Non-Newtonian flows over an oscillating plate with variable suction

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THE FLOW OF second order fluid due to an oscillating infinite plate in the presence of a transverse magnetic field for two forms of time-dependent suction are considered. The analytical solutions of the governing boundary value problems are obtained. It is found that an external magnetic field and normal stress coefficient on the flow has opposite effects.

1. Introduction

IN THE PAST few years there has been a considerable interest in the oscillating flows due to possible applications in engineering. The study of such flows was first initiated by LIGHTHILL [1] who studied the effects of free-stream oscillations on the boundary layer flow of a viscous, incompressible fluid past an infinite plate. Thereafter STUART [2] extended it to study a two-dimensional flow past an infinite, porous plate with constant suction when the free-stream oscillates in time about a constant mean. The boundary layer suction is a very efficient method for the prevention of separation. The effects of different arrangements and configurations of the suction holes and slots on the undesired phenomenon of separation have been studied extensively by various scholars and have been compiled by LACHMANN [3].

Due to the development of practical boundary layer control systems, interest in problems concerning suction have been renewed. This problem has also been useful in the study of unsteady flow. Watson [4] generalized the Stuart's problem to the case of an arbitrary free-stream velocity. Later, KALONI [5] and MESSIHA [6] extended Stuart's problem to the case of constant and variable suction respectively. Further, SOUNDALGEKAR and PURI [7] discussed the fluctuating flow of an elastico-viscous fluid past an infinite plate with variable suction.

Using the viscous fluid model, the flow of a fluid near a porous oscillating infinite plane has been investigated in SCHLICHTING [8]. RAJAGOPAL [9, 10]

discussed the flows of second and third order fluids due to a rigid plate oscillating in its own plane. Later, FOOTE *et al.* [11] examined the flow of an oscillating porous plate for an elastico-viscous fluid. PURI [12] studied an oscillating rotating flow of an elastico-viscous fluid. Morerecently, HAYAT *et al.* [13–15] analyzed some periodic flows of a second order fluid. TURBATU *et al.* [16] generalized the viscous fluid flow problem of an oscillating flat plate in two directions. They first considered the oscillating flat plate with superimposed blowing or suction. The second generalization is concerned with an increasing or decreasing velocity amplitude of the oscillating flat plate.

On the other hand in view of the increasing technical applications using the magnetohydrodynamic effect, it is desirable to extend many of the available hydrodynamic solutions to include the effects of magnetic fields for those cases when the fluid is electrically conducting. Flow past a flat plate has been studied by ROSSOW [17]. He has considered transverse magnetic field on the flow. SURYAPRAKASRAO [18, 19] investigated the effects of transverse magnetic field on the fluctuating free-stream velocity when the plate is subjected to a constant suction velocity. Boundary layer flows of fluids of small electrical conductivity are important particularly in the field of aeronautical engineering. Further, in technological fields boundary layer phenomenon in non-Newtonian fluids is also being studied extensively. Therefore, it is of interest to analyze the effects of magnetic field on the flow of second order, incompressible and electrically conducting fluid over an infinite oscillating plate with variable suction.

The object of Sec. 2 is to investigate the effect of the variable suction velocity of the form $v'_0(1+ \in Ae^{i\omega't'})$ as assumed by MESSIHA [6]. It is of interest to study how second order results get modified due to the conducting fluid over an oscillating porous plate. In Sec. 3 we assumed the suction velocity of the form $v_0[1+\delta(e^{i\omega t}+e^{-i\omega t})]$ as in KELLY [20]. Detailed study is made in order to extend the Kelly's results [20] of viscous fluid past an infinite plate with time dependent suction to the second order and electrically conducting fluid over an oscillating porous plate. Thus, in this section the combined effects of second order fluid and a magnetic field are considered.

2. Problem formulation

Consider the two dimensional flow of an incompressible and electrically conducting second order fluid over a porous oscillating plate of infinite extent, which occupies the plane $y' = 0$. The geometry of the problem is shown in the Fig. 1. Let u' and v' be the velocity components parallel and normal to the plate respectively. We look for a solution for the velocities which is independent of x', the distance parallel to the plate. Then the continuity equation requires that v' is at most a function of time and therefore retains its value at the plate throughout

the flow. Hence, following MESSIHA [6] and SOUNDALGEKAR [7] we consider v' for the first boundary value problem as

$$(2.1) \qquad v' = -v'_\circ(1 + \in Ae^{i\omega' t'}),$$

where $v'_\circ$ is a non-zero constant mean suction velocity, ω' is the angular frequency, $\in$ is small and A is real positive constant such as $\in A \ll 1$. By neglecting higher powers of $\in$ approximate solutions are obtained for the velocity field in the boundary layer. The negative sign in Eq. (2.1) shows that the suction velocity normal to the wall is directed towards the wall. Further, the conducting fluid is permeated by an imposed uniform magnetic field $\mathbf{B} = [0, B_\circ, 0]$ which acts in the positive y'-direction normal to the sheet. In the low magnetic Reynolds number approximation (SHERCLIFF [22]), in which the induced magnetic field can be ignored, the magnetic body force $\mathbf{j} \times \mathbf{B}$ becomes $\sigma(\mathbf{V} \times \mathbf{B}) \times \mathbf{B}$ when imposed and induced electric fields are negligible and only the magnetic field $\mathbf{B}$ contributes to the current $\mathbf{j} = \sigma(\mathbf{V} \times \mathbf{B})$. Here, σ is the electrical conductivity of the fluid, which has density ρ'. The constitutive equation of a homogeneous incompressible fluid of second order is

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2,$$

where $\mathbf{T}$ is the Cauchy stress tensor, $\mathbf{A}_1$ and $\mathbf{A}_2$ are the well known first two Rivlin-Ericken tensors, μ is the dynamic viscosity, α_1 and α_2 are normal stress moduli and p is the pressure. In view of $\mathbf{T}$ the momentum equation in absence of modified pressure gradient gives

$$(2.2) \qquad \frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + \alpha^* \left[\frac{\partial^3 u'}{\partial y'^2 \partial t'} + v'\frac{\partial^3 u'}{\partial y'^3} \right] - \frac{\sigma B_\circ^2 u'}{\rho'},$$

where

$$\nu = \frac{\mu}{\rho'}, \qquad \alpha^* = \frac{\alpha_1}{\rho'}.$$

In above equation α_1 is the material constant. For fluids to have motions which are compatible with thermodynamics in the sense of Clausius-Duhem inequality and the condition that the Helmholtz free energy be a minimum when the fluid is at rest, the following conditions must be satisfied [23]

$$\mu \geq 0, \qquad \alpha_1 \geq 0, \qquad \alpha_1 + \alpha_2 = 0.$$

The relevant boundary conditions for the problem are

$$(2.3) \qquad u'(0, t') = U'_\circ \in e^{i\omega' t'},$$

$$(2.4) \qquad \underset{y' \to \infty}{\text{Limit}} \ u'(y', t') = 0.$$
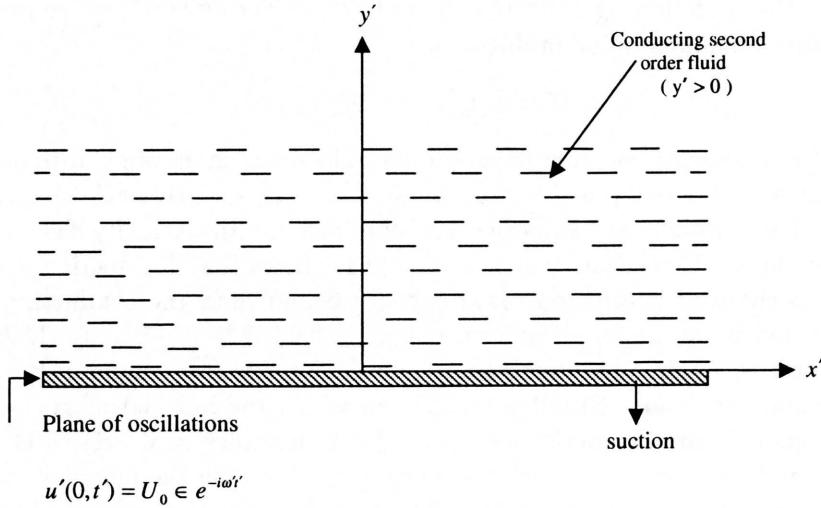
$$u'(0,t') = U_0 \in e^{-i\omega't'}$$

Fig. 1. Physical model under consideration.

It should be noted that for $\alpha_1 = B_0 = 0$ we are left with the equations governing the flow of a non-conducting Newtonian fluid over an oscillating porous plate.

2.1. Solution of the first boundary value problem

Following [7] we take a solution of the form

$$(2.5) \qquad u'(y', t') = U_0' \left[f_1(y') + \in e^{i\omega't'} f_2(y') \right].$$

Now using Eqs. (2.1) and (2.5) in Eqs. (2.2) to (2.4), comparing harmonic and non-harmonic terms and neglecting coefficients of $\in^2$, we get

$$(2.6) \qquad \alpha \frac{d^3 f_1}{d\eta^3} - \frac{d^2 f_1}{d\eta^2} - \frac{df_1}{d\eta} + N f_1 = 0,$$

$$(2.7) \qquad \alpha \frac{d^3 f_2}{d\eta^3} - \left(1 + \frac{1}{4} i\omega\alpha\right) \frac{d^2 f_2}{d\eta^2} - \frac{df_2}{d\eta} + \left(\frac{i\omega}{4} + N\right) f_2 = A \frac{df_1}{d\eta} - \alpha A \frac{d^3 f_1}{d\eta^3},$$

$$(2.8) \qquad \begin{aligned} f_1 &= 0, & f_2 &= 1, & \text{at } \eta = 0, \\ f_1 &= 0, & f_2 &= 0, & \text{as } \eta \to \infty, \end{aligned}$$

where

(2.9)
$$\eta = \frac{y'v_\circ'}{\nu}, \qquad t = \frac{v_\circ'^2 t'}{4\nu}, \qquad \omega = \frac{4\nu\omega'}{v_\circ'^2},$$
$$u = \frac{u'}{U_\circ'}, \qquad \alpha = \frac{\alpha^* v_\circ'^2}{\nu^2}, \qquad N = \frac{\sigma\nu B_\circ^2}{\rho' v_\circ'^2}.$$

During the past three decades there have been several studies of boundary layer flows of non-Newtonian fluids. These investigations have been for non-Newtonian fluids of the differential type [24]. In the case of fluids of differential type, the equations of motion are an order higher than the Navier-Stokes equations and thus the adherence boundary condition is insufficient to determine the solution completely (see [25–27] for a detailed discussion of the relevant issues). The same is also true for the approximate boundary layer approximations of motion. In the absence of a clear means of obtaining additional boundary conditions, BEARD and WALTERS [28], in their study of an incompressible fluid of elastico-viscous suggested a method for overcoming this difficulty. They suggested a perturbation approach in which the velocity and the pressure field were expanded in a series in terms of small parameter. This parameter in question multiplied the highest order spatial derivatives in their equation. Though this approximation reduces the order of the equation, it treats a singular perturbation problem as a regular perturbation problem.

In 1991, GARG and RAJAGOPAL suggested that it would be preferable to overcome the difficulty associated with the paucity of boundary conditions by augmenting them on the basis of physically reasonable assumptions. They thought that it is possible to do this in the case of flows which take place in unbounded domains by using the fact that either the solution is bounded or the solution has certain smoothness at infinity. To demonstrate this, GARG and RAJAGOPAL [29] studied the stagnation flow of a fluid of second order by augmenting the boundary conditions. Their result agreed well with the result of RAJESWARI and RATHNA [30] who studied the problem based on the perturbation approach for a small value of the perturbation parameter.

Before proceeding with the solution, we note that Eqs. (2.6) and (2.7) are the third-order differential equations when $\alpha \neq 0$ and for the classical viscous case ($\alpha = 0$), we encounter differential equations of order two. Hence the presence of the material constant of the fluid increases the order of the governing equations from two to three. It would, therefore, seem that an additional boundary condition must be imposed in order to get a unique solution. The difficulty, in the present case, in however, removed by seeking a solution of the form [28]

(2.10)
$$f_1 = f_{01} + \alpha f_{11} + O\left(\alpha^2\right),$$
$$f_2 = f_{02} + \alpha f_{12} + O\left(\alpha^2\right),$$

which is valid for small values of α. Putting Eqs. (2.10) in Eqs. (2.6), (2.7) and boundary conditions (2.8) and equating the coefficients of α and then solving the resulting boundary value problems, the velocity field is given by

$$(2.11) \qquad u = (1 + \alpha L \eta) \in e^{-h\eta + i\omega t},$$

where

$$h = h_r + i h_i = \frac{1 + \sqrt{1 + 4N + i\omega}}{2},$$

$$(2.12) \qquad L = L_r + i L_i = \frac{h^2 \left(h + \dfrac{i\omega}{4} \right)}{\sqrt{1 + 4N + i\omega}}.$$

Knowing the velocity field, we now calculate the shearing stress which in terms of η is given by

$$(2.13) \qquad p_{xy} = \frac{p'_{x'y'}}{U'_o v'_o \rho'} = \frac{\partial u}{\partial \eta} + \frac{\alpha}{4} \left[\frac{\partial^2 u}{\partial \eta \partial t} - 4 \left(1 + \in A e^{i\omega t} \right) \frac{\partial^2 u}{\partial \eta^2} \right].$$

From Eqs. (2.11) and (2.13) we get

$$(2.14) \qquad (p_{xy})_{\eta \to 0} = \in e^{i\omega t} \left[\alpha L - h - \frac{i\omega}{4} \alpha h - \alpha h^2 \right].$$

Now from Eqs. (2.11) and (2.14) we have

$$(2.15) \qquad u(y, t) = (M_r \cos \omega t - M_i \sin \omega t),$$

$$(2.16) \qquad p_{xy} = |B| \cos (\omega t + \beta),$$

where

$$(2.17) \qquad M_r = \in e^{-h_r \eta} \left[\cos h_i \eta + \alpha \eta \left(L_r \cos h_i \eta + L_i \sin h_i \eta \right) \right],$$

$$(2.18) \qquad M_i = - \in e^{-h_r \eta} \left[\sin h_i \eta - \alpha \eta \left(L_i \cos h_i \eta - L_r \sin h_i \eta \right) \right],$$

$$B = B_r + i B_i, \qquad \beta = \arctan \frac{B_i}{B_r},$$

$$(2.19) \qquad \begin{aligned} B_r &= \in \left[\alpha L_r - h_r + \frac{1}{4} \omega \alpha h_i - \alpha \left(h_r^2 - h_i^2 \right) \right], \\ B_i &= \in \left[\alpha L_i - h_i - \frac{1}{4} \omega \alpha h_r - 2\alpha h_r h_i \right], \end{aligned}$$

$$(2.20) \quad \begin{aligned} h_i &= \frac{1}{2}\left[\frac{1}{2}\left\{\sqrt{(1+4N)^2+\omega^2}-1-4N\right\}\right]^{1/2}, \\ h_r &= \frac{1}{2}+\frac{1}{2}\left[\frac{1}{2}\left\{\sqrt{(1+4N)^2+\omega^2}+1+4N\right\}\right]^{1/2}, \end{aligned}$$

$$(2.21) \quad \begin{aligned} L_r &= \frac{1}{2}+\frac{N}{2}+\frac{a}{2}+\frac{N\omega b}{4r}-\frac{Na}{2r}-\frac{\omega^2 a}{16r}, \\ L_i &= \frac{\omega}{4}+\frac{b}{2}+\frac{N\omega a}{4r}+\frac{Nb}{2r}+\frac{\omega^2 b}{16r}, \end{aligned}$$

$$(2.22) \quad r = a^2+b^2 = \sqrt{(1+4N)^2+\omega^2},$$

$$(2.23) \quad \begin{aligned} a &= \sqrt{\frac{\sqrt{(1+4N)^2+\omega^2}+1+4N}{2}}, \\ b &= \sqrt{\frac{\sqrt{(1+4N)^2+\omega^2}-1-4N}{2}}. \end{aligned}$$

3. Second boundary value problem

In this section geometry of the problem is the same as that in the previous section except the form of the variable suction velocity. Thus, following the notation of [20], the boundary layer equation with no pressure gradient is given by

$$(3.1) \quad \frac{\partial u}{\partial t}+v_\circ\left[1+\delta\left(e^{i\omega t}+e^{-i\omega t}\right)\right]\frac{\partial u}{\partial y}$$

$$= \nu\frac{\partial^2 u}{\partial y^2}+\alpha^*\left[\frac{\partial^3 u}{\partial y^2\partial t}+v_\circ\left\{1+\delta\left(e^{i\omega t}+e^{-i\omega t}\right)\right\}\frac{\partial^3 u}{\partial y^3}\right]-\frac{\sigma B_\circ^2 u}{\rho},$$

where $\nu=\mu/\rho$, $\alpha^*=\alpha_1/\rho$, $v_\circ<0$ is the suction at the wall and the coupling parameter is the non-dimensional amplitude, say, δ. Note that in writing Eq. (3.1) we have used the variable suction velocity equal to $v_\circ[1+\delta(e^{i\omega t}+e^{-i\omega t})]$ from KELLY [20]. We further note that for $\alpha^*=0=B_\circ$ Eq. (3.1) reduces to KELLY [20]. For the problem under consideration the boundary conditions are

$$(3.2) \quad u(0,\,t) = 2U_\circ\cos\omega t,$$

$$(3.3) \quad u(\infty,\,t) = 0.$$

3.1. Solution of the second boundary value problem

We shall assume a solution of the form

$$(3.4) \qquad u(y,\, t) = u_0(y) + \sum_{n=1}^{\infty} u_n(y)\, e^{in\omega t} + \sum_{n=1}^{\infty} \widetilde{u}_n(y)\, e^{-in\omega t},$$

where $\widetilde{u}_n(y)$ is the complex conjugate of $u_n(y)$. Substituting Eq. (3.4) in (3.1) to (3.3) and then introducing

$$(3.5) \qquad \eta = \frac{|v_0|\, y}{\nu}, \qquad u_n = U_0 \phi_n,$$

into the resulting equations and the boundary conditions we arrive at the following boundary value problems

$$(3.6) \qquad \frac{v_0}{|v_0|} \frac{d\phi_0}{d\eta} + \frac{\delta v_0}{|v_0|} \left(\frac{d\phi_1}{d\eta} + \frac{d\widetilde{\phi}_1}{d\eta} \right)$$

$$= \frac{d^2\phi_0}{d\eta^2} + \alpha \frac{v_0}{|v_0|} \frac{d^3\phi_0}{d\eta^3} + \alpha \frac{\delta v_0}{|v_0|} \left(\frac{d^3\phi_1}{d\eta^3} + \frac{d^3\widetilde{\phi}_1}{d\eta^3} \right) - N_1 \phi_0,$$

$$(3.7) \qquad \phi_0(0) = 0, \qquad \phi_0(\infty) = 0,$$

$$(3.8) \qquad in\lambda\phi_n + \frac{v_0}{|v_0|} \frac{d\phi_n}{d\eta} + \frac{\delta v_0}{|v_0|} \left(\frac{d\phi_{n-1}}{d\eta} + \frac{d\phi_{n+1}}{d\eta} \right)$$

$$= \frac{d^2\phi_n}{d\eta^2} + \alpha \frac{\delta v_0}{|v_0|} \left(\frac{d^3\phi_{n-1}}{d\eta^3} + \frac{d^3\phi_{n+1}}{d\eta^3} \right)$$

$$+ \alpha \frac{v_0}{|v_0|} \frac{d^3\phi_n}{d\eta^3} + in\alpha\lambda \frac{d^2\phi_n}{d\eta^2} - N_1\phi_0, \quad n \geq 1,$$

$$(3.9) \qquad \phi_1(0) = 1, \qquad \phi_1(\infty) = 0,$$

$$(3.10) \qquad \phi_n(0) = 0, \qquad \phi_n(\infty) = 0, \qquad n \geq 2,$$

where

$$(3.11) \qquad \alpha = \frac{\alpha^* |v_0|^2}{\nu^2}, \qquad \lambda = \frac{\omega \nu}{|v_0|^2}, \qquad N_1 = \frac{\nu \sigma B_0^2}{\rho |v_0|^2}.$$

Equations similar to (3.8) to (3.10) result for $\widetilde{\phi}_1$ and $\widetilde{\phi}_n$.

For $\delta \ll 1$, the equations are weakly coupled and an expansion is performed in terms of powers of δ. Hence, we define

$$(3.12) \qquad \phi_n(\eta) = \sum_{j=0}^{\infty} \phi_{nj}(\eta)\, \delta^j.$$

Making use of (3.11) in (3.6) to (3.10) and then comparing the powers of δ we get

$$(3.13) \qquad \alpha \frac{d^3 \phi_{00}}{d\eta^3} - \frac{d^2 \phi_{00}}{d\eta^2} + \frac{v_\circ}{|v_\circ|} \frac{d\phi_{00}}{d\eta} + N_1 \phi_{00} = 0,$$

$$\phi_{00}(0) = 0, \qquad \phi_{00}(\infty) = 0,$$

$$(3.14) \qquad \alpha \frac{d^3 \phi_{10}}{d\eta^3} - (1 + i\alpha\lambda) \frac{d^2 \phi_{10}}{d\eta^2} - \frac{d\phi_{10}}{d\eta} + (i\lambda + N_1)\phi_{10} = 0,$$

$$\phi_{10}(0) = 1, \qquad \phi_{10}(\infty) = 0,$$

$$(3.15) \qquad \begin{aligned} \alpha \frac{d^3 \phi_{01}}{d\eta^3} &- \frac{d^2 \phi_{01}}{d\eta^2} - \frac{d\phi_{01}}{d\eta} + N_1 \phi_{01} \\ &= \left(\frac{d\phi_{10}}{d\eta} + \frac{d\tilde{\phi}_{10}}{d\eta} \right) - \alpha \left(\frac{d^3 \phi_{10}}{d\eta^3} + \frac{d^3 \tilde{\phi}_{10}}{d\eta^3} \right), \end{aligned}$$

$$\phi_{01}(0) = 0, \qquad \phi_{01}(\infty) = 0,$$

$$(3.16) \qquad \begin{aligned} \alpha \frac{d^3 \phi_{11}}{d\eta^3} &- (1 + i\alpha\lambda) \frac{d^2 \phi_{11}}{d\eta^2} - \frac{d\phi_{11}}{d\eta} + (i\lambda + N_1)\phi_{11} \\ &= -\alpha \left(\frac{d^3 \phi_{00}}{d\eta^3} + \frac{d^3 \phi_{20}}{d\eta^3} \right) + \left(\frac{d\phi_{00}}{d\eta} + \frac{d\phi_{20}}{d\eta} \right), \end{aligned}$$

$$\phi_{11}(0) = 0, \qquad \phi_{11}(\infty) = 0,$$

$$(3.17) \qquad \begin{aligned} \alpha \frac{d^3 \phi_{02}}{d\eta^3} &- \frac{d^2 \phi_{02}}{d\eta^2} - \frac{d\phi_{02}}{d\eta} + N_1 \phi_{02} \\ &= \left(\frac{d\phi_{11}}{d\eta} + \frac{d\tilde{\phi}_{11}}{d\eta} \right) - \alpha \left(\frac{d^3 \phi_{11}}{d\eta^3} + \frac{d^3 \tilde{\phi}_{11}}{d\eta^3} \right), \end{aligned}$$

$$\phi_{02}(0) = 0, \qquad \phi_{02}(\infty) = 0.$$

Similar to (2.10) we can write

$$\phi_{00} = \phi_{00,1} + \alpha\phi_{00,2} + O\left(\alpha^2\right),$$

$$\phi_{10} = \phi_{10,1} + \alpha\phi_{10,2} + O\left(\alpha^2\right),$$

(3.18) $$\phi_{01} = \phi_{01,1} + \alpha\phi_{01,2} + O\left(\alpha^2\right),$$

$$\phi_{11} = \phi_{11,1} + \alpha\phi_{11,2} + O\left(\alpha^2\right),$$

$$\phi_{02} = \phi_{02,1} + \alpha\phi_{02,2} + O\left(\alpha^2\right).$$

Substituting (3.18) in (3.13) to (3.16), equating the coefficients of α and then solving the resulting systems we arrive at

(3.19) $$\phi_{n0} = 0, \qquad n = 0 \qquad \text{and} \qquad n \geq 2,$$

(3.20) $$\phi_{10} = (1 + \alpha S\eta)\, e^{-g\eta},$$

(3.21) $$\phi_{11} = 0,$$

(3.22) $$\phi_{02} = 0,$$

(3.23) $$\phi_{01} = -\left(\frac{2}{\lambda}g_i + \alpha A_0 + \frac{2}{\lambda}\alpha\eta g_i\right) e^{-\eta} + Q_r \cos g_i\eta + Q_i \sin g_i\eta,$$

where

$$A_\circ = -\frac{2}{\lambda^2}\left[(2 + g_r)\lambda^2 + \lambda\left(3g_i - S_i\right) - g_r + \left(S_r g_r - S_i g_i\right)\right],$$

$$g = g_r + ig_i = \frac{1 + \sqrt{1 + 4N_1 + 4i\lambda}}{2},$$

$$S = S_r + iS_i = \frac{g^2\left(g + i\lambda\right)}{\sqrt{1 + 4N_1 + 4i\lambda}},$$

$$Q_r = \frac{2}{\lambda^2}e^{-g_r\eta}\left[-\alpha\lambda^2\left(2 + g_r\right) + \lambda g_i - \alpha\lambda\{(3g_i - S_i)\right.$$

$$\left. - \eta\left(S_r g_i + S_i g_r\right)\} + \alpha g_r - \alpha\left(S_r g_r - S_i g_i\right)\right],$$

$$Q_i = \frac{2}{\lambda^2} e^{-g_r \eta} \Big[-\alpha\lambda^2 g_i - \lambda g_r + \alpha\lambda\{3g_r - S_r + 1$$

$$- \eta\left(S_r g_r - S_i g_i\right)\} - \alpha g_i + \alpha\left(S_r g_i + S_i g_r\right)\Big].$$

Hence from (3.4), (3.5), (3.12) and (3.19) to (3.23); the velocity field in the boundary layer is given by

(3.24) $u = U_\circ [\{(-\frac{2}{\lambda}g_i - \alpha A_\circ - \frac{2}{\lambda}\alpha\eta g_i)e^{-\eta}$

$$+ Q_r \cos g_i \eta + Q_i \sin g_i \eta\}\delta + e^{-g_r \eta} \cos\left(g_i \eta - \omega t\right)$$

$$+ e^{-g_r \eta}\alpha\eta\{S_r \cos\left(g_i \eta - \omega t\right) + S_i \sin\left(g_i \eta - \omega t\right)\}].$$

The expression for the shearing stress in term of η is given by

(3.25) $P_{xy} = \rho U_\circ |v_\circ| \left[\dfrac{\partial\phi}{\partial\eta} + \alpha\left[\dfrac{\lambda}{\omega}\dfrac{\partial^2\phi}{\partial\eta\partial t} - v_\circ\left\{1 + \delta(e^{i\omega t} + e^{-i\omega t})\right\}\dfrac{\partial^2\phi}{\partial\eta^2}\right]\right].$

Using (3.5) and (3.24) in (3.25) and neglecting $O\left(\delta^2\right)$ terms we get

(3.26) $(P_{xy})_{\eta\to 0} = \rho U_\circ |v_\circ| \left[|E| \cos\left(\omega t + \gamma\right) - 2\delta\right],$

where

$$E^2 = E_1^2 + E_2^2, \qquad \gamma = \arctan\frac{E_2}{E_1},$$

$$E_1 = -g_r + \alpha\left(S_r + \lambda g_i - g_r\right),$$

$$E_2 = g_i - \alpha\left(S_i + \lambda g_r + g_i + \lambda\right),$$

(3.27)

$$g_i = \frac{1}{2}\left[\frac{1}{2}\left\{\sqrt{(1 + 4N_1)^2 + 16\lambda^2} - 1 - 4N_1\right\}\right]^{\frac{1}{2}},$$

$$g_r = \frac{1}{2} + \frac{1}{2}\left[\frac{1}{2}\left\{\sqrt{(1 + 4N_1)^2 + 16\lambda^2} + 1 + 4N_1\right\}\right]^{\frac{1}{2}},$$

$$S_r = \frac{1}{2} + \frac{N_1}{2} + \frac{a_1}{2} + \frac{N_1\lambda b_1}{r_1} - \frac{N_1 a_1}{2r_1} - \frac{\lambda^2 a_1}{r_1},$$

$$S_i = \lambda + \frac{b_1}{2} + \frac{N_1\lambda a_1}{r_1} + \frac{N_1 b_1}{2r_1} + \frac{\lambda^2 b_1}{r_1},$$

$$r_1 = a_1^2 + b_1^2 = \sqrt{(1 + 4N_1)^2 + 16\lambda^2},$$

$$a_1 = \sqrt{\frac{\sqrt{(1 + 4N_1)^2 + 16\lambda^2} + 1 + 4N_1}{2}},$$

$$b_1 = \sqrt{\frac{\sqrt{(1 + 4N_1)^2 + 16\lambda^2} - 1 - 4N_1}{2}}.$$

4. Discussions

In order to investigate the effects of the material parameter on the flow we have plotted u against η in Figs. 2 to 7.

- In Fig. 2 we note that the boundary layer thickness decreases with increase in frequency. It is further noted form Fig. 3 that velocity is negative for higher values of ω when $N = 100$.

- Figure 4 indicates the variation of the velocity profile for various values of α. It is observed that as α increases, the value of the velocity decreases. That is, increasing the normal stress coefficient has the effect of increasing the boundary layer thickness. Further, comparison of Figs. 4 and 5 show that layer thickness decreases drastically with increase of N. It appears that the electromagnetic force makes the layer thicknesses thinner. It is likely that the magnetic field provides some mechanism to control the growth of the boundary layer thickness. Moreover, Fig. 5 also illustrates that u is negative for $\omega t = \pi/2$, $\in = 0.5$, $\omega = 10$, $N = 100$, $\alpha = 0.025, 0.05, 0.075, 0.1$.

- In Figs. 6 and 7, the effect of material parameter is shown for second problem when $N = 0$ and $N \neq 0$ respectively. It is also clear from Fig. 6 that u decreases with increase of α first and then increases. With $N \neq 0$ in Fig. 7, the velocity is less in comparison to the velocity in Fig. 6.

- In Figs. 8 and 9 the fluctuating parts are shown for comparison purposes when $N = 5, \in = 0.5$ and $\omega = 100$. Figure 9 is particularly interesting because it illustrates the effects of α at large ω on M_i. In the case of fluids with material parameter, at $\omega = 100$, there is a sudden rise and fall of M_i near the wall. Also, from Figs. 8 and 9 one can conclude that an increase in α leads to much increase in M_i than M_r.
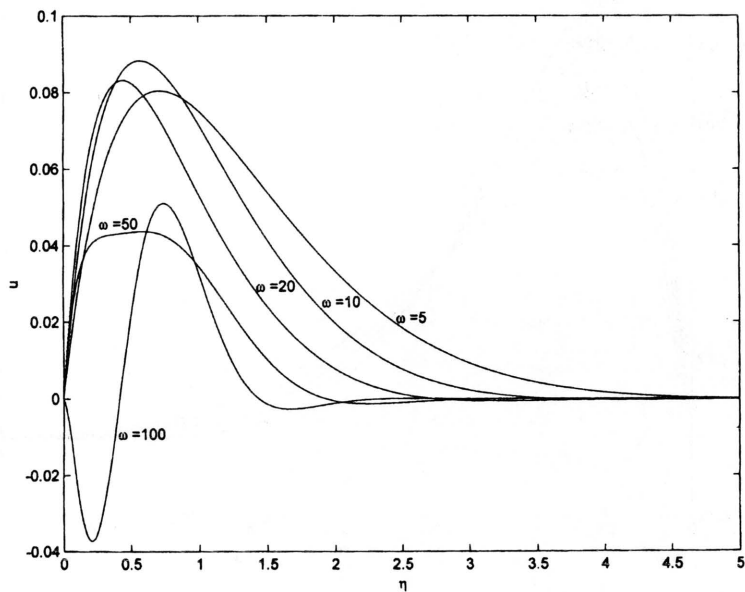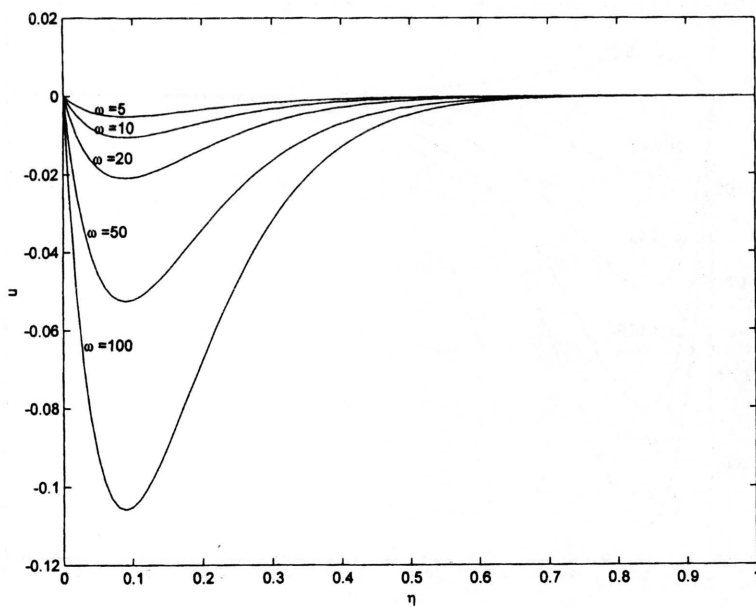
Fig. 2. Normalized velocity profiles for first BVP $\omega t = \pi/2$, $\varepsilon = 0.5$, $\alpha = 0.05$, $N = 0$.

Fig. 3. Normalized velocity profiles for first BVP $\omega t = \pi/2$, $\varepsilon = 0.5$, $\alpha = 0.05$, $N = 100$.
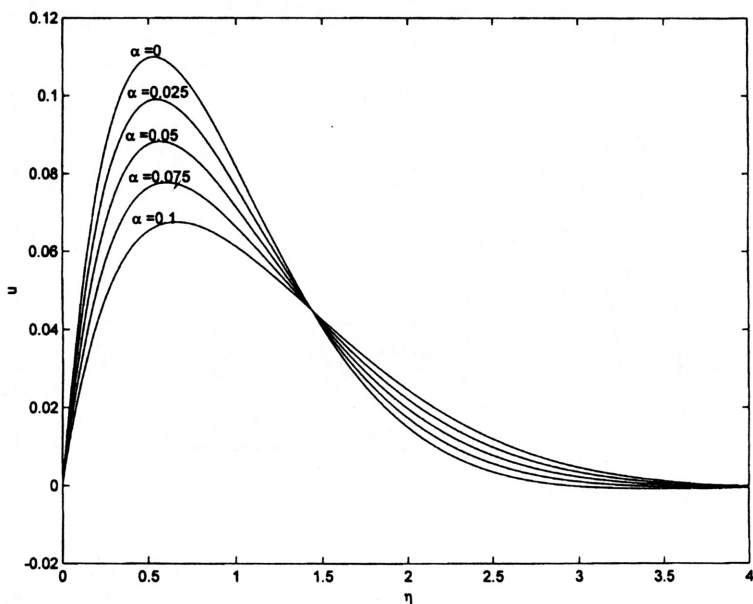
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FIG. 4. Normalized velocity profiles for first BVP $\omega t = \pi/2$, $\varepsilon = 0.5$, $\omega = 10$, $N = 0$.
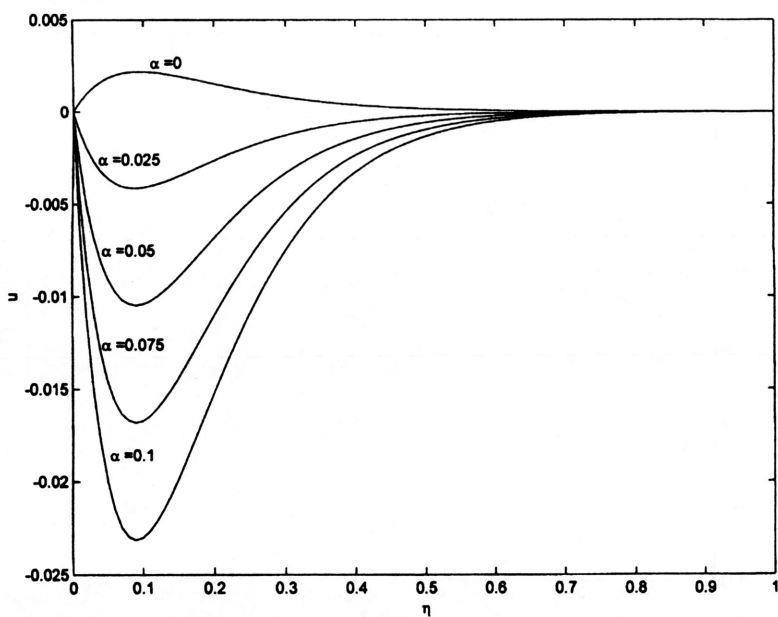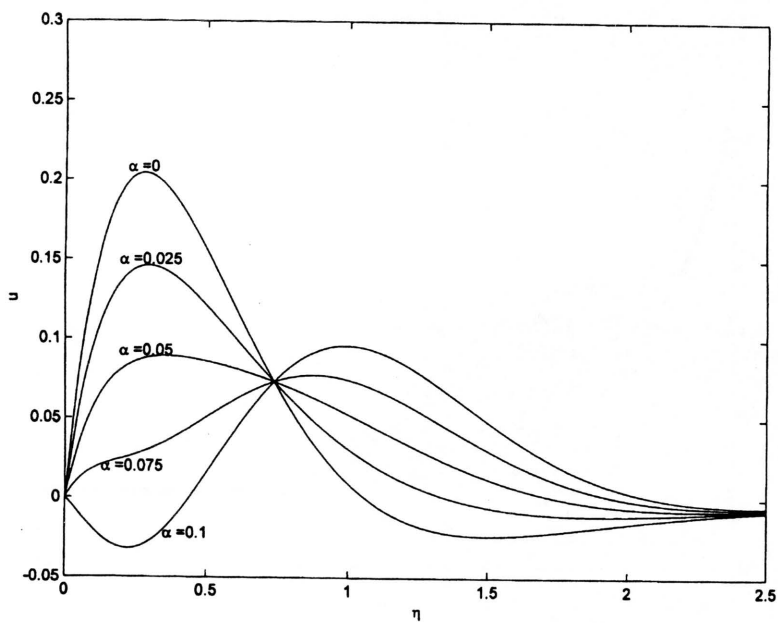

FIG. 5. Normalized velocity profiles for first BVP $\omega t = \pi/2$, $\varepsilon = 0.5$, $\omega = 10$, $N = 100$.

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FIG. 6. Normalized velocity profiles for second BVP $\omega t = \pi/2$, $\delta = 0.2$, $\lambda = 10$, $N = 0$.
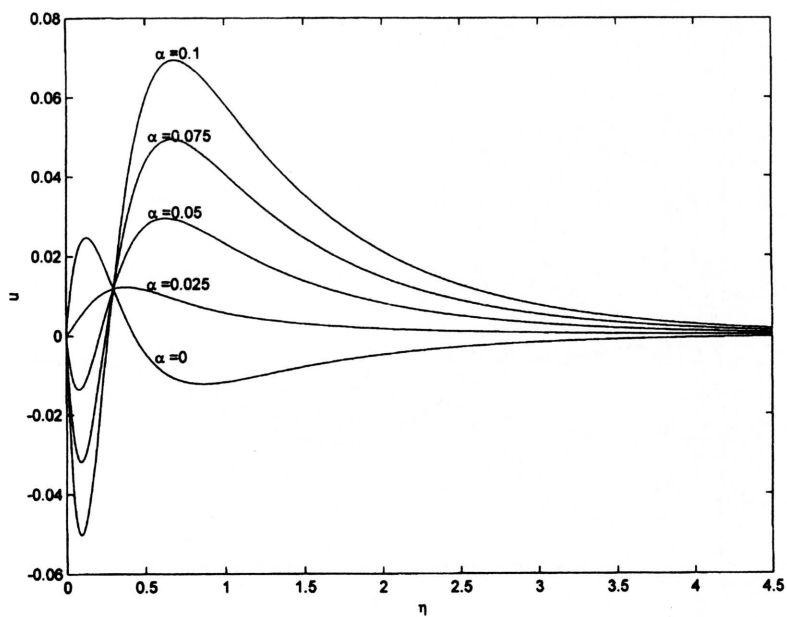

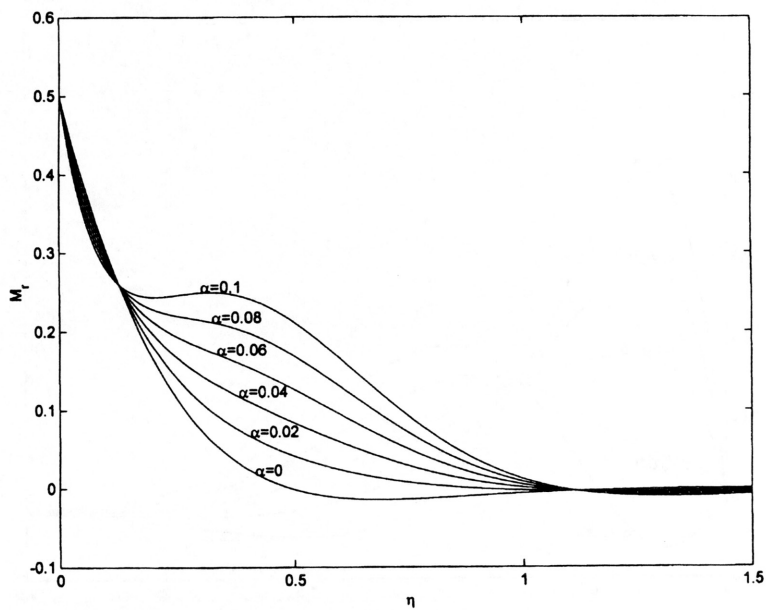FIG. 7. Normalized velocity profiles for second BVP $\omega t = \pi/2$, $\delta = 0.2$, $\lambda = 10$, $N = 30$.

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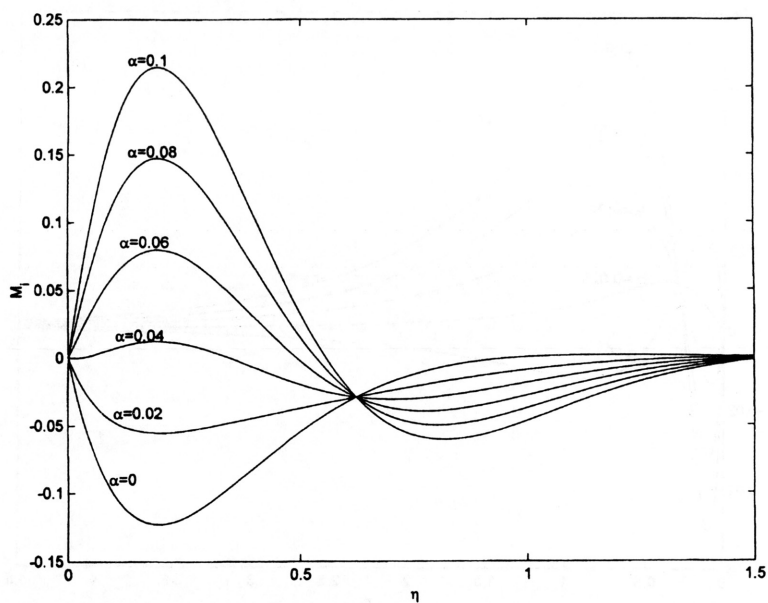FIG. 8. Fluctuating part M_r at $\varepsilon = 0.5$, $\omega = 100$, $N = 5$.

FIG. 9. Fluctuating part M_i at $\varepsilon = 0.5$, $\omega = 100$, $N = 5$.

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5. Concluding remarks

Exact solutions for the Stokes problem on a porous plate for an second order fluid are obtained in the presence of a magnetic field. From Eqs. (2.15), (2.17), (2.18), (3.24) and (3.27), it is found that the penetration depth decreases with fundamental frequency. This is not surprising; if we slowly oscillate a plate in a sticky fluid, we expect to drag large masses of fluid along with the plate; on the other hand, if we move the plate rapidly in a fluid of low viscosity, we expect the fluid essentially to ignore the plate, except in a thin boundary layer. Further, we note from these solutions that an increase of the magnetic field reduces the velocity within the boundary layer and also to reduce the boundary layer thickness.

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