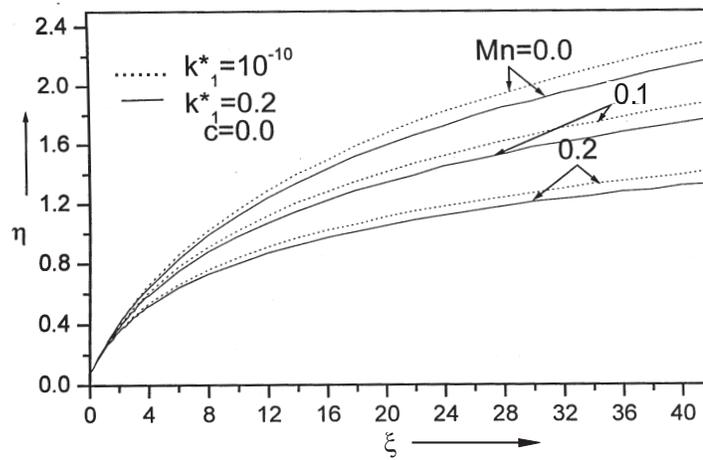


FIG. 2. Streamline patterns for different values of k_1^* and M_n when $v_w^* = 0.07$, $b_0 = 0.2$ and $k_2 = 0.1$.

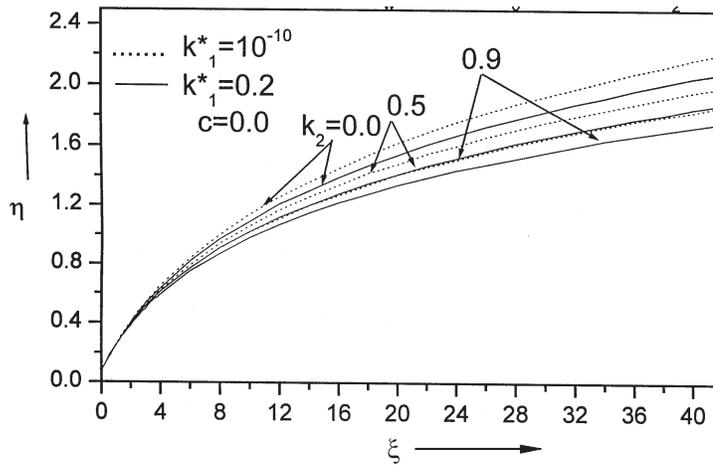


FIG. 3. Streamline patterns for different values of k_1^* and k_2 when $v_w^* = 0.07$, $b_0 = 0.2$ and $M_n = 0.5$.

Figure 2 demonstrates the streamline patterns for different values of viscoelastic parameter k_1^* and Hartmann number M_n . Analysis of the graph reveals that the effect of increasing values of viscoelastic parameter k_1^* is shifting of the location of streamline towards the stretching surface.

The effect of increasing values of the Hartmann number M_n is also to shift the location of the streamline towards the stretching boundary. The combined effect of viscoelastic parameter k_1^* and magnetic parameter M_n is suppression of the flow in the boundary layer region which clearly manifests the thinning of the boundary layer.

The streamline patterns for different values of viscoelastic parameter k_1^* and permeability parameter k_2 are shown in the Fig. 3. Analysis of the figure shows that the effect of permeability parameter k_2 is to shift the streamline towards the stretching sheet. This means that the boundary layer flow is suppressed and hence the boundary layer thickness decreases. The combined effect of viscoelastic parameter k_1^* and permeability parameter k_2 is seen to suppress the flow largely.

Streamline patterns are depicted in the Fig. 4 for different values of viscoelastic parameter k_1^* and linear mass flux parameter b_0 (that is also the effect of quadratic velocity parameter α). The figure reveals the effect of linear mass flux parameter b_0 in addition to a similar effect of viscoelastic parameter k_1^* on the streamline pattern. The effect of increasing values of the linear mass flux parameter b_0 is shifting of the location of the streamline away from the stretching sheet. This means that the boundary layer flow is enhanced and the boundary layer thickness increases. This result is quite consistent as there is an addition of mass flux through the porous stretching boundary. This change of flow pattern may also be attributed to the quadratic part of the stretching velocity, α , as δ (i.e, b_0) and α appear simultaneously. When the linear mass flux parameter b_0 takes the zero value then slope of the streamline is also zero, which means that the boundary layer flow is completely undisturbed. When the linear mass flux parameter b_0 is negative, the slope of the streamline is also negative which means that the boundary layer flow is suppressed by annihilation of the fluid through the porous boundary.

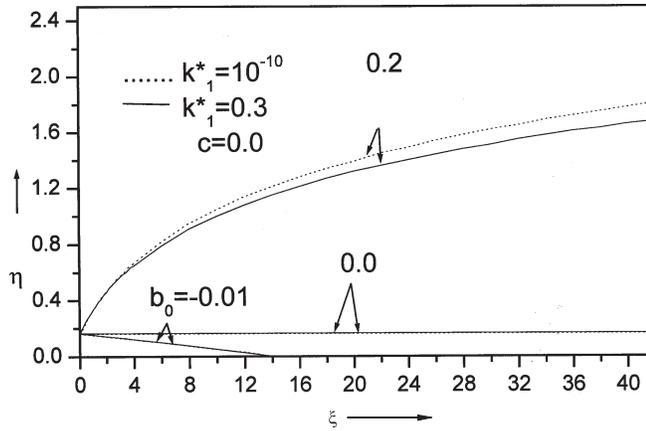


FIG. 4. Streamline patterns for different values $k_1 = 0.1$ and b_0 when $v_w^* = 0.14$, $k_2 = 0.9$ and $M_n = 1.0$.

The combined effect of permeability parameter k_2 and linear mass flux parameter b_0 can be seen from the Fig. 5. From this figure it follows that increasing positive values of linear mass flux parameter b_0 is to enhance the flow and shift

the streamline away from the boundary significantly having a positive slope. Whereas, the negative value of the linear mass flux parameter suppresses the flow and hence, the streamlines take negative slope. The effect of permeability parameter k_2 on the streamline patterns is not significant when linear mass flux parameter b_0 takes negative values. The effect of constant mass flux parameter v_w^* and linear mass flux parameter b_0 on the streamline patterns may be obtained by the analysis of the Fig. 6. Here we observe that the effect of the parameter b_0 , i.e. also of the parameter α , is the same as that seen in the Figs. 4–5.

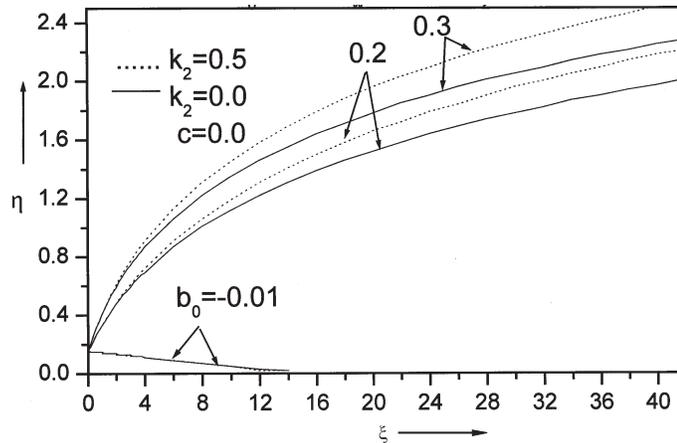


FIG. 5. Streamline patterns for different values k_2 and b_0 when $k_1^* = 0.2$, $v_w = 0.14$ and $M_n = 1.0$.

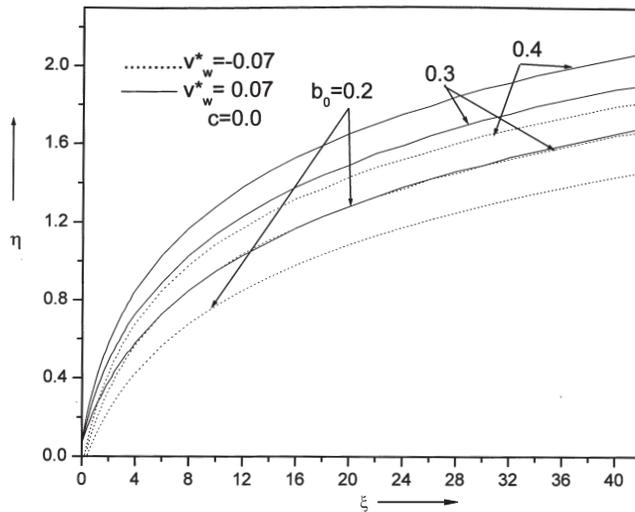


FIG. 6. Streamline patterns for different values of v_w^* and b_0 when $k_1^* = 0.2$, $k_2 = 0.5$ and $M_n = 1.0$.

The effect of constant mass flux parameter v_w^* is to dislocate the streamlines by shifting them away from the stretching sheet. The combined effect of constant mass flux parameter v_w^* and linear mass flux parameter b_0 is to enhance the shifting of position of the streamlines from the stretching sheet. This behaviour occurs due to blowing of the boundary layer flow by addition of the mass flux through the porous stretching sheet.

The values of skin friction parameter C_f for the values of physical parameters k_1^* , k_2 , v_w^* , b_0 , Re_x and M_n are recorded in the Table 1. A careful analysis of the table reveals that the effect of increasing values of permeability parameter k_2 and Hartmann number M_n is to increase the skin friction parameter C_f . The effect of linear mass flux parameter b_0 , i.e. also of α , is also the increase of the skin friction parameter in the case of a viscous fluid flow. The effect of constant flux parameter v_w^* and local Reynolds number Re_x is to reduce the skin friction parameter C_f . These are essentially due to the increase of the boundary layer thickness as a result of addition of external mass the flux through the porous boundary and increased rate of linear stretching resulting in an increase of velocity, respectively.

Hence, in order to minimize the skin friction parameter which we usually look for in an industrial application, one needs to decrease the values of the permeability parameter k_2 , magnetic parameter i.e. the Hartmann number M_n , linear mass flux parameter b_0 and to increase the viscoelastic parameter k_1^* , the constant mass flux parameter v_w^* and the local Reynolds number Re_x .

5. Conclusions

Some of the important findings of the mathematical and diagrammatic analysis of the present problem may be listed as follows.

1. The combined effect of viscoelastic parameter k_1^* and magnetic parameter M_n and permeability parameter k_2 is to shift the location of the streamline towards the stretching sheet.
2. Streamline attains positive, negative and zero slope depending upon the value of the linear mass flux parameter b_0 , positive, negative and zero, respectively.
3. The effect of permeability parameter k_2 is not significant when the linear mass flux parameter b_0 takes zero or negative values.
4. The flow is enhanced significantly by the positive values of linear mass flux parameter b_0 appearing due to quadratic stretching of the boundary. The flow is suppressed significantly by negative values of the linear mass flux parameter.
5. Reduction of the values of the permeability parameter, the Hartmann number and the linear mass flux parameter can be used as means to minimize the skin friction in industrial applications.

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References

1. B.C. SAKIDIS, *Boundary layer behavior on continuous solid surfaces*, Am. Inst. Chem. Eng. J., **7**, 26–28, 1961.
2. H. MARKOVITZ and B.D. COLEMAN, *Advances in Applied Mechanics*, **8**, Academic Press, 1964.
3. K.R. RAJAGOPAL, T.Y. NA and A.S. GUPTA, *Flow of a viscoelastic fluid over a stretching sheet*, Rheol. Acta., **23**, 213–215, 1984.
4. K.R. RAJAGOPAL, T.Y. NA and A.S. GUPTA, *A non-similar boundary layer on a stretching sheet in a non-Newtonian fluid with uniform free stream*, J. Math. Phys., Sc., **21**, 2, 189–200, 1987.
5. B.S. DANDAPAT and A.S. GUPTA, *Flow and heat transfer in a viscoelastic fluid over a stretching sheet*, Int. J. Nonlinear Mech., **24**, 3, 215–219, 1989.
6. D. ROLLINS and K. VAJRAVELU, *Heat transfer in a second order fluid over a continuous stretching surface*, Acta Mech., **89**, 167–178, 1991.
7. H.I. ANDERSSON, *MHD flow of a viscoelastic fluid past a stretching surface*, Acta Mech., **95**, 227–230, 1992.
8. P.S. LAWRENCE and B.N. RAO, *Heat transfer in the flow of a viscoelastic fluid over a stretching sheet*, Acta Mech., **93**, 53–61, 1992.
9. M.I. CHAR, *Heat and mass transfer in a hydromagnetic flow of viscoelastic fluid over a stretching sheet*, J. of Math. Anal. Appl., **186**, 674–689, 1994.
10. B.N. RAO, Technical note, *Flow of a fluid of second grade over a stretching sheet*, Int. J. Nonlinear Mech., **31**, 4, 547–550, 1996.
11. K. VAJRAVELU and J. NAYFEH, *Convective heat transfer at a stretching sheet*, Acta Mech., **96**, 47–54, 1993.
12. K. VAJRAVELU, *Convection heat transfer at a stretching sheet with suction or blowing*, J. of Math. Anal. Appl., **188**, 1002–1011, 1994.
13. N. AHMAD and A. MUBEEN, *Boundary layer flow and heat transfer for the stretching plate with suction*, Int. Comm. Heat Mass Transfer., **22**, 6, 895–906, 1995.
14. T.C. CHIAM, *Magnetohydrodynamic heat transfer over a non-isothermal stretching sheet*, Acta Mechanica, **122**, 169–179, 1997.
15. K.A. YIH, *Note: Blowing suction/effect on non-Darcy forced convection flow about a flat plate with variable wall temperature in porous media*, Acta Mechanica, **131**, 255–265, 1998.
16. M. ACHARYA, L.P. SING and G.C. DASH, *Heat and mass transfer over an accelerating surface with heat source in presence of suction and blowing*, Int. J. Eng. Sc., **37**, 189–211, 1999.

17. K. V. PRASAD, M.S. ABEL and S.K. KHAN, *Momentum and heat transfer in viscoelastic fluid flow in a porous medium over a non-isothermal stretching sheet.*, Int. J. of Num. Method for Heat and Fluid Flow, **10**, 8, 786–801, 2000.
18. R.M. SONTI S.K. KHAN M.S. ABEL and K.V. PRASAD, *Heat and mass transfer in a viscoelastic fluid flow over an accelerating surface with heat source/sink and viscous dissipation*, Heat and Mass Transfer., **38**, 213–220, 2002.
19. B.S. DANDAPAT, L.E. HOLMEDAL and H.I. ANDERSSON, *Stability of flow of a viscoelastic fluid over a stretching sheet*, Arch. Mech., **46**, 6, 829–838, 1994.
20. B.S. DANDAPAT, L.E. HOLMEDAL and H.I. ANDERSSON, *On the stability of MHD flow of a viscoelastic fluid past a stretching sheet*, Acta Mechanica, **130**, 143–146, 1998.
21. P.S. GUPTA and A.S GUPTA, *Heat and mass transfer on a stretching sheet with suction or blowing*, Canad. J. of Chem. Engg., **55**, 744–746, 1977.
22. V. KUMARAN and G. RAMANAIAH, *A note on the flow over a stretching sheet*, Acta Mech., **116**, 229–233, 1996.
23. GUPTA and T. SRIDHAR, *Viscoelastic effect in non-Newtonian flows through porous media*, Rheol Acta., **24**, 148–151, 1985.
24. D. W. BEARD, K. WALTERS, *Elastico – viscous boundary layer flows. 1. Two dimensional flow near a stagnation point*, Proc. Comb. Phil. Soc., **60**, 667–674, 1964.
25. W.C TROY, E.A OVERMAN., H.G.B. ERMENTROUT and J.P. KEENER, *Uniqueness of flow of a second order fluid past a stretching sheet*, Quart. Appl. Math., **44**, 753–755, 1987.
26. WEN-DONG CHANG, *The non-uniqueness of the flow of a viscoelastic fluid over a stretching sheet*, Quart. Appl. Math., **47**, 2, 365–366, 1989.

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