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On couple-stress fluid permeated with suspended particles heated from below

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A LAYER of a couple-stress fluid heated from below and permeated with suspended particles is considered. For the case of stationary convection, the couple-stress has a stabilizing effect whereas suspended particles have a destabilizing effect on the couple-stress fluid permeated with suspended particles, heated from below. Graphs have been plotted by giving numerical values to the parameters, to depict the stability characteristics. The principle of exchange of stabilities is found to hold true for the couple-stress fluid in the presence of suspended particles, heated from below.

Key words: couple-stress fluid, heated from below, suspended particles.

1. Introduction

THE THEORY of Bénard convection in a viscous, Newtonian fluid layer heated from below has been given by CHANDRASEKHAR [1]. CHANDRA [2] observed that in an air layer, convection occurred at much lower gradients than those predicted if the layer depth was less than 7mm, and called this motion, "Columnar instability". However, for layers deeper than 10mm, a Bénard-type cellular convection was observed. Thus there is a contradiction between the theory and the experiment. SCANLON and SEGEL [3] have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure fluid was supplemented by that of the particles.

The study of a layer of fluid heated from below is motivated theoretically and by its practical applications in engineering. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. STOKES [4] formulated the theory of couple-stress fluid. One of the applications of couple-stress fluid is its use to the study of the mechanisms of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze-film action is capable of providing considerable protection to the cartilage surface. The shoulder, hip, knee and ankle joints are the loaded-bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear.

Normal synovial fluid is clear or yellowish and is a viscous, non-Newtonian fluid. According to the theory of STOKES [4], couple-stresses are found to appear in noticeable magnitudes in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluid, WALICKI and WALICKA [5] modeled synovial fluids as a couple-stress fluid in human joints. Environmental pollution is the main cause of dust to enter into the human body. The metal dust, which filters into the blood stream of those working near the furnace, causes extensive damage to the chromosomes and the genetic mutations so observed are likely to breed cancer or malformations in the coming progeny. Therefore it is very essential to study the blood flow with dust particles. Considering blood as a couple-stress fluid and dust particles as microorganisms, RATHOD and THIPPESWAMY [6] have studied the gravity flow of pulsatile blood through closed rectangular inclined channel with micro-organisms.

Keeping in mind the importance of non-Newtonian fluids and convection in fluid layer heated from below; the present paper attempts to study the couple-stress fluid, permeated with suspended particles, heated from below.

2. Formulation of the problem and perturbation equations

Here we consider an infinite, horizontal, incompressible couple-stress fluid layer of thickness d, heated from below so that the temperatures and densities at the bottom surface $z = 0$ are T_0 and ρ_0, and at the upper surface $z = d$ are T_d and ρ_d respectively, and that uniform temperature gradient $\beta\left(= |dT/dz|\right)$ is maintained. This layer is acted on by the gravity field $\mathbf{g}$=(0, 0, - g) pervading the system.

Let ρ, p, T and $\mathbf{v}$=(u, v, w) denote respectively the fluid density, pressure, temperature, and filter velocity, $\mathbf{v}\left(\bar{x}, t\right)$ and $N\left(\bar{x}, t\right)$ denote the velocity and number density of the particles, respectively. Then the equations of motion and continuity of a couple-stress fluid (Stokes [4], Joseph [8]) are

$$(2.1)\qquad \left[\frac{\partial \mathbf{v}}{\partial t} + \left(\mathbf{v}.\nabla\right)\mathbf{v}\right] = -\nabla\left(\frac{p}{\rho_0}\right) + \mathbf{g}\left(1+\frac{\delta\rho}{\rho_0}\right) + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\mathbf{v} + \frac{KN}{\rho_0}\left(\mathbf{v}_d - \mathbf{v}\right),$$

$$(2.2)\qquad \nabla.\mathbf{v} = 0,$$

where the suffix zero refers to values at the references level $z = 0$ and in writing (2.1), use has been made of the Boussinesq approximation which states that the density variations are ignored in all terms in the equation of motion except the external force term; $\mathbf{g}$ is acceleration due to gravity, $\bar{\mathbf{x}} = (x, y, z)$ and $K = 6\pi\mu\eta', \eta'$ being particle radius, is the Stoke's drag coefficient. The kinematic viscosity ν, couple-stress viscosity μ', thermal diffusivity κ and coefficient of thermal expansion α are all assumed to be constants.

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. The buoyancy force and pressure force acting on the particles are neglected. Inter-particle reactions are also not considered for we assume that distances between particles are quite large compared with their diameters. If mN is the mass of the particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions, are

$$(2.3)\qquad mN\left[\frac{\partial \mathbf{v}_d}{\partial \mathrm{t}} + \left(\mathbf{v}_d.\nabla\right)\mathbf{v}_d\right] = KN\left(\mathbf{v} - \mathbf{v}_d\right),$$

$$(2.4)\qquad \frac{\partial N}{\partial \mathrm{t}} + \nabla.\left(N\,\mathbf{v}_d\right) = 0.$$

Let c_v, c_p, c_{pt}, T, and q denote respectively, the heat capacity of fluid at constant volume, heat capacity of fluid at constant pressure, heat capacity of particles, temperature, and "effective thermal conductivity" of the clean fluid. Since the volume fraction of the particles is assumed to be small, the effective properties of the suspension are taken to be those of the clean fluid. If we assume that the particles and the fluid are in thermal equilibrium, the equation of heat gives

$$(2.5)\qquad \rho c_v\frac{\partial T}{\partial \mathrm{t}} + \rho\, c_v\left(\mathbf{v}.\nabla\right)T + mNc_{pt}\left(\frac{\partial}{\partial\,\mathrm{t}} + \mathbf{v}_d.\nabla\right)T = q\nabla^2 T.$$

The equation of state for the fluid is given by

(2.6)
$$\rho = \rho_0\Big[1 - \alpha\big(T - T_0\big)\Big].$$

The basic motionless solution is

(2.7)
$$\mathbf{v} = \Big(0, 0, 0\Big), \quad \mathbf{v}_d = \Big(0, 0, 0\Big), \quad T = -\beta z + T_0,$$
$$\rho = \rho_0\Big(1 + \alpha\beta z\Big), \quad N = N_0, \quad \text{a constant.}$$

Assume small perturbations around the basic solution, and let $\delta\rho$, δp, θ, $\mathbf{v} = (u, v, w)$, $\mathbf{v}_d = (\ell, r, s)$ and N denote respectively the perturbations in fluid density ρ_0, pressure p_0, temperature T, couple-stress fluid velocity (0, 0, 0), suspended particles velocity (0, 0, 0) and suspended particles number density N_0. The change in density $\delta\rho$, caused mainly by the perturbation θ in temperature, is given by

(2.8)
$$\delta\rho = -\alpha\rho_0\theta.$$

Then the linearized perturbation equations of the couple-stress fluid and particles are

(2.9)
$$\frac{\partial \mathbf{v}}{\partial \mathrm{t}} = \frac{1}{\rho_0}(\nabla\delta p) - \mathbf{g}\alpha\theta + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\mathbf{v} + \frac{KN_0}{\rho_0}\Big(\mathbf{v}_d - \mathbf{v}\Big),$$

(2.10)
$$\nabla.\mathbf{v} = 0,$$

(2.11)
$$\big(1 + h\big)\frac{\partial\theta}{\partial \mathrm{t}} = \beta(w + hs) + \kappa\nabla^2\theta.$$

(2.12)
$$mN_0\frac{\partial \mathbf{v}_d}{\partial \mathrm{t}} = KN_0\big(\mathbf{v} - \mathbf{v}_d\big),$$

(2.13)
$$\frac{\partial N}{\partial \mathrm{t}} + \nabla.\Big(N_0\mathbf{v}_d\Big) = 0.$$

Here $h = f\dfrac{c_{pt}}{c_v}$, $f = \dfrac{mN_0}{\rho_0}$, $\kappa = \dfrac{q}{\rho_0 c_v}$.

Eliminating $\mathbf{v}_d$ in (2.9) with the help of (2.12), writing the scalar components of Eq. (2.9) and eliminating $u, v, \delta p$ between them, by using (2.10), we obtain

$$\left[\left\{\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)+\frac{mN_0}{\rho_0}\right\}\frac{\partial}{\partial t}-\left\{\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\right\}\nabla^2\right]\nabla^2 w - \left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left\{g\alpha\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\theta\right\}=0. \tag{2.14}$$

Eliminating s with the help of (2.12), equation (2.11) yields

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[(1+h)\frac{\partial}{\partial t}-\kappa\nabla^2\right]\theta=\beta\left(\frac{m}{K}\frac{\partial}{\partial t}+1+h\right)w. \tag{2.15}$$

3. The dispersion relation

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$\Big[w,\theta\Big]=\Big[W(z),\Theta(z)\Big]\exp\Big(ik_x x+ik_y y+nt\Big), \tag{3.1}$$

where k_x, k_y are the wave numbers along the x- and y- directions respectively, $k=\sqrt{\left(k_x^2+k_y^2\right)}$ is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Expressing the coordinates x, y, z in the new unit of length d, time t in the new unit of length $\frac{d^2}{\kappa}$ and let $a=kd$, $\sigma=\frac{nd^2}{\nu}$, $p_1=\frac{\nu}{\kappa}$, $\mathrm{F}=\frac{1}{\nu}\frac{\mu'}{(\rho_0 d^2)}$, $\sigma'=\frac{n'd^2}{\nu}$, H=h+1, $\tau=\frac{m\kappa}{KD^2}$, $n'=n\left[1+\frac{mN_0K/\rho_0}{mn+K}\right]$ and $D=\frac{d}{dz}$; Eqs. (2.14), and (2.15), using (3.1), yield

$$\left[\sigma'-\left\{1-F(D^2-a^2)\right\}(D^2-a^2)\right](D^2-a^2)W=-\frac{g\alpha d^2}{\nu}a^2\Theta, \tag{3.2}$$

$$\left(\frac{\tau\nu}{d^2}\sigma+1\right)\left\{D^2-a^2-(1+h)p_1\sigma\right\}\Theta=-\frac{\beta d^2}{\kappa}\left(H+\frac{\tau\nu}{d^2}\sigma\right)W. \tag{3.3}$$

Eliminating Θ between Eqs. (3.2) - (3.3), we obtain

$$\left(1+\frac{\nu\tau}{d^2}\sigma\right)\left[\sigma'-\left\{1-F\left(D^2-a^2\right)\right\}\left(D^2-a^2\right)\right]\left(D^2-a^2\right) \tag{3.4}$$

(3.4) [cont.]
$$\left\{D^2 - a^2 - (1+h)p_1\sigma\right\}W = Ra^2\left(H + \frac{\nu\tau}{d^2}\sigma\right)W,$$

where $R = \dfrac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number.

Consider the case where both boundaries are free as well as perfect conductors of heat, while the adjoining medium is perfectly conducting. The case of two free boundaries is somewhat artificial but relevant for stellar atmospheres (SPIEGEL [7]). However, it enables us to find analytical solutions and to make some qualitative conclusions. The boundary conditions, with respect to which Eq. (3.4) must be solved, are

(3.5)
$$W = D^2W = 0, \;\; \Theta = 0, \;\; \text{at } z = 0 \text{ and } 1.$$

The constitutive equation for the couple-stress fluid are

(3.6)
$$\tau_{ij} = \left(2\mu - 2\mu'\nabla^2\right)e_{ij}, \;\; e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right).$$

The condition on a free surface are

(3.7)
$$\tau_{xz} = \tau_{yz} = 0,$$

which yield

(3.8)
$$(\mu - \mu'\nabla^2)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0,$$

(3.9)
$$(\mu - \mu'\nabla^2)\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0.$$

Since w vanishes (for all x and y) on the bounding surface, it follows from Eqs. (3.8) and (3.9) that

(3.10)
$$\left(\mu - \mu'\nabla^2\right)\left(\frac{\partial u}{\partial z}\right) = 0, \;\; \left(\mu - \mu'\nabla^2\right)\left(\frac{\partial v}{\partial z}\right) = 0.$$

From the equation of continuity (2.2) differentiated with respect of z, we conclude that

(3.11)
$$\left[\mu - \mu'\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\right]\frac{\partial^2 w}{\partial z^2} = 0,$$

which implies that

(3.12)
$$\frac{\partial^2 w}{\partial z^2} = 0, \;\; \frac{\partial^4 w}{\partial z^4} = 0, \;\; \text{on } z = 0 \text{ and } z = d.$$

Using expression (3.1), the boundary conditions (3.12), in non-dimensional form, yield additional boundary condition

$$D^4 w = 0 \text{ on } z = 0 \text{ and } z = 1. \tag{3.13}$$

Equations (3.2) and (3.3), using boundary conditions (3.5) and (3.13), yield

$$D^6 w = 0 \text{ on } z = 0 \text{ and } z = 1. \tag{3.14}$$

Using (3.5), (3.13) and (3.14), Eq. (3.4), yields

$$D^8 w = 0 \text{ on } z = 0 \text{ and } z = 1. \tag{3.15}$$

Differentiating Eq. (3.4) twice, four times,.....w.r.t. z and using the preceding boundary conditions (29), it can be shown that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$, and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{3.16}$$

where W_0 is a constant. Substituting the proper solution (3.16) in Eq. (3.4), we obtain the dispersion relation

$$R_1 = \frac{\mathbf{A}}{\mathbf{B}} \tag{3.17}$$

where

$$\mathbf{A} = (1+x_1)(1+x_1+iHp_1\sigma_1)\left(1+i\frac{\nu\tau\pi^2}{d^2}\sigma_1\right)\left[i\sigma_1' + \left\{1+\pi^2 F(1+x_1)\right\}(1+x_1)\right],$$

$$\mathbf{B} = x_1\left(H + i\frac{\nu\tau\pi^2}{d^2}\sigma_1\right),$$

and, where $x_1 = \dfrac{a^2}{\pi^2}$, $i\sigma_1 = \dfrac{\sigma}{\pi^2}$ and $R_1 = \dfrac{R}{\pi^4}$.

4. The stationary convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (3.17) reduces to

$$R_1 = \frac{(1+x_1)^3\left\{1+\pi^2 F(1+x_1)\right\}}{x_1 H}. \tag{4.1}$$

To study the effects of couple-stress parameter and suspended particles, we examine the natures of $\frac{dR_1}{dF}$ and $\frac{dR_1}{dH}$ analytically. Equation (4.1) yields

$$\frac{dR_1}{dF} = \frac{\pi^2(1+x_1)^4}{x_1 H^2}, \tag{4.2}$$

$$\frac{dR_1}{dH} = -\frac{(1+x_1)^3\{1+\pi^2 F(1+x_1)\}}{x_1 H^2}, \tag{4.3}$$

which imply that the couple-stress a stabilizing effect whereas suspended particles have a destabilizing effects on the onset of convection in couple-stress fluid permeated with suspended particles heated from below.

The dispersion relation (4.1) is analysed numerically. In Fig. 1, R_1 is plotted against x_1 for $F = 1, 2, 3$ and $H = 10$. It is clear that the couple-stress has a stabilizing effect on the onset of convection as the Rayleigh number increases with the increase in couple-stress parameter. In Fig. 2, R_1 is plotted against x_1 for $H = 10, 20, 30$ and $F = 2$. Here we find the suspended particles have a destabilizing effect, as the Rayleigh number decreases with the increase in suspended particles parameter, on couple-stress fluid permeated with suspended particles heated from below.

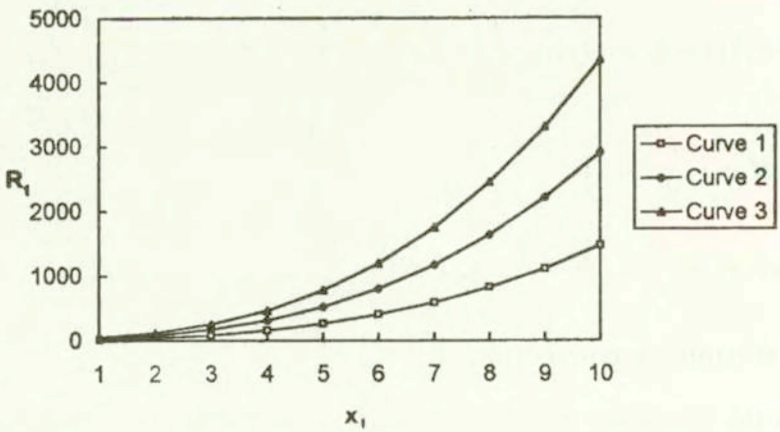

FIG. 1. The variation of Rayleigh number (R_1) with wave number (x_1) for H = 10, F=1 for Curve 1, F=2 for Curve 2 and F=3 for Curve 3.

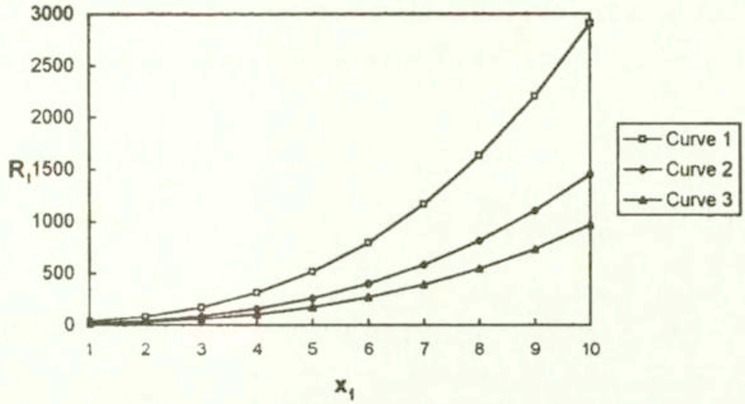

FIG. 2. The variation of Rayleigh number (R_1) with wave number (x_1) for F = 2, H=10 for Curve 1, H=20 for Curve 2 and H=30 for Curve 3.

5. Principle of exchange of stabilities

Multiplying (3.2) by W^*, the complex conjugate of W, and using (3.3) together with the boundary conditions (3.5) and (3.13), we obtain

$$FI_1 + I_2 + \sigma' I_3 - \left(\frac{g\alpha\kappa a^2}{\nu\beta}\right)\left(\frac{d^2+\nu\tau\sigma^*}{Hd^2+\nu\tau\sigma^*}\right)\Big[I_4 + Hp_1\sigma^* I_5\Big] = 0, \tag{5.1}$$

where

$$I_1 = \int_0^1 \left(|D^3W|^2 + 3a^2|D^2W|^2 + 3a^4|DW|^2 + a^6|W|^2\right)dz,$$

$$I_2 = \int_0^1 \left(|D^2W|^2 + 2a^2|DW|^2 + +a^4|W|^2\right)dz,$$

$$I_3 = \int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz, \tag{5.2}$$

$$I_4 = \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz$$

$$I_5 = \int_0^1 \left|\Theta|^2\right)dz$$

The integrals $I_1, ..., I_5$ are all positive definite. Since $\sigma = \sigma_r + i\sigma_i$, putting $\sigma = i\sigma_i$, $f = \dfrac{mN_0}{\rho_0}$ where σ_i is real and equating the imaginary parts of Eq. (5. 1), we obtain

$$(5.3)\quad \sigma_i\left[\left(1+\frac{f}{1+p_1^2\tau^2\sigma_1^2}\right)I_3 + \frac{g\alpha\kappa a^2}{\nu\beta\left(H^2d^2+\nu^2\tau^2\sigma^2\right)}\left\{d^2\nu\tau h I_4 + \left(Hd^4+\nu^2\tau^2\sigma^2\right)Hp_1 I_5\right\}\right]=0$$

But the quantity inside the brackets is positive definite. Hence

$$(5.4)\qquad \sigma_i = 0.$$

This shows that whenever $\sigma_r = 0$ implies that $\sigma_i = 0$, then the stationary (cellular) pattern of flow prevails on the onset of instability. In other words, the principle of exchange of stabilities is valid for the couple-stress fluid permeated with suspended particles, heated from below.

6. Conclusion

The presence of small amounts of additives in a lubricant can improve the bearing performance by increasing the lubricant viscosity and thus producing an increase in the load capacity. These additives in a lubricant also reduce the coefficient of friction and increase the temperature range in which the bearing can operate. A number of theories of the microcontinuum have been postulated and applied (STOKES [4], LAI *et al.* [9], WALICKA [10]). The theory due to STOKES [4] allows for polar effects such as the presence of couple stresses and body couples. STOKES [4] theory has been applied to the study of some simple lubrication problems (see e.g. SINHA *et al.* [11], BUJURKE and JAYARAMAN [12], LIN [13]).

A layer of a couple-stress fluid heated from below and permeated with suspended particles is considered. Here we use linearized stability theory and normal mode analysis method. We consider the case where both boundaries are free as well as perfect conductors of heat, while the adjoining medium is perfectly conducting. For the case of stationary convection, the couple-stress has a stabilizing effect whereas suspended particles have a destabilizing effect on the couple-stress fluid, permeated with suspended particles, heated from below. Graphs have been plotted by giving numerical values to the parameters, to depict the stability characteristics. It is clear that the couple-stress has a stabilizing effect on the onset of convection as the Rayleigh number increases with the increase in the couple-stress parameter. Also here we find that the suspended particles have destabilizing effect as the Rayleigh number decreases with the increase in suspended particles parameter on the couple-stress fluid, permeated with suspended particles, heated from below. We also see that oscillatory modes are not allowed due to the presence of kinematic viscoelasticity and suspended particles, since whenever $\sigma_r = 0$ implies that $\sigma_i = 0$, then the stationary (cellular) pattern of flow prevails on the onset of instability. In other words, the principle of exchange of stabilities is valid for the couple-stress fluid permeated with suspended particles, heated from below.

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