

## Thermosolutal convection in ferromagnetic fluid

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THE THERMOSOLUTAL CONVECTION in ferromagnetic fluid is considered for a fluid layer heated and soluted from below in the presence of uniform vertical magnetic field. For the case of two free boundaries, an exact solution is obtained using a linear stability analysis. For the case of stationary convection, magnetization has a destabilizing effect, whereas stable solute gradient has a stabilizing effect on the onset of instability. Graphs have been plotted by giving numerical values to various parameters, to depict the stability characteristics. The principle of exchange of stabilities is found to hold true for the ferromagnetic fluid heated from below in porous medium in the absence of stable solute gradient. The oscillatory modes are introduced due to the presence of the stable solute gradient, which were non-existent in its absence. A sufficient condition for non-existence of the overstability is also obtained.

**Key words:** thermosolutal instability, ferromagnetic fluid, uniform magnetic field, magnetization.

### 1. Introduction

FERROHYDRODYNAMICS (FHD) deals with the mechanics of fluid motions influenced by strong forces of magnetic polarization. Ferrohydrodynamics concerns usually non-conducting liquids with magnetic properties and constitutes an entire field of physics close to magnetohydrodynamics but still different. The polarization force and the body couple are the two main features that distinguish ferromagnetic fluids from ordinary fluids. Magnetic fluids, called also “ferromagnetic fluids”, are electrically non-conducting colloidal suspensions of solid ferromagnetic particles in a non-electrically conducting carrier fluid like water, kerosene, hydrocarbon etc. A typical ferromagnetic fluid contains  $10^{23}$  particles per cubic meter. These fluids behave as a homogeneous continuum and exhibit a variety of interesting phenomena. Ferromagnetic fluids are not found in nature but are artificially synthesized.

Soon after the method of formation of ferromagnetic fluids in the early or mid-1960s, the importance of ferrohydrodynamics was realized. Due to the wide ranges of applications of ferromagnetic fluid to instrumentation, lubrication, printing, vacuum technology, vibration damping, metals recovery, acoustics and medicine, its commercial usage includes vacuum feed throughs for semiconductor manufacturing and related uses (MOSKOWITZ [1]), pressure seals for compressors and blowers (ROSENSWEIG [2]). It is also used in liquid-cooled loudspeakers which involves small bulk quantities of the ferromagnetic fluid to conduct heat away from the speaker coils (HATHAWAY [3]). This innovation increases the amplifying power of the coil, and hence, it leads to the loudspeaker to produce high-fidelity sound. In order to bring the drugs to a target site in human body, a magnetic field can pilot the path of a drop of ferromagnetic fluid in the human body (MORIMOTO *et al.* [4]). The novel zero-leakage rotating-shaft seals are used in computer disk drives (BAILEY [5]).

Experimental and theoretical physicists and engineers gave significant contributions to ferrohydrodynamics and its applications (ODENBACH [6]). During the last half century, research on magnetic liquids has been very productive in many fields. Strong efforts have been undertaken to synthesize stable suspensions of magnetic particles with different performances in magnetism, fluid mechanics or physical chemistry.

An authoritative introduction to this fascinating subject has been discussed in detail in the celebrated monograph by ROSENSWEIG [7]. This monograph reviews several applications of heat transfer through ferrofluids. One such phenomenon is enhanced convective cooling having a temperature-dependent magnetic moment due to magnetization of the fluid. This magnetization, in general, is a function of the magnetic field, temperature, salinity and density of the fluid. Any variation of these quantities can induce a change of body force distribution in the fluid. This leads to convection in ferromagnetic fluids in the presence of magnetic field gradient. This mechanism is known as ferroconvection, which is similar to Bénard convection (CHANDRASEKHAR [8]). In our analysis, we assume that the magnetization is aligned with the magnetic field. Convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of uniform vertical magnetic field has been considered by FINLAYSON [9]. He explained the concept of thermo-mechanical interaction in ferromagnetic fluids. Thermoconvective stability of ferromagnetic fluids without considering buoyancy effects has been investigated by LALAS and CARMÍ [10], whereas SHLIOMIS [11] analyzed the linearized relation for magnetized perturbed quantities at the limit of instability.

Ferromagnetic fluids are mostly organic solvent carriers having a ferromagnetic salt acting as a solute. The effects of temperature, rotation and porous medium on ferromagnetic fluids as a single-component fluid has been studied by

SEKAR *et al.* [12], SEKAR and VAIDYANATHAN [13], GUPTA and GUPTA [14] and FINLAYSON [9]. SHARMA *et al.* [15] and SUNIL *et al.* [16] have studied thermosolutal instability problems of non-Newtonian fluid in porous medium. They found that the stable solute gradient has a stabilizing effect when it is salted from below. Normally, ferromagnetic fluids are suspension of magnetic salts in carrier organic fluids and hence, it is appropriate to study the convective stability in two-component fluids in which ferric salts are treated as solute and organic carrier as solvents. Hence, the study of convection in the two-component ferromagnetic fluid will now throw more light on convective instability. VAIDYANATHAN *et al.* [17] have studied ferroconvective instability of two-component fluid heated from below and soluted from above. They found that the salinity of ferromagnetic fluid enables the fluid to get destabilized more when it is salted from above. The present paper, therefore, deals with the thermosolutal convection in ferromagnetic fluid heated and soluted from below in the presence of a uniform vertical magnetic field.

## 2. Mathematical formulation of the problem

Here we consider an infinite, horizontal layer of thickness  $d$  of an electrically non-conducting incompressible ferromagnetic fluid, heated and soluted from below. A uniform magnetic field  $H_0$  acts along the vertical direction which is taken as the  $z$ -axis. The temperature and solute concentration at the bottom and top surfaces  $z = \pm \frac{1}{2}d$  are  $T_0, T_1$  and  $C_0, C_1$ , respectively, and a uniform temperature gradient  $\beta \left( = \left| \frac{dT}{dz} \right| \right)$  and a uniform solute gradient  $\beta' \left( = \left| \frac{dC}{dz} \right| \right)$  are maintained (see Fig. 1). The gravity field  $\mathbf{g}(0, 0, -g)$ , pervades the system.

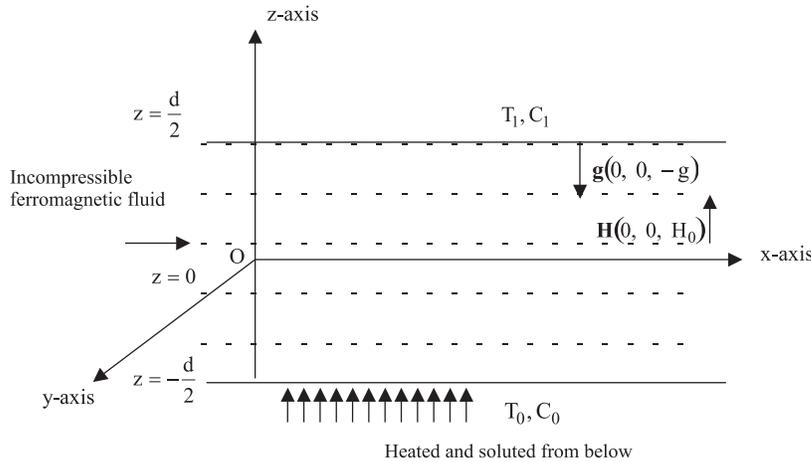


FIG. 1. Geometrical configuration.

The mathematical equations governing the motion of ferromagnetic fluids for the above model are as follows:

The continuity equation for an incompressible ferromagnetic fluid is

$$(2.1) \quad \nabla \cdot \mathbf{q} = 0.$$

The momentum equation is

$$(2.2) \quad \rho_0 \frac{D\mathbf{q}}{Dt} = -\nabla p + \rho \mathbf{g} + \nabla \cdot (\mathbf{H}\mathbf{B}) + \mu \nabla^2 \mathbf{q}.$$

The equations expressing the conservation of temperature and solute concentration are

$$(2.3) \quad \left[ \rho_0 C_{V,H} - \mu_0 \mathbf{H} \cdot \left( \frac{\partial \mathbf{M}}{\partial T} \right)_{V,\mathbf{H}} \right] \frac{DT}{Dt} + \mu_0 T \left( \frac{\partial \mathbf{M}}{\partial T} \right)_{V,\mathbf{H}} \cdot \frac{D\mathbf{H}}{Dt} = K_1 \nabla^2 T + \Phi_T,$$

$$(2.4) \quad \left[ \rho_0 C_{V,H} - \mu_0 \mathbf{H} \cdot \left( \frac{\partial \mathbf{M}}{\partial C} \right)_{V,\mathbf{H}} \right] \frac{DC}{Dt} + \mu_0 C \left( \frac{\partial \mathbf{M}}{\partial C} \right)_{V,\mathbf{H}} \cdot \frac{D\mathbf{H}}{Dt} = K'_1 \nabla^2 C + \Phi_S.$$

The density equation of state is taken as:

$$(2.5) \quad \rho = \rho_0 [1 - \alpha(T - T_a) + \alpha'(C - C_a)],$$

where  $\rho$ ,  $\rho_0$ ,  $\mathbf{q}$ ,  $t$ ,  $p$ ,  $\mu$ ,  $\mu_0$ ,  $H$ ,  $\mathbf{B}$ ,  $C_{V,H}$ ,  $T$ ,  $C$ ,  $\mathbf{M}$ ,  $K_1$ ,  $K'_1$ ,  $\alpha$ ,  $\alpha'$ ,  $\Phi_T$  and  $\Phi_S$  are the fluid density, reference density, velocity, time, pressure, dynamic viscosity (constant), magnetic permeability, magnetic field, magnetic induction, specific heat at constant volume and magnetic field, temperature, solute concentration, magnetization, thermal conductivity, solute conductivity, thermal expansion coefficient, an analogous solvent coefficient of expansion, viscous dissipation factor containing second-order terms in velocity and viscous dissipation factor analogous to  $\Phi_T$  but corresponding to the solute, respectively.  $\Phi_T$  and  $\Phi_S$  being small of second order may be neglected.  $T_a$  is the average temperature given by  $T_a = \frac{(T_0 + T_1)}{2}$ , where  $T_0$  and  $T_1$  are the constant average temperatures of the lower and upper surfaces of the layer and  $C_a$  is the average concentration given by  $C_a = \frac{(C_0 + C_1)}{2}$ , where  $C_0$  and  $C_1$  are the constant average concentrations of the lower and upper surfaces of the layer. In the Eq. (2.2), we assume

that the viscosity is isotropic and independent of the magnetic field. We also use the Boussinesq approximation by allowing the density to change only in the gravitational body force term.

Maxwell's equations, simplified for a non-conducting fluid with no displacement currents, become

$$(2.6) \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = \mathbf{0}.$$

In the Chu formulation of electrodynamics (PENFIELD and HAUS, [18]), the magnetic field, magnetization and the magnetic induction are related by

$$(2.7) \quad \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}).$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field, temperature and salinity, so that

$$(2.8) \quad \mathbf{M} = \frac{\mathbf{H}}{H} M(H, T, C).$$

The magnetic equation of state is linearized about the magnetic field,  $H_0$ , the average temperature,  $T_a$  and the average salinity  $C_a$  to become

$$(2.9) \quad M = M_0 + \chi(H - H_0) - K_2(T - T_a) + K_3(C - C_a),$$

where  $H_0$  is the uniform magnetic field of the fluid layer when placed in an external magnetic field  $\mathbf{H} = \hat{\mathbf{k}} H_0^{\text{ext}}$ ,  $\chi = \left( \frac{\partial M}{\partial H} \right)_{H_0, T_a}$  is the magnetic susceptibility,

$K_2 = - \left( \frac{\partial M}{\partial T} \right)_{H_0, T_a}$  is the pyromagnetic coefficient and  $K_3 = \left( \frac{\partial M}{\partial C} \right)_{H_0, C_a}$  is the magnetic salinity coefficient,  $H$  is magnitude of  $\mathbf{H}$  and  $M_0 = M(H_0, T_a, C_a)$ .

The basic state is assumed to be a quiescent state and is given by

$$(2.10) \quad \begin{aligned} \mathbf{q} &= \mathbf{q}_b = \mathbf{0}, & p &= p_b(z), \\ T &= T_b(z) = -\beta z + T_a, & C &= C_b(z) = -\beta' z + C_a, \\ \beta &= \frac{T_1 - T_0}{d}, & \beta' &= \frac{C_1 - C_0}{d}, \\ \mathbf{H}_b &= \left[ H_0 + \frac{K_2(T_b - T_a)}{1 + \chi} - \frac{K_3(C_b - C_a)}{1 + \chi} \right] \hat{\mathbf{k}}, \\ \mathbf{M}_b &= \left[ M_0 - \frac{K_2(T_b - T_a)}{1 + \chi} + \frac{K_3(C_b - C_a)}{1 + \chi} \right] \hat{\mathbf{k}}, & H_0 + M_0 &= H_0^{\text{ext}}, \end{aligned}$$

where  $\hat{\mathbf{k}}$  is unit vector in the  $z$ -direction.

### 3. The perturbation equations

We shall analyze the stability of the basic state by introducing the following perturbations:

$$(3.1) \quad \begin{aligned} \mathbf{q} &= \mathbf{q}_b + \mathbf{q}', & p &= p_b(z) + \delta p, \\ T &= T_b(z) + \theta, & C &= C_b(z) + \gamma, \\ \mathbf{H} &= \mathbf{H}_b(z) + \mathbf{H}', & \mathbf{M} &= \mathbf{M}_b(z) + \mathbf{M}' \end{aligned}$$

where  $\mathbf{q}' = (u, v, w)$ ,  $\delta p, \theta, \gamma$ ,  $\mathbf{H}'$  and  $\mathbf{M}'$  are perturbations in velocity, pressure, temperature, concentration, magnetic field and magnetization. These perturbations are assumed to be small and then the linearized perturbation equations become

$$(3.2) \quad \rho_0 \frac{\partial u}{\partial t} = -\frac{\partial}{\partial x} \delta p + \mu_0 (M_0 + H_0) \frac{\partial H'_1}{\partial z} + \mu \nabla^2 u,$$

$$(3.3) \quad \rho_0 \frac{\partial v}{\partial t} = -\frac{\partial}{\partial y} \delta p + \mu_0 (M_0 + H_0) \frac{\partial H'_2}{\partial z} + \mu \nabla^2 v,$$

$$(3.4) \quad \begin{aligned} \rho_0 \frac{\partial w}{\partial t} &= -\frac{\partial}{\partial z} \delta p + \mu_0 (M_0 + H_0) \frac{\partial H'_3}{\partial z} \\ &\quad + \mu \nabla^2 w - \frac{\mu_0 K_2 \beta}{1 + \chi} (H'_3 (1 + \chi) - K_2 \theta) \\ &\quad + \frac{\mu_0 K_3 \beta'}{1 + \chi} (H'_3 (1 + \chi) + K_3 \gamma) - \frac{\mu_0 K_2 K_3}{1 + \chi} (\beta' \theta + \beta \gamma) + g \rho_0 (\alpha \theta - \alpha' \gamma), \end{aligned}$$

$$(3.5) \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

$$(3.6) \quad \rho C_1 \frac{\partial \theta}{\partial t} - \mu_0 T_0 K_2 \frac{\partial}{\partial t} \left( \frac{\partial \Phi'_1}{\partial z} \right) = K_1 \nabla^2 \theta + \left[ \rho C_1 \beta - \frac{\mu_0 T_0 K_2^2 \beta}{(1 + \chi)} \right] w,$$

where

$$(3.7) \quad \rho C_1 = \rho_0 C_{V,H} + \mu_0 K_2 H_0,$$

$$(3.8) \quad \rho C_2 \frac{\partial \gamma}{\partial t} - \mu_0 C_0 K_3 \frac{\partial}{\partial t} \left( \frac{\partial \Phi'_2}{\partial z} \right) = K'_1 \nabla^2 \gamma + \left[ \rho C_2 \beta' - \frac{\mu_0 C_0 K_3^2 \beta'}{(1 + \chi)} \right] w,$$

where

$$(3.9) \quad \rho C_2 = \rho_0 C_{V,H} - \mu_0 K_3 H_0.$$

Equations (2.8) and (2.9) yield

$$(3.10) \quad \begin{aligned} H'_3 + M'_3 &= (1 + \chi) H'_3 - K_2 \theta, \\ H'_3 + M'_3 &= (1 + \chi) H'_3 + K_3 \gamma, \\ H'_i + M'_i &= \left(1 + \frac{M_0}{H_0}\right) H'_i \quad (i = 1, 2). \end{aligned}$$

Here we have assumed  $K_2 \beta d \ll (1 + \chi) H_0$ ,  $K_3 \beta' d \ll (1 + \chi) H_0$ . Equation (2.6)<sub>2</sub> means that we can write  $\mathbf{H}' = \nabla (\Phi'_1 - \Phi'_2)$ , where  $\Phi'_1$  is the perturbed magnetic potential and  $\Phi'_2$  is the perturbed magnetic potential analogous to the solute.

The vertical component of Eq. (2.2) becomes

$$(3.11) \quad \begin{aligned} \rho_0 \frac{\partial}{\partial t} \nabla^2 w &= \mu \nabla^2 (\nabla^2 w) - \frac{\mu_0 K_2 \beta}{1 + \chi} \nabla_1^2 \left( (1 + \chi) \frac{\partial}{\partial z} (\Phi'_1 - \Phi'_2) - K_2 \theta \right) \\ &\quad + \frac{\mu_0 K_3 \beta'}{1 + \chi} \nabla_1^2 \left( (1 + \chi) \frac{\partial}{\partial z} (\Phi'_1 - \Phi'_2) + K_3 \gamma \right) \\ &\quad - \frac{\mu_0 K_2 K_3}{1 + \chi} \nabla_1^2 (\beta' \theta + \beta \gamma) + g \rho_0 \nabla_1^2 (\alpha \theta - \alpha' \gamma). \end{aligned}$$

From (3.10), we have

$$(3.12) \quad (1 + \chi) \frac{\partial^2 \Phi'_1}{\partial z^2} + \left(1 + \frac{M_0}{H_0}\right) \nabla_1^2 \Phi'_1 - K_2 \frac{\partial \theta}{\partial z} = 0,$$

$$(3.13) \quad (1 + \chi) \frac{\partial^2 \Phi'_2}{\partial z^2} + \left(1 + \frac{M_0}{H_0}\right) \nabla_1^2 \Phi'_2 - K_3 \frac{\partial \gamma}{\partial z} = 0.$$

#### 4. Normal mode analysis method

Analyzing the disturbances of normal modes, we assume that the perturbation quantities are of the form

$$(4.1) \quad (w, \theta, \gamma, \Phi'_1, \Phi'_2) = [W(z), \Theta(z), \Gamma(z), \Phi_1(z), \Phi_2(z)] \exp i(k_x x + k_y y),$$

where  $k_x, k_y$  are the wave numbers along the  $x$ - and  $y$ -directions respectively,  $k = \sqrt{(k_x^2 + k_y^2)}$  is the resultant wave number.

Equations (3.11), (3.6), (3.8), (3.12) and (3.13) using Eq. (4.1), become

$$(4.2) \quad \rho_0 \frac{\partial}{\partial t} \left( \frac{\partial^2}{\partial z^2} - k^2 \right) W = \mu \left( \frac{\partial^2}{\partial z^2} - k^2 \right)^2 W \\ + \frac{\mu_0 K_2 \beta}{1 + \chi} \left[ (1 + \chi) \frac{\partial}{\partial z} (\Phi_1 - \Phi_2) - K_2 \Theta \right] k^2 \\ - \frac{\mu_0 K_3 \beta'}{1 + \chi} \left[ (1 + \chi) \frac{\partial}{\partial z} (\Phi_1 - \Phi_2) + K_3 \Gamma \right] k^2 \\ + \frac{\mu_0 K_2 K_3}{1 + \chi} \left[ \beta' \Theta + \beta \Gamma \right] k^2 - \rho_0 g \left( \alpha \Theta - \alpha' \Gamma \right) k^2,$$

$$(4.3) \quad \rho C_1 \frac{\partial \Theta}{\partial t} - \mu_0 T_0 K_2 \frac{\partial}{\partial t} \left( \frac{\partial \Phi_1}{\partial z} \right) \\ = K_1 \left( \frac{\partial^2}{\partial z^2} - k^2 \right) \Theta + \left( \rho C_1 \beta - \frac{\mu_0 T_0 K_2^2 \beta}{1 + \chi} \right) W,$$

$$(4.4) \quad \rho C_2 \frac{\partial \Gamma}{\partial t} - \mu_0 C_0 K_3 \frac{\partial}{\partial t} \left( \frac{\partial \Phi_2}{\partial z} \right) \\ = K_1' \left( \frac{\partial^2}{\partial z^2} - k^2 \right) \Gamma + \left( \rho C_2 \beta' - \frac{\mu_0 C_0 K_3^2 \beta'}{1 + \chi} \right) W,$$

$$(4.5) \quad (1 + \chi) \frac{\partial^2 \Phi_1}{\partial z^2} - \left( 1 + \frac{M_0}{H_0} \right) k^2 \Phi_1 - K_2 \frac{\partial \Theta}{\partial z} = 0,$$

$$(4.6) \quad (1 + \chi) \frac{\partial^2 \Phi_2}{\partial z^2} - \left( 1 + \frac{M_0}{H_0} \right) k^2 \Phi_2 - K_3 \frac{\partial \Gamma}{\partial z} = 0.$$

Equations (4.2) – (4.6) give the following dimensionless equations:

$$(4.7) \quad \frac{\partial}{\partial t^*} (D^2 - a^2) W^* = (D^2 - a^2)^2 W \\ + aR^{1/2} \left[ (M_1 - M_4) D\Phi_1^* - (1 + M_1 - M_4) T^* \right] \\ + aS^{1/2} \left[ (M_1' - M_4') D\Phi_2^* + (1 - M_1' + M_4') C^* \right],$$

$$(4.8) \quad P_r \frac{\partial T^*}{\partial t^*} - P_r M_2 \frac{\partial}{\partial t^*} (D\Phi_1^*) = (D^2 - a^2) T^* + aR^{1/2} (1 - M_2) W^*,$$

$$(4.9) \quad P_r' \frac{\partial C^*}{\partial t^*} - P_r' M_2' \frac{\partial}{\partial t^*} (D\Phi_2^*) = (D^2 - a^2) C^* + aS^{1/2} (1 - M_2') W^*,$$

$$(4.10) \quad D^2 \Phi_1^* - a^2 M_3 \Phi_1^* - DT^* = 0,$$

$$(4.11) \quad D^2 \Phi_2^* - a^2 M_3 \Phi_2^* - DC^* = 0,$$

where the following non-dimensional parameters are introduced:

$$(4.12) \quad \begin{aligned} t^* &= \frac{\nu t}{d^2}, & W^* &= \frac{Wd}{\nu}, \\ \Phi_1^* &= \frac{(1 + \chi) K_1 a R^{1/2}}{K_2 \rho C_1 \beta \nu d^2} \Phi_1, & \Phi_2^* &= \frac{(1 + \chi) K_1' a S^{1/2}}{K_3 \rho C_2 \beta' \nu d^2} \Phi_2, \\ R &= \frac{g \alpha \beta d^4 \rho C_1}{\nu K_1}, & S &= \frac{g \alpha' \beta' d^4 \rho C_2}{\nu K_1'}, \\ T^* &= \frac{K_1 a R^{1/2}}{\rho C_1 \beta \nu d} \Theta, & C^* &= \frac{K_1' a S^{1/2}}{\rho C_2 \beta' \nu d} \Gamma, \\ a &= kd, & z^* &= \frac{z}{d}, & D &= \frac{\partial}{\partial z^*}, \\ P_r &= \frac{\nu}{K_1} \rho C_1, & P_r' &= \frac{\nu}{K_1'} \rho C_2, \\ M_1 &= \frac{\mu_0 K_2^2 \beta}{(1 + \chi) \alpha \rho_0 g}, & M_1' &= \frac{\mu_0 K_3^2 \beta'}{(1 + \chi) \alpha' \rho_0 g}, \\ M_2 &= \frac{\mu_0 T_0 K_2^2}{(1 + \chi) \rho C_1}, & M_2' &= \frac{\mu_0 C_0 K_3^2}{(1 + \chi) \rho C_2}, \\ M_3 &= \frac{\left(1 + \frac{M_0}{H_0}\right)}{(1 + \chi)}, & M_4 &= \frac{\mu_0 K_2 K_3 \beta'}{(1 + \chi) \alpha \rho_0 g}, \\ M_4' &= \frac{\mu_0 K_2 K_3 \beta}{(1 + \chi) \alpha' \rho_0 g}, & M_5 &= \frac{M_4}{M_1} = \frac{M_1'}{M_4'} = \frac{K_3 \beta'}{K_2 \beta}. \end{aligned}$$

## 5. Exact solution for free boundaries

Consider the case where both boundaries are free as well as perfect conductors of heat. The case of two free boundaries is of little physical interest, but it is

mathematically important because it enables us to find the exact solutions, whose properties guide our analysis below.

The boundary conditions are

$$(5.1) \quad W^* = D^2W^* = T^* = C^* = D\Phi_1^* = D\Phi_2^* = 0 \quad \text{at} \quad z = \pm \frac{1}{2}.$$

Following the analysis of Finlayson, the exact solutions satisfying the boundary conditions are given by

$$(5.2) \quad \begin{aligned} W^* &= A_0 e^{\sigma t^*} \cos \pi z^*, & T^* &= B_0 e^{\sigma t^*} \cos \pi z^*, & D\Phi_1^* &= C_0 e^{\sigma t^*} \cos \pi z^*, \\ \Phi_1^* &= \left( \frac{C_0}{\pi} \right) e^{\sigma t^*} \sin \pi z^*, & \Phi_2^* &= \left( \frac{E_0}{\pi} \right) e^{\sigma t^*} \sin \pi z^*, \\ D\Phi_2^* &= E_0 e^{\sigma t^*} \cos \pi z^*, & C^* &= F_0 e^{\sigma t^*} \cos \pi z^*, \end{aligned}$$

where  $A_0, B_0, C_0, E_0, F_0$  are constants and  $\sigma$  is the growth rate which is, in general, a complex constant.

Substituting Eq. (5.2) in Eqs. (4.7)–(4.11) and dropping asterisks for convenience, we get the following five linear, homogeneous algebraic equations

$$(5.3) \quad \begin{aligned} (\pi^2 + a^2) (\pi^2 + a^2 + \sigma) A_0 - aR^{1/2} (1 + M_1 - M_4) B_0 \\ + aR^{1/2} (M_1 - M_4) C_0 + aS^{1/2} (1 - M'_1 + M'_4) F_0 \\ + aS^{1/2} (M'_1 - M'_4) E_0 = 0, \end{aligned}$$

$$(5.4) \quad aR^{1/2} (1 - M_2) A_0 - (\pi^2 + a^2 + P_r \sigma) B_0 + (P_r M_2 \sigma) C_0 = 0,$$

$$(5.5) \quad aS^{1/2} (1 - M'_2) A_0 - (\pi^2 + a^2 + P'_r \sigma) F_0 + (P'_r M'_2 \sigma) E_0 = 0,$$

$$(5.6) \quad -\pi^2 B_0 + (\pi^2 + a^2 M_3) C_0 = 0,$$

$$(5.7) \quad -\pi^2 F_0 + (\pi^2 + a^2 M_3) E_0 = 0.$$

For existence of non-trivial solutions of the above equations, the determinant of the coefficients of  $A_0, B_0, C_0, E_0, F_0$  in Eqs. (5.3)–(5.7) must vanish. This determinant after simplification yields

$$(5.8) \quad -iU\sigma_1^3 - V\sigma_1^2 + iW\sigma_1 + X = 0,$$

where

$$(5.9) \quad U = (1+x) P_r P_r' \left[ \{(1-M_2) + xM_3\} \left\{ (1-M_2') + xM_3 \right\} \right]$$

$$(5.10) \quad V = (1+x)^2 \begin{bmatrix} (1+xM_3) \left[ P_r' \left\{ (1-M_2') + xM_3 \right\} \right. \\ \left. + P_r \{(1-M_2) + xM_3\} \right] \\ \left. + P_r P_r' \{(1-M_2) + xM_3\} \left\{ (1-M_2') + xM_3 \right\} \right] \end{bmatrix},$$

$$(5.11) \quad W = (1+x)^3 (1+xM_3) \left[ (1+xM_3) \right. \\ \left. + \left[ P_r' \left\{ (1-M_2') + xM_3 \right\} + P_r \left\{ (1-M_2) + xM_3 \right\} \right] \right] \\ \left. + xS_1 P_r (1-M_2') \left\{ (1-M_2) + xM_3 \right\} \right. \\ \left. \cdot \left\{ (M_1' - M_4') + (1+xM_3) (1-M_1' + M_4') \right\} \right. \\ \left. - xR_1 P_r' (1-M_2) \left\{ (1-M_2') + xM_3 \right\} \left\{ 1+xM_3 (1+M_1 - M_4) \right\}, \right]$$

$$(5.12) \quad X = (1+x) (1+xM_3) \begin{bmatrix} (1+x)^3 (1+xM_3) \\ + xS_1 (1-M_2') \left\{ (M_1' - M_4') \right. \\ \left. + (1+xM_3) (1-M_1' + M_4') \right\} \\ \left. - xR_1 (1-M_2) \left\{ 1+xM_3 (1+M_1 - M_4) \right\} \right] \end{bmatrix},$$

where  $R_1 = \frac{R}{\pi^4}$ ,  $S_1 = \frac{S}{\pi^4}$ ,  $x = \frac{a^2}{\pi^2}$  and  $i\sigma_1 = \frac{\sigma}{\pi^2}$ .

## 6. The case of stationary convection

When the instability sets in as stationary convection ( $M_2 \cong 0$  and  $M_2' \cong 0$ ), the marginal state will be characterized by  $\sigma_1 = 0$ , then the Rayleigh number

is given by

$$(6.1) \quad R_1 = \frac{(1+x)^3(1+xM_3)}{x\{(1+xM_3)+xM_3M_1(1-M_5)\}} + \frac{S_1M_1' \left[ \frac{1}{M_1'} + x\frac{M_3}{M_1'} + xM_3 \left\{ \frac{1}{M_5} - 1 \right\} \right]}{\{(1+xM_3)+xM_3M_1(1-M_5)\}},$$

which expresses the modified Rayleigh number  $R_1$  as a function of the dimensionless wave number  $x$ , the magnetization parameter  $M_3$ , stable solute gradient parameter  $S_1$  and the ratio of the salinity effect on the magnetic field to pyromagnetic coefficient  $M_5$ .

To investigate the effects of magnetization and stable solute gradient, we examine the behaviour of  $dR_1/dM_3$ ,  $dR_1/dS_1$  analytically. Eq. (6.1) yields

$$(6.2) \quad \frac{dR_1}{dM_3} = -\frac{(1+x)^3}{x} \left[ \frac{xM_1(1-M_5)}{\{1+xM_3+xM_3M_1(1-M_5)\}^2} \right] - S_1M_1' \left[ \frac{x(1-M_5) \left\{ \frac{M_1}{M_1'} - \frac{1}{M_5} \right\}}{\{1+xM_3+xM_3M_1(1-M_5)\}^2} \right],$$

$$(6.3) \quad \frac{dR_1}{dS_1} = \frac{M_1' \left[ \frac{1}{M_1'} + x\frac{M_3}{M_1'} + xM_3 \left\{ \frac{1}{M_5} - 1 \right\} \right]}{\{(1+xM_3)+xM_3M_1(1-M_5)\}}.$$

This shows that, for a stationary convection, the magnetization has a destabilizing effect, whereas the stable solute gradient has a stabilizing effect on the onset of instability.

The critical Rayleigh number for the onset of instability is determined by the condition  $dR_1/dx = 0$ . When  $M_1 = 0$  and  $M_1' = 0$ , then from Eq. (6.1), we get

$$(6.4) \quad x_c = \frac{1}{2} \quad \text{with} \quad R_c = \frac{27}{4} + S_1.$$

For  $M_1$  sufficiently large, we obtain the results for the magnetic mechanism:

$$(6.5) \quad N = R_1M_1 = \frac{(1+x)^3(1+xM_3)}{x^2M_3(1-M_5)} + \frac{S_1 \left\{ 1 + xM_3 + xM_1'M_3 \left( \frac{1}{M_5} - 1 \right) \right\}}{xM_3(1-M_5)},$$

where  $N$  is the magnetic thermal Rayleigh number.

The critical magnetic Rayleigh number for the onset of instability is determined by the condition  $dN/dx = 0$ .

The critical magnetic Rayleigh number,  $N_c$ , depends on the magnetization parameter  $M_3$ , ratio of the salinity effect on magnetic field to pyromagnetic coefficient  $M_5$  and stable solute gradient  $S_1$ . In the particular case

$$x_c = 1, \quad N_c = 17.78 \quad \text{for} \quad M_3 = 1, \quad S_1 = 0$$

and

$$x_c = \frac{1}{2}, \quad N_c = 7.5, \quad \text{for} \quad M_3 \rightarrow \infty, \quad S_1 = 0$$

and intermediate values for intermediate  $M_3$ .

The dispersion relation (6.1) is analyzed numerically. In Fig. 2,  $R_1$  is plotted against the wave number  $x$  for  $M_1 = 1000$ ,  $M'_1 = 0.5$ ,  $M_5 = 0.1$ ,  $S_1 = 500$ ;  $M_3 = 1, 3, 5, 7$ . In Fig. 3,  $R_1$  is plotted against the wave number  $x$  for  $M_1 = 1000$ ,  $M'_1 = 0.5$ ,  $M_5 = 0.5$ ,  $M_3 = 1$ ;  $S_1 = 100, 200, 300, 400$ . It is clear that the magnetization has destabilizing effects as the Rayleigh number decreases with the increase of the magnetization parameter whereas stable solute gradient has a stabilizing effect as the Rayleigh number increases with the increase in stable solute parameter. In Fig. 4,  $R_1$  is plotted against the wave number  $x$  for  $M_1 = 1000$ ,  $M'_1 = 0.5$ ,  $M_3 = 1$ ,  $S_1 = 500$ ;  $M_5 = 0.1, 0.2, 0.3, 0.4$ . The increase in  $M_5$ , which represents the ratio of the salinity effect on magnetic field to pyromagnetic coefficient, reduced  $R_1$ . Therefore, magnetic parameter  $M_5$  has also a destabilizing effect on the system.

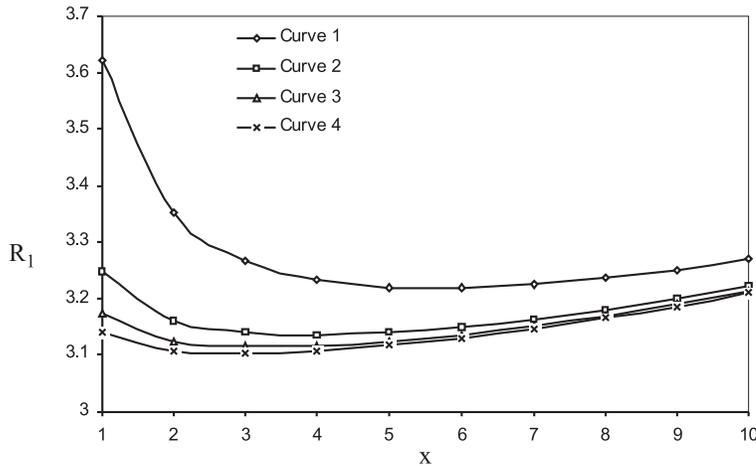


FIG. 2. Variation of Rayleigh number ( $R_1$ ) with wave number ( $x$ ) for  $M_1 = 1000$ ,  $M'_1 = 0.5$ ,  $M_5 = 0.1$ ,  $S_1 = 500$ ;  $M_3 = 1$  for Curve 1,  $M_3 = 3$  for Curve 2,  $M_3 = 5$  for Curve 3, and  $M_3 = 7$  for Curve 4.

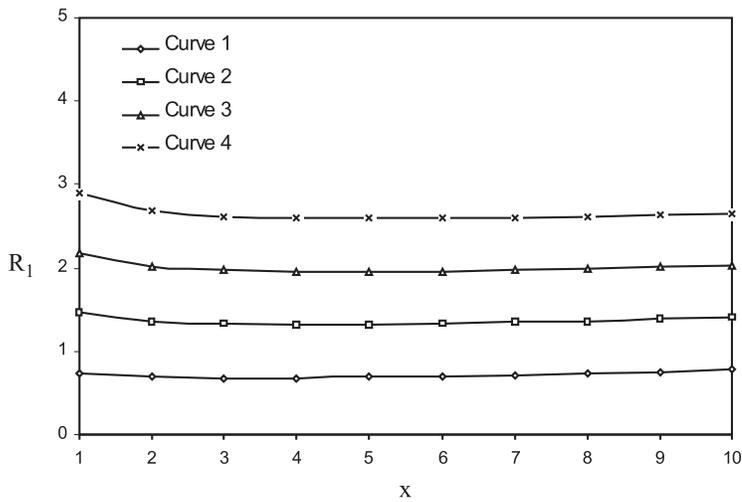


FIG. 3. Variation of Rayleigh number ( $R_1$ ) with wave number ( $x$ ) for  $M_1 = 1000$ ,  $M_1' = 0.5$ ,  $M_5 = 0.1$ ,  $M_3 = 1$ ;  $S_1 = 100$  for Curve 1,  $S_1 = 200$  for Curve 2,  $S_1 = 300$  for Curve 3 and  $S_1 = 400$  for Curve 4.

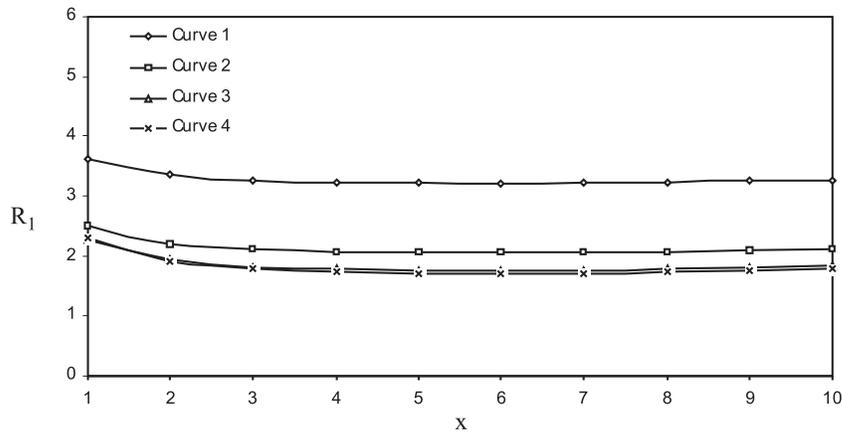


FIG. 4. Variation of Rayleigh number ( $R_1$ ) with wave number ( $x$ ) for  $M_1 = 1000$ ,  $M_1' = 0.5$ ,  $S_1 = 500$ ,  $M_3 = 1$ ;  $M_5 = 0.1$  for Curve 1,  $M_5 = 0.2$  for Curve 2,  $M_5 = 0.3$  for Curve 3 and  $M_5 = 0.4$  for Curve 4.

Suggestion by FINLAYSON [9] and GUPTA and GUPTA [14] have also been taken for variation of these parametric values. There are many types of ferrofluids formed by changing ferric oxides and carrier organic fluids. In the present analysis, the range of values pertaining to ferric oxide, kerosene and other organic carriers are chosen. With the same ferric oxide, the different carriers like alcohol, hydrocarbon, ester, halocarbon, silicon could be chosen.

For such fluids,  $M_2$  is assumed to have a negligible value and hence it is taken to be zero (SEKAR and VAIDYANATHAN [13]).  $M_3$  is varied from 1 to 25 because  $M_3$  can not take a value smaller than 1 (VAIDYANATHAN *et al.* [19]).  $M'_1$  (the effect on magnetization due to salinity) is allowed to vary from 0.1 to 0.5, taking values smaller than the magnetization parameter  $M_3$ .  $M_5$  represents the ratio of the salinity effect on magnetic field to pyromagnetic coefficient. This varies between 0.1 and 0.5. The salinity Rayleigh number  $S_1$  varies from  $-500$  to  $+500$ . In the above analysis, the magnetization parameter  $M_1$  is taken to be 1000. These values are used for analysis in the present paper.

## 7. Principle of exchange of stabilities

Here we examine the possibility of oscillatory modes, if any, on the stability problem due to the presence of stable solute parameter and magnetization parameter. Equating the imaginary parts of Eq. (5.8), we obtain

$$(7.1) \quad \sigma_1 \left[ \begin{array}{l} - (1+x)^3 (1+xM_3) \left\{ (1+xM_3) + \left[ P'_r \left\{ (1-M'_2) + xM_3 \right\} \right. \right. \\ \left. \left. + P_r \left\{ (1-M_2) + xM_3 \right\} \right] \right\} \\ - xS_1 P_r (1-M'_2) \left\{ (1-M_2) + xM_3 \right\} \left\{ (M'_1 - M'_4) \right. \\ \left. + (1+xM_3) (1-M'_1 + M'_4) \right\} \\ + xR_1 P'_r (1-M_2) \left\{ (1-M'_2) + xM_3 \right\} \\ \left. \cdot \left\{ 1+xM_3 (1+M_1 - M_4) \right\} \right. \\ \left. + \sigma_1^2 \left[ P_r P'_r (1+x) \left\{ (1-M_2) + xM_3 \right\} \left\{ (1-M'_2) + xM_3 \right\} \right] \right] = 0. \end{array} \right.$$

It is clear from Eq. (7.1) that  $\sigma_1$  may be either zero or non-zero, meaning that the modes may be either non-oscillatory or oscillatory. In the absence of stable solute gradient [ $S_1 = 0$  and consequently  $P'_r = 0$ ], Eq. (7.1) reduces to

$$(7.2) \quad \sigma_1 [(1+x)(1+xM_3) + P_r \{(1-M_2) + xM_3\}] = 0.$$

Here the quantity inside the brackets is positive definite because the typical values of  $M_2$  are  $+10^{-6}$  (FINLAYSON, [9]). Thus  $\sigma_1 = 0$ , which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied in the absence of stable solute gradient. The oscillatory modes are introduced due to the presence of the stable solute gradient, which were non-existent in its absence.

## 8. The case of overstability

The present section is devoted to determine the possibility as to whether instability may occur as overstability. Since we wish to determine the Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (5.8) will admit the solutions with real values of  $\sigma_1$ .

Equating real and imaginary parts of (5.8) and eliminating  $R_1$  between them, we obtain

$$(8.1) \quad A_1 \sigma_1^2 + A_0 = 0,$$

where

$$(8.2) \quad A_1 = (1+x) P_r'^2 \left\{ (1-M_2') + xM_3 \right\}^2 \left[ (1+xM_3) + P_r \{ (1-M_2) + xM_3 \} \right],$$

$$(8.3) \quad A_0 = (1+xM_3) \left[ \begin{array}{l} (1+x)^3 (1+xM_3) \left\{ (1+xM_3) \right. \\ \quad \left. + P_r \{ (1-M_2) + xM_3 \} \right\} \\ \quad + xS_1 (1-M_2') \left\{ (M_1' - M_4') \right\} \\ \quad \left. + (1+xM_3) (1-M_1' + M_4') \right\} \\ \left[ P_r \{ (1-M_2) + xM_3 \} - P_r' \left\{ (1-M_2') + xM_3 \right\} \right] \end{array} \right].$$

Since  $\sigma_1$  is real for overstability, both values of  $\sigma_1$  are positive. But  $\sigma_1^2$  is always negative if  $A_0$  is positive (because  $A_1 > 0$ ). It is clear from Eq. (8.3) that  $A_0$  is always positive if

$$(8.4) \quad P_r > P_r' \quad \text{and} \quad P_r > \frac{P_r'}{(1-M_2)},$$

which implies that

$$(8.5) \quad K_1' > K_1 \left[ \frac{\rho C_2}{\rho C_1 \left\{ 1 - \frac{\mu_0 T_0 K_2^2}{(1+\chi) \rho C_1} \right\}} \right],$$

however  $P_r > P_r'$  is already satisfied in the above condition.

Thus, for  $K'_1 > K_1 \left[ \frac{\rho C_2}{\rho C_1 \left\{ 1 - \frac{\mu_0 T_0 K_2^2}{(1 + \chi) \rho C_1} \right\}} \right]$ , overstability cannot occur

and the principle of the exchange of stabilities is valid. Hence the above condition is a sufficient condition for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability, whereas in absence of the magnetic parameters, the above condition, as expected, reduces to  $K'_1 > K_1$ , i.e. the solute conductivity is greater than thermal conductivity.

## 9. Conclusions

In this paper, we have studied the thermosolutal convection in ferromagnetic fluid heated and soluted from below in the presence of a uniform vertical magnetic field. In the preceding sections, we have investigated the effects of magnetization and stable solute gradient on the onset of convection. The principal conclusions from the analysis of this paper are as follows:

1. For the case of stationary convection, the magnetization speeds up the onset of convection what is evident from Eq. (6.2). The results obtained for different values of magnetization parameter are depicted in Fig. 2. It is seen that the magnetization accelerates the onset of convection and this is in agreement with the analytical result.
2. Figure 4 is a plot of  $R_1$  versus the wave number  $x$  for various values of  $M_5$ .  $M_5$  is the ratio of the salinity effect on magnetic field to pyromagnetic coefficient. As  $M_5$  increases  $R_1$  decreases. Hence  $M_5$  has also a destabilizing effect on the onset of instability. From this figure we also observe that the ferromagnetic fluid layer is slightly destabilized.
3. From Eq. (6.3), we observe that the stable solute gradient postpones the onset of instability. Figure 3 illustrates the stabilizing effect of stable solute gradient. The stabilizing effect of stable solute gradient is accounted by VERONIS [20] and is found to be valid also for a ferromagnetic fluid.
4. The principle of exchange of stabilities is found to hold true for the ferromagnetic fluid heated from below in the absence of stable solute gradient. The oscillatory modes are introduced due to the presence of the stable solute gradient, which were non-existent in its absence.

5. The condition, i.e.  $K'_1 > K_1 \left[ \frac{\rho C_2}{\rho C_1 \left\{ 1 - \frac{\mu_0 T_0 K_2^2}{(1 + \chi) \rho C_1} \right\}} \right]$  is the sufficient

condition for non-existence of overstability. In absence of the magnetic

parameters, the above condition, as expected, reduces to  $K_1' > K_1$  i.e. solute conductivity is greater than thermal conductivity.

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