Arch. Mech., **53**, 6, pp. 643–651, Warszawa 2001

Heat transfer over an exponentially stretching continuous surface with suction

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SIMILARY SOLUTIONS of the laminar boundary layer equations describing heat and flow in a quiescent fluid driven by an exponentially stretching surface subject to suction are examined numerically. The direction and amount of heat flow were found to be dependent on the magnitude of "γ" (parameter of temperature) for the same Prandtl number. Nusselt number increases with increasing "γ" and the Prandtl number. The effect of decreasing suction parameter is found to be significant particularly for the Prandtl number.

1. Introduction

THE CONTINUOUS SURFACE heat transfer problem has many practical applications in industrial manufacturing processes. Such processes are hot rolling, wire drawing, glass fiber production, and paper production. Since the pioneering work of SAKIADIS [1], various aspects of the problem have been investigated by many authors. Most studies have been concerned with constant surface velocity and temperature (see, for example TSOU *et al.* [2]) but for many practical applications the surface undergoes stretching and cooling or heating that cause surface velocity and temperature variations. CRANE [3] and VLEGGAAR [4] have analysed the stretching problem with constant surface temperature while SOUNDALGEKAR and RAMANA MURTY [5] investigated the constant surface velocity case with power law temperature. ALI [6] has examined flow and heat transfer characteristics on a stretched surface subject to a power velocity and temperature. Recently MGYARI and KELLER [7] have analysed the exponential stretching problem by discussing a further type of similarity solution of the governing equations. These solutions involve an exponential dependence of the similarity variable as well as of the stretching velocity and temperature distribution on the coordinate in the direction parallel to that of the stretching.

Suction or injection of a stretched surface was introduced by ERICKSON *et al.*, [8] and FOX *et al.*, [9] for uniform surface velocity and temperature. GAUPTA and GAUPTA [10] extended Erickson's work in which the surface was moving with a linear speed for various values of parameters. Furthermore, stretching surface subject to suction or injection was studied by ALI [11] for uniform and variable surface temperature while ELBASHBESHY [12] investigated the uniform and variable surface heat flux.

The present work analyses the heat transfer over an exponentially stretching continuous surface with suction.

2. Formulation of the problem

The laminar velocity and thermal boundary layers on a continuous stretching surface with velocity $U_w \equiv U_w(x)$ and temperature $T_w \equiv T_w(x)$ moving axially through a stationary incompressible fluid with constant physical properties and temperature T_∞ may be described using the normal boundary approximations by the following continuity, momentum and energy equations [7, 11]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\,\frac{\partial u}{\partial x} + \nu\,\frac{\partial u}{\partial y} = \nu\,\frac{\partial^2 u}{\partial y^2}, \tag{2.2}$$

$$u\,\frac{\partial T}{\partial x} + \nu\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial^2 T}{\partial y^2}, \tag{2.3}$$

with the boundary conditions

$$\begin{aligned} &u = U_w(x) = U_0 \exp(x/L), \quad \nu = -V_w, \quad &&T = T_w \quad \text{at } y = 0, \\ &u = 0, &&T = T_\infty \quad \text{at } y \to \infty. \end{aligned} \tag{2.4}$$

The x-axis runs along the continuous surface in the direction of motion and the y-axis is perpendicular to it, u and v are the velocity components in the directions of x and y respectively, ν is the kinematic viscosity, T is the temperature, α is the thermal diffusivity, T_∞ is the free stream temperature, U_0 is a constant, L is the reference length, w is the condition at the surface and $y \to \infty$ concerns the condition at the ambient medium.

The solution of Eq. (2.1) may be written in terms of the function $\psi(x, y)$ defined by the relations

$$u = \frac{\partial \psi}{\partial y}, \qquad \nu = -\frac{\partial \psi}{\partial x}.$$

Introducing the usual similarity transformation and dimensionless temperature

$$\eta = y\sqrt{\frac{U_0}{2\nu L}}\,\exp(x/2L), \tag{2.5}$$

$$\psi(x,\, y) = \sqrt{2\nu\, LU_0}\, f(\eta)\, \exp\,(x/2L)\,, \tag{2.6}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_0 \exp(\gamma x/2L)}\,. \tag{2.7}$$

Where γ and T_0 are parameters of the temperature distribution in the stretching surface.

The momentum Eq. (2.2) and energy Eq. (2.3) can be written as

$$f''' + f\,f'' - 2f'^2 = 0\,, \tag{2.8}$$

$$\theta'' + \Pr\,(f\,\theta' - \gamma\, f'\,\theta) = 0\,, \tag{2.9}$$

with the boundary conditions

$$\begin{aligned} &f(0) = I\,,\;\; f'(0) = 1\,,\;\; \theta(0) = 1\,,\\ &f'(\infty) = 0\,,\;\; \theta(\infty) = 0\,, \end{aligned} \tag{2.10}$$

where the prime denotes differentiation with respect to η, and $I = V_w \sqrt{\dfrac{2L}{\nu U_0}}$.

3. Numerical solution

Equations (2.8) and (2.9) can be written in the integral form

$$f'' + f\,f' = S + I + 3\int_0^\eta f'^2(\eta_1)\, d\eta_1\,, \tag{3.1}$$

$$\theta' + \Pr\, f\theta = H + (\gamma + 1)\, \Pr\, \theta(\eta) \int_0^\eta f'^2(\eta_1)\, d\eta_1\,, \tag{3.2}$$

where $S = f''(0)$ and $H = \theta'(0)$.
For $\eta \to \infty$

$$I + S = -3\int_0^\eta f'^2(\eta)\, d\eta\,, \tag{3.3}$$

$$H = -(\gamma + 1)\, \Pr \int_0^\infty \theta(\eta)\, f'(\eta)\, d\eta\,. \tag{3.4}$$

For $\eta = 0$

$$f''(0) = 2 - SI, \tag{3.5}$$

$$\theta''(0) = \gamma \Pr . \tag{3.6}$$

We return to the integral Eq. (3.1). By integrating this equation once more, we get

$$f' + \frac{1}{2} f^2 = \frac{1}{2} I^2 + 1 + I\eta + S\eta + 3 \int_0^{\eta} \left(\int_0^{\eta_2} f'^2(\eta_1) d\eta_1 \right) d\eta_2 . \tag{3.7}$$

The iteration algorithm has to be started by substituting on the right hand side (RHS) as adequate zero-order approximation $f_0'(\eta)$ for $f'(\eta)$. By so doing, the procedure is reduced to the sequential solution of the Riccati-type equations:

$$f_n' + \frac{1}{2} f_n^2 = \mathrm{RHS}\,(f_{n-1}'), \qquad n = 1,\ 2,\ \ldots . \tag{3.8}$$

We suggest for the initiating function of the iteration scheme the expression

$$f_0'(\eta) = \exp(-S\eta), \tag{3.9}$$

yielding

$$f_0(\eta) = I + \left[\frac{1 - \exp(-S\eta)}{S} \right] . \tag{3.10}$$

By substituting this into the right hand side of Eq. (3.10) and by requiring that the first iteration f_1 on the left-hand side satisfies the boundary conditions (2.10), one obtains in the zero-order approximation

$$S = S_0 = \frac{I \pm \sqrt{I^2 + 6}}{2}, \quad f_0''(0) = S_0 .$$

The equation for the first-order iteration f_1 becomes

$$f_1' + \frac{1}{2} f_1^2 = 1 + \frac{1}{2} I^2 + \left(\frac{3}{2} I + \frac{\sqrt{I^2+6}}{2} - \frac{3}{I + \sqrt{I^2+6}} \right) \eta$$
$$- \left(\frac{3}{6 + 2I^2 + 2I\sqrt{I^2+6}} \right) \left(1 - \exp\left[-\left(I + \sqrt{I^2+6} \right) \eta \right] \right).$$

We now turn to solving the energy Eq. (2.9) by using f and f' the zero-order approximation. Introducing a new variable ξ as

$$\xi = -\,\mathrm{Pr}\,\exp(-S\eta),$$

and substituting the solution for f into Eq. (2.9) gives

$$\xi\,\frac{\partial^2\theta}{\partial\xi^2} + \left[1 - \mathrm{Pr}^* - \frac{\xi}{S^2}\right]\frac{\partial\theta}{\partial\xi} + \gamma^*\theta = 0\,, \tag{3.11}$$

where $\mathrm{Pr}^* = \left(\dfrac{1+I}{S^2}\right)\mathrm{Pr}$, $\gamma^* = \dfrac{\gamma}{S^2}$ with the approximation boundary conditions

$$\theta(-\mathrm{Pr}) = 1, \qquad\qquad \theta(0) = 0\,.$$

It can be readly demonstrated that the solution (3.11) in terms of Kummer's function [14] is

$$\theta = \left(\frac{-\xi}{\mathrm{Pr}^*}\right)^{\mathrm{Pr}^*}\frac{M(\mathrm{Pr}^* - \gamma,\ \mathrm{Pr}^* + 1,\ \xi)}{M(\mathrm{Pr}^* - \gamma,\ \mathrm{Pr}^* - 1,\ \mathrm{Pr}^*)} \tag{3.12}$$

where

$$M(a,\ b,\ z) = 1 + \sum_{n=1}^{\infty}\frac{a_n}{b_n}\frac{z^n}{a!}\,,$$

$$a_n = a\,(a+1)\,(a+2)\ \ldots\ (a+n-1)\,,$$

$$b_n = b\,(b+1)\,(b+2)\ \ldots\ (b+n-1)\,.$$

The local dimensionless surface temperature gradient corresponding to Eq. (3.2) is

$$\theta'(0) = -\mathrm{Pr}^* + \frac{\mathrm{Pr}^*\,(\mathrm{Pr}^* - \gamma)}{(\mathrm{Pr}^* + 1)}\,\frac{M(\mathrm{Pr}^* - \gamma + 1,\ \mathrm{Pr}^* + 2,\ -\mathrm{Pr}^*)}{M(\mathrm{Pr}^* - \gamma,\ \mathrm{Pr}^* + 1,\ -\mathrm{Pr}^*)}\,. \tag{3.13}$$

The accuracy of the numerical solutions has been verified – (for the case $I = 0$) – by comparing them with published results [7], (see Table 1).

Table 1. Results for – $f''(0)$ for different values of I

I	0	0.2	0.4	0.6
$f''(0)$	1.28181 1.28180*	1.37889	1.4839	1.59824

* results by ref. [7].

4. Results and discussion

The shear stress on the surface is defined by

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}, \qquad (\mu \text{ is the viscosity})$$

$$\tau_w = \mu \frac{U_o}{L} \sqrt{\frac{\mathrm{Re}}{2}} \exp\left(\frac{3x}{2L}\right) f''(0), \tag{4.1}$$

where $f''(0)$ is the friction coefficient and $\mathrm{Re} = L\,U_o/\,\nu$ is the Reynolds number. The total Nusselt number for heat transfer in the present case is defined by

$$\mathrm{Nu} = x \left. \frac{\partial T}{\partial y} \right|_{y=0} \Big/ (T_w - T_\infty)\,,$$

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = \sqrt{\frac{x}{2L}}\, \theta'(0), \qquad \mathrm{Re} = \frac{U_w\, x}{\nu}. \tag{4.2}$$

Results for the dimensionless temperature profiles and Nusselt numbers are obtained for various values of Prandtl numbers 0.72, 1, 3 and 10 for different values of γ and I. The nondimensional shear stress at the stretched surface presented by Eq. (4.1) is shown in Table 1 for various values of I. It is clear from the table that the friction coefficient increases as suction decreases. Also from Fig. 1, it is seen that the velocity decreases with suction.

Table 2. Results for $-\theta'(0)$ for different values of I, Pr and γ

		$-\theta'(0)$					
I	Pr	$\gamma=-1.5$	$\gamma=-1.0$	$\gamma=-0.5$	$\gamma=0.0$	$\gamma=1.0$	$\gamma=3.0$
0.0	0.72	-0.304049	0.0	0.234344	0.434717	0.767778	1.274760
	1.0	-0.377410	0.0	0.299874	0.549641	0.954779	1.560290
	3.0	- 0.923855	0.0	0.634114	1.122090	1.869070	2.938530
	10.0	-2.200988	0.0	1.308610	2.257430	3.660370	5.628200
0.6	0.72		-0.235096	0.595220	0.7453955	1.014517	1.463863
	1.0		-0.265921	0.802771	0.9872843	1.313957	1.851375
	3.0		-290646	2.166029	2.4933760	3.063438	3.986393
	10.0		-0.111818	6.560464	7.0727307	7.987197	9.518049

From Table 2, it is observed that for $\gamma = -1$ and no suction ($I = 0$), there is no heat transfer between the continuous surface and the medium, and for

$\gamma > -1$ and $I > 0$ it was found that the heat is transferred to the moving surface. For $\gamma > 0$, $I > 0$ the heat is transferred from the surface to the medium.

From Fig. 2, it is clear that the suction decreases the thermal boundary layer. In other words, the suction can be used as a means for cooling.

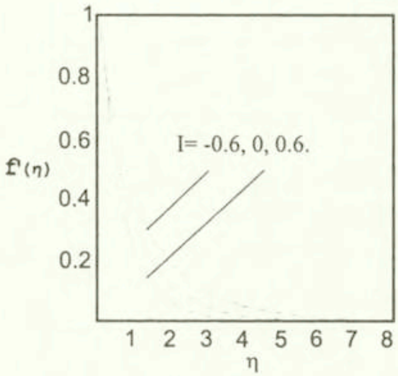

FIG. 1. Velocity profiles for various values of suction

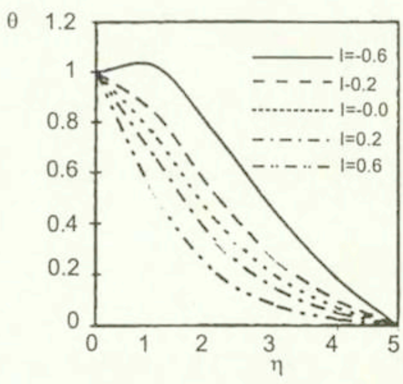

FIG. 2. Temperature profiles aganis η for selected values of I at Pr = 0.72

Sample of the boundary layer temperature for $\gamma = 1$ are presented in Fig. 3. The effect of Prandtl number is such that the thermal boundary layer decreases sharply with increasing Prandtl number. Figure 4 is constructed to present the effect of increasing γ (parameter of temperature) on temperature profile for Pr = 0.72 and the heat is transferred from the continuously stretching surface to the fluid medium.

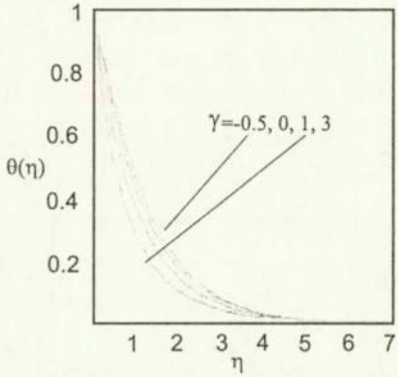

FIG. 3. Teperature profiles for I=0.6, Pr = 0.72 at γ = -0.5, 0, 1, 3.

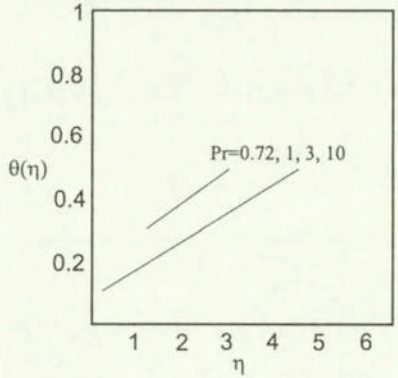

FIG. 4. Temperature profiles for I=0.6, γ =1, at Pr = 0.72, 1. 3, 10.

5. Conclusions

The heat transfer over an exponentially stretching continuous surface with suction have been examined and compared with the well known results. The heat transfer characteristics for the suction parameter I, temperature parameter γ and the Prandtl number are analyses. The magnitude of γ in the presence of suction affects the direction and quantity of heat flow. For $\gamma = -1$ and no suction ($I = 0$), there is no heat transfer occurring between the moving surface and medium. In other words, the suction enhance heat transfer coefficient and friction coefficient.

The suction can be used as means for cooling the moving continuous surface.

The thickness of the thermal boundary layer decreases with increasing parameter of temperature and suction for all the Prandtl number.

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Received May 15, 2000; revised version May 7, 2001.