Arch. Mech., 51, 1, pp. 47–58, Warszawa 1999

Domain of influence theorem in the theory of bending of micropolar elastic plates with stretch

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IN THE CONTEXT OF LINEAR THEORY of bending of micropolar plates with stretch, a domain of dependence inequality associated with the initial-boundary value problem is derived and a domain of influence theorem is established. It is shown that for a finite time, a solution corresponding to the data of a bounded support vanishes outside a bounded domain.

1. Introduction

THE THEORY OF MICROPOLAR ELASTIC SOLIDS with stretch has been introduced by ERINGEN in [1, 2] as a generalization of the micropolar theory [3] and a special case of the micromorphic theory [4]. Such a theory takes into consideration microstructural expansions and contractions of the material particles. Micropolar continuum with stretch is a model for Bravais lattice with basis on the atomic level, and two-phase dipolar solid with a core on the macroscopic level. The theory of micropolar elastic solids with stretch characterizes composite materials reinforced with chopped elastic fibers, porous media with pores filled with gas or inviscid liquid, asphalt and other elastic inclusions and solid-liquid crystals.

By using the method described by ERINGEN in [5], CIARLETTA [6] presented a theory of micropolar elastic plates with stretch. Within the context of bending of micropolar elastic plates with stretch, a spatial decay estimate of Saint-Venant type is also derived and a reciprocal theorem is established which leads to a uniqueness theorem with no definiteness assumptions on the elastic coefficients.

In the present paper we continue the study of the theory of micropolar elastic plates with stretch developed in [6] by establishing a domain of influence theorem of the type discussed in [7, 8]. To this aim we first establish a domain of dependence inequality associated with the initial-boundary value problem of micropolar plate with stretch, in the sense of [9]. Then we prove that, provided the given data of the initial-boundary value problem have a bounded support on the time interval $[0, t]$, the corresponding dynamic process vanishes outside a certain bounded domain.

Finally, we note that the domain of influence theorem has been studied in connection with various theories of continua (see e.g. [10] – [14]).

2. Basic equations

We consider a homogeneous and isotropic micropolar elastic solid with stretch that at time $t = 0$ occupies the right cylinder $\overline{B}$ of length $2h$ with cross-section $\overline{\Sigma}$ and the smooth lateral boundary Π. We call B and Σ be the interiors of $\overline{B}$ and $\overline{\Sigma}$ and denote by n_i the components of the outward unit normal to the boundary of $\overline{B}$. We assume that Σ is a simply connected region and we denote by L the boundary of Σ.

We refer the motion of continuum to the system of rectangular Cartesian axes Ox_k $(k = 1, 2, 3)$ chosen in such a way that

$$B = \{\tilde{\mathbf{x}} = (x_1, x_2, x_3) : (x_1, x_2, 0) \in \Sigma, \ -h < x_3 < h\},$$

and

$$\Pi = \{\tilde{\mathbf{x}} = (x_1, x_2, x_3) : (x_1, x_2, 0) \in L, \ -h < x_3 < h\}.$$

We denote the tensor components of order $p \geqslant 1$ by Latin subscripts, ranging over $\{1, 2, 3\}$, or by Greek subscripts, ranging over $\{1, 2\}$. Summation over repeated subscripts is implied. Superposed dots or subscripts preceded by a comma mean partial derivative with respect to the time or the corresponding coordinates.

In the context of the theory of micropolar elastic solids with stretch, we consider the following set of the independent variables:

$$\mathcal{U} = (u_i, \varphi_i, \psi)$$

where u_i are the components of the displacement vector, φ_i are the components of the microrotation vector and ψ is the microstretch function. We suppose that the fields $u_i, \varphi_i, \psi \in C^{2,2}(B \times T) \cap C^{1,1}(\overline{B} \times [0, \infty))$, where $T = (0, \infty)$.

In the considered theory, the equations of motion are

$$t_{ji,j} + f_i = \rho \ddot{u}_i,$$

$$m_{ji,j} + \varepsilon_{irs} t_{rs} + g_i = j \ddot{\varphi}_i,$$

$$\lambda_{j,j} - s + H = J \ddot{\psi}, \qquad \text{on } B \times T,$$

where t_{ij} is the stress tensor, m_{ij} is the couple stress tensor, $\lambda_i/3$ is the microstress vector, $s/3$ is the microstress function, f_i, g_i and H are the body loads, ρ is the reference mass density, j is a coefficient of inertia, $J = 3j/2$ and ε_{irs} is the alternating symbol. We assume that ρ and j are given strictly positive constants and that the surface tractions $t_i = t_{ji}n_j$, $m_i = m_{ji}n_j$, $p = \lambda_j n_j$ at regular points are assigned on the surfaces $x_3 = \pm h$, i.e. the functions t_{3i}, m_{3i}, λ_3 are prescribed.

We call a state of bending on $B \times T$ a process $\mathcal{U}$ that satisfies the following relations [5]:

$$
\begin{aligned}
u_\alpha(x_1, x_2, x_3, t) &= -u_\alpha(x_1, x_2, -x_3, t),\\
u_3(x_1, x_2, x_3, t) &= u_3(x_1, x_2, -x_3, t),\\
\varphi_\alpha(x_1, x_2, x_3, t) &= \varphi_\alpha(x_1, x_2, -x_3, t),\\
\varphi_3(x_1, x_2, x_3, t) &= -\varphi_3(x_1, x_2, -x_3, t),\\
\psi(x_1, x_2, x_3, t) &= -\psi(x_1, x_2, -x_3, t), \qquad (x_1, x_2, x_3, t) \in B \times T.
\end{aligned}
$$

In accordance with the theories established by ERINGEN in [1, 2] and CIARLETTA in [6], we assume that the body loads obey the relations

$$
\begin{aligned}
f_\alpha(x_1, x_2, x_3, t) &= -f_\alpha(x_1, x_2, -x_3, t),\\
f_3(x_1, x_2, x_3, t) &= f_3(x_1, x_2, -x_3, t),\\
g_\alpha(x_1, x_2, x_3, t) &= g_\alpha(x_1, x_2, -x_3, t),\\
g_3(x_1, x_2, x_3, t) &= -g_3(x_1, x_2, -x_3, t),\\
H(x_1, x_2, x_3, t) &= -H(x_1, x_2, -x_3, t).
\end{aligned}
$$

In the context of the theory of bending of micropolar elastic plates with stretch, we have the following independent variables (see [5, 6] for physical meaning of various quantities):

$$
\begin{aligned}
v_\alpha &= \frac{1}{I}\int_{-h}^{h} x_3 u_\alpha dx_3, \qquad w = \frac{1}{2h}\int_{-h}^{h} u_3\, dx_3,\\
\psi_\alpha &= \frac{1}{2h}\int_{-h}^{h} \varphi_\alpha\, dx_3, \qquad u = \frac{1}{I}\int_{-h}^{h} x_3 \psi dx_3,
\end{aligned}
$$

where $I = \dfrac{2}{3}h^3$.

Following [6], we consider the state of bending characterized by

$$u_\alpha = x_3 v_\alpha(x_1,x_2,t), \qquad u_3 = w(x_1,x_2,t),$$
$$\varphi_\alpha = \psi_\alpha(x_1,x_2,t), \qquad \varphi_3 = 0, \qquad \psi = x_3 u(x_1,x_2,t), \qquad \text{on } B\times T.$$

On the basis of the theory established in [6], we obtain the fundamental system of field equations consisting of the equations of motion

$$\begin{aligned} \tau_{\alpha 3,\alpha} + F &= \rho\ddot{w}, \\ \mu_{\beta\alpha,\beta} + \varepsilon_{\beta\alpha 3}(\tau_{3\beta} - \tau_{\beta 3}) + K_\alpha &= j\ddot{\psi}_\alpha, \\ M_{\beta\alpha,\beta} - 2h\tau_{3\alpha} + H_\alpha &= \rho I \ddot{v}_\alpha, \\ Q_{\alpha,\alpha} - 2h\pi_3 - S + P &= \zeta\ddot{u}, \qquad \text{on } \Sigma\times T, \end{aligned} \tag{2.1}$$

the constitutive equations

$$\begin{aligned} \tau_{\alpha 3} &= (\mu+\kappa)\epsilon_{\alpha 3} + \mu\epsilon_{3\alpha}, \qquad \tau_{3\alpha} = (\mu+\kappa)\epsilon_{3\alpha} + \mu\epsilon_{\alpha 3}, \\ \mu_{\alpha\beta} &= \alpha\eta_{\rho\rho}\delta_{\alpha\beta} + \beta\eta_{\beta\alpha} + \gamma\eta_{\alpha\beta}, \\ M_{\alpha\beta} &= I[\lambda\epsilon_{\rho\rho}\delta_{\alpha\beta} + (\mu+\kappa)\epsilon_{\alpha\beta} + \mu\epsilon_{\beta\alpha} + au\delta_{\alpha\beta}], \\ Q_\alpha &= \sigma I\xi_\alpha, \qquad \pi_3 = \sigma u, \qquad S = I(a\epsilon_{\rho\rho} + bu), \end{aligned} \tag{2.2}$$

and the geometrical equations

$$\begin{aligned} \epsilon_{\alpha\beta} &= v_{\beta,\alpha}, \qquad \epsilon_{\alpha 3} = w_{,\alpha} + \varepsilon_{3\alpha\beta}\psi_\beta, \qquad \epsilon_{3\alpha} = v_\alpha - \varepsilon_{3\alpha\beta}\psi_\beta, \\ \eta_{\alpha\beta} &= \psi_{\beta,\alpha}, \qquad \xi_\alpha = u_{,\alpha}. \end{aligned} \tag{2.3}$$

In these equations we have used the notations [5, 6]

$$\begin{aligned} \tau_{ij} &= \frac{1}{2h}\int_{-h}^{h} t_{ij}\,dx_3, \qquad \mu_{ij} = \frac{1}{2h}\int_{-h}^{h} m_{ij}\,dx_3, \\ M_{ij} &= \int_{-h}^{h} x_3 t_{ij}dx_3, \qquad Q_i = \int_{-h}^{h} x_3\lambda_i dx_3, \\ \pi_i &= \frac{1}{2h}\int_{-h}^{h}\lambda_i\,dx_3, \qquad S = \int_{-h}^{h} x_3 s dx_3, \qquad \zeta = \frac{3}{2}jI; \end{aligned} \tag{2.4}$$

the loads F, K_α, H_α and P are defined by

$$
\begin{aligned}
F &= F_3 + q, & q &= \frac{1}{h} t_{33}(x_1, x_2, h, t), & F_3 &= \int_{-h}^{h} f_3 dx_3,\\
K_\alpha &= G_\alpha + \eta_\alpha, & \eta_\alpha &= \frac{1}{h} m_{3\alpha}(x_1, x_2, h, t), & G_\alpha &= \int_{-h}^{h} g_\alpha dx_3,\\
H_\alpha &= q_\alpha + L_\alpha, & q_\alpha &= 2h t_{3\alpha}(x_1, x_2, h, t), & L_\alpha &= \int_{-h}^{h} x_3 f_\alpha dx_3,\\
P &= \chi + R, & \chi &= 2h\lambda_3(x_1, x_2, h, t), & R &= \int_{-h}^{h} x_3 H dx_3;
\end{aligned}
$$

moreover, the coefficients λ, μ, κ, a, $\alpha\,\beta$, γ, σ and b are constitutive constants. The loads F, K_α, H_α and P are prescribed.

Together with the system of field equations (2.1) – (2.3) in the variables v_α, w, ψ_α, u, we consider the following initial-boundary conditions:

$$
\begin{aligned}
v_\alpha(x_1, x_2, 0) &= v_\alpha^0(x_1, x_2), & w(x_1, x_2, 0) &= w^0(x_1, x_2),\\
\psi_\alpha(x_1, x_2, 0) &= \psi_\alpha^0(x_1, x_2), & u(x_1, x_2, 0) &= u^0(x_1, x_2),\\
\dot{v}_\alpha(x_1, x_2, 0) &= \nu_\alpha(x_1, x_2), & \dot{w}(x_1, x_2, 0) &= \omega(x_1, x_2),\\
\dot{\psi}_\alpha(x_1, x_2, 0) &= \chi_\alpha(x_1, x_2), & \dot{u}(x_1, x_2, 0) &= v(x_1, x_2), \qquad \text{on } \overline{\Sigma},
\end{aligned}
\tag{2.5}
$$

and

$$
\begin{aligned}
M_{\beta\alpha} n_\beta &= \tilde{M}_\alpha, & \tau_{\alpha 3} n_\alpha &= \tilde{\tau},\\
\mu_{\beta\alpha} n_\beta &= \tilde{\mu}_\alpha, & Q_\alpha n_\alpha &= \tilde{Q}, \qquad \text{on } L \times T.
\end{aligned}
\tag{2.6}
$$

The terms on the right-hand sides of the equations (2.5) and (2.6) stand for the (sufficiently smooth) assigned function; with F, K_α, H_α, P, these are the (external) data of the mixed problem considered. An array field $\mathcal{Q} = \{v_\alpha, w, \psi_\alpha, u\}$ satisfying all equations (2.1) – (2.3) and (2.5) – (2.6), for some assignment of the data, will be referred to as a (regular) *solution* of the problem of bending.

Of interest in the sequel will be the internal energy W of the plate due to bending (see [6])

$$(2.7)\qquad 2W = 2\int_{-h}^{h} W^* \, dx_3 = M_{\alpha\beta}\epsilon_{\alpha\beta} + Q_\alpha \xi_\alpha + Su$$

$$+2h(\tau_{\alpha 3}\epsilon_{\alpha 3} + \tau_{3\alpha}\epsilon_{3\alpha} + \pi_3 u + \mu_{\alpha\beta}\eta_{\alpha\beta}),$$

in which W^* is the strain energy density of the micropolar elastic solid with stretch [2]. In what follows, we suppose that W^* is a positive definite quadratic form; thus, there exists a strictly positive constant ξ (it is maximum elastic modulus) such that [15]

$$(2.8)\qquad \frac{1}{\xi}(t_{ij}t_{ij} + m_{ij}m_{ij} + \lambda_i\lambda_i + s^2) \leqslant 2W^*,$$

and [6]

$$(2.9)\qquad \frac{1}{\xi}\Big[\frac{1}{I}(M_{\alpha\beta}M_{\alpha\beta} + Q_\alpha Q_\alpha) + 2h(\tau_{\alpha 3}\tau_{\alpha 3} + \mu_{\alpha\beta}\mu_{\alpha\beta})\Big] \leqslant 2W.$$

3. Domain of influence

Having fixed a solution $\mathcal{Q}$ of the problem (2.1) – (2.3) and (2.5) – (2.6) and a time $t \in T$, we denote by $D_0(t)$ the support of initial and boundary data and body loads of the problem in concern; $D_0(t)$ is the set of the point $\mathbf{x} \in \bar{\Sigma}$ such that

(i) if $\mathbf{x} \in \Sigma$, then $v^0_\alpha(\mathbf{x}) \neq 0$ or $w^0(\mathbf{x}) \neq 0$ or $\psi^0_\alpha(\mathbf{x}) \neq 0$ or $u^0(\mathbf{x}) \neq 0$ or $\nu_\alpha(\mathbf{x}) \neq 0$ or $\omega(\mathbf{x}) \neq 0$ or $\chi_\alpha(\mathbf{x}) \neq 0$ or $v(\mathbf{x}) \neq 0$ or there exists $\tau \in [0, t]$ such that $F(\mathbf{x}, \tau) \neq 0$ or $K_\alpha(\mathbf{x}, \tau) \neq 0$ or $H_\alpha(\mathbf{x}, \tau) \neq 0$ or $P(\mathbf{x}, \tau) \neq 0$;

(ii) if $\mathbf{x} \in L$, then there exists $\tau \in [0, t]$ such that $\tilde{M}_\alpha(\mathbf{x}, \tau) \neq 0$ or $\tilde{\tau}(\mathbf{x}, \tau) \neq 0$ or $\tilde{\mu}_\alpha(\mathbf{x}, \tau) \neq 0$ or $\tilde{Q}(\mathbf{x}, \tau) \neq 0$.

By a domain of influence of the data at time t for mixed problem (2.1) – (2.3) and (2.5) – (2.6) we mean the set

$$(3.1)\qquad D(t) = \{\mathbf{x}_0 \in \bar{\Sigma} : D_0(t) \cap \overline{S(\mathbf{x}_0, ct)} \neq \emptyset\},$$

where $\overline{S(\mathbf{x}_0, d)}$ is the closed disc of radius d and the center $\mathbf{x}_0$, and c is given by

$$(3.2)\qquad c = \left\{\frac{\xi}{m}\right\}^{1/2} \qquad \text{with} \qquad m = \min\{\rho, j\}.$$

We put

$$\Sigma(\mathbf{x}_0, d) = \Sigma \cap S(\mathbf{x}_0, d), \qquad L(\mathbf{x}_0, d) = L \cap S(\mathbf{x}_0, d),$$

in which $S(\mathbf{x}_0, d)$ is the interior of $\overline{S(\mathbf{x}_0, ct)}$.

In order to approach the proof of the domain of influence theorem, we have to show first the

THEOREM. *Let $\mathcal{Q}$ be a solution of the initial-boundary value problem (2.1) - (2.3) and (2.5) - (2.6), and U be the function defined by*

$$U = W + \frac{1}{2}(\rho I \dot{v}_\alpha \dot{v}_\alpha + \zeta \dot{u}^2 + 2h\rho \dot{w}^2 + 2hj\dot{\psi}_\alpha \dot{\psi}_\alpha). \tag{3.3}$$

Then, for each $t \in T$ and any positive constant R we have

$$\int\limits_{\Sigma(\mathbf{x}_0,R)} U(\mathbf{x},t)\, da \leqslant \int\limits_{\Sigma(\mathbf{x}_0,R+ct)} U(\mathbf{x},0)\, da + \int\limits_0^t \int\limits_{\Sigma(\mathbf{x}_0,R+c(t-r))} (H_\beta \dot{v}_\beta + P\dot{u} + F\dot{w}_i + K_\beta \dot{\psi}_\beta)\, da\, dr + \int\limits_0^t \int\limits_{L(\mathbf{x}_0,R+c(t-r))} (M_{\beta\alpha}\dot{v}_\alpha + Q_\beta \dot{u} + 2h(\tau_{\beta 3}\dot{w} + \mu_{\beta\alpha}\dot{\psi}_\alpha))n_\beta\, ds\, dr, \tag{3.4}$$

where c is given by (3.2).

Proof. At a fixed $(\mathbf{x}_0, t) \in \mathbb{R}^2 \times T$ and a positive constant R, we introduce the function

$$G(\mathbf{x}, r) = G_\delta \left\{ \frac{1}{c}[R + c(t-r) - |\mathbf{x} - \mathbf{x}_0|] \right\}, \qquad (\mathbf{x}, r) \in \mathbb{R}^2 \times T,$$

where G_δ is a smooth, nondecreasing function on $\mathbb{R}$, such that

$$G_\delta(\iota) = 0 \quad \text{if } \iota \in (-\infty, 0], \quad \text{and} \quad G_\delta(\iota) = 1 \quad \text{if } \iota \in [\delta, \infty).$$

The support of G is $\Omega = \bigcup\limits_{r \leqslant t} S\Big(\mathbf{x}_0\,,\ R + c(t-r)\Big)$. We may prove that G is identically equal to 1 on the set $\Omega_0 = \bigcup\limits_{r \leqslant t} S(\mathbf{x}_0\,,\ R + ct - cr - c\delta)$ and, consequently, $\operatorname{grad} G$ vanishes on Ω_0.

Of course, we must choose δ small enough to assume that $R+ct-cr-c\delta>0$ for any $r\in[0,t]$.

From the equations (2.2) and (2.3), we can note that

$$(3.5)\qquad G\dot U = G\Big(M_{\beta\alpha}\dot v_{\alpha,\beta} + Q_\beta \dot u_{,\beta} + S\dot u + 2h(\tau_{\beta 3}\dot w_{,\beta} + \varepsilon_{3\beta\alpha}\tau_{\beta 3}\dot\psi_\alpha + \tau_{3\alpha}\dot v_\alpha - \varepsilon_{3\beta\alpha}\tau_{3\beta}\dot\psi_\alpha + \pi_3\dot u + \mu_{\beta\alpha}\dot\psi_{\alpha,\beta}) + \rho I\ddot v_\alpha \dot v_\alpha + \zeta\ddot u\dot u + 2h\rho\ddot w\dot w + 2hj\ddot\psi_\alpha\dot\psi_\alpha\Big),$$

and, taking into account the equations (2.1) and (3.5), we can write

$$(3.6)\qquad \int_0^t\int_\Sigma \frac{\partial A}{\partial r}\,da\,dr = \int_0^t\int_\Sigma \Big(U\frac{\partial G}{\partial r}\Big)(\mathbf{x},r)\,da\,dr + \int_0^t\int_\Sigma G\Big(H_\alpha\dot v_\alpha + P\dot u + 2hF\dot w + 2hK_\alpha\dot\psi_\alpha\Big)\,da\,dr + \int_0^t\int_\Sigma G\Big[M_{\beta\alpha}\dot v_\alpha + Q_\beta\dot u + 2h(\tau_{\beta 3}\dot w + \mu_{\beta\alpha}\dot\psi_\alpha)\Big]_{,\beta}\,da\,dr,$$

where $A=GU$. Using the equation (3.6) and the divergence theorem, we get

$$(3.7)\qquad \int_\Sigma A(\mathbf{x},t)\,da = \int_\Sigma A(\mathbf{x},0)\,da + \int_0^t\int_\Sigma \Big(U\frac{\partial G}{\partial r}\Big)(\mathbf{x},r)\,da\,dr + \int_0^t\int_\Sigma G\Big(H_\alpha\dot v_\alpha + P\dot u + 2hF\dot w + 2hK_\alpha\dot\psi_\alpha\Big)\,da\,dr - \int_0^t\int_\Sigma G_{,\beta}\Big[2h(\tau_{\beta 3}\dot w + \mu_{\beta\alpha}\dot\psi_\alpha) + M_{\beta\alpha}\dot v_\alpha + Q_\beta\dot u\Big]\,da\,dr + \int_0^t\int_L G\Big[2h(\tau_{\beta 3}\dot w + \mu_{\beta\alpha}\dot\psi_\alpha) + M_{\beta\alpha}\dot v_\alpha + Q_\beta\dot u\Big]n_\beta\,ds\,dr.$$

If we denote by

$$G'_\delta = \frac{dG_\delta(y)}{dy},\quad y=\frac{1}{c}[R+c(t-r)-|\mathbf{x}-\mathbf{x}_0|],$$

then we can write

$$G_{,\beta} = -\zeta_\beta \frac{1}{c} G'_\delta, \qquad \zeta_\beta = \frac{(x_\beta - x^0_\beta)}{|\mathbf{x} - \mathbf{x}_0|}, \tag{3.8}$$

and

$$\frac{\partial G}{\partial r} = -G'_\delta. \tag{3.9}$$

Thus, we have

$$-G_{,\beta}\Big[M_{\beta\alpha}\dot{v}_\alpha + Q_\beta \dot{u} + 2h(\tau_{\beta 3}\dot{w} + \mu_{\beta\alpha}\dot{\psi}_\alpha)\Big] = G'_\delta\Big[M_{\beta\alpha}\zeta_\beta\frac{\dot{v}_\alpha}{c} + Q_\beta\zeta_\beta\frac{\dot{u}}{c} + 2h\left(\tau_{\beta 3}\zeta_\beta\frac{\dot{w}}{c} + \mu_{\beta\alpha}\zeta_\beta\frac{\dot{\psi}_\alpha}{c}\right)\Big]. \tag{3.10}$$

If we apply the Schwarz inequality and the arithmetic-geometric inequality, the equation (3.10) implies that

$$-G_{,\beta}\Big[M_{\beta\alpha}\dot{v}_\alpha + Q_\beta \dot{u} + 2h(\tau_{\beta 3}\dot{w} + \mu_{\beta\alpha}\dot{\psi}_\alpha)\Big] \leqslant \frac{1}{2}G'_\delta\Big[\frac{1}{I\xi}M_{\beta\alpha}M_{\beta\alpha} + \frac{I\xi}{c^2}\dot{v}_\alpha\dot{v}_\alpha + \frac{1}{I\xi}Q_\beta Q_\beta + \frac{I\xi}{c^2}\dot{u}^2 + \frac{2h}{\xi}\tau_{\beta 3}\tau_{\beta 3} + \frac{2h\xi}{c^2}\dot{w}^2 + \frac{2h}{\xi}\mu_{\beta\alpha}\mu_{\beta\alpha} + \frac{2h\xi}{c^2}\dot{\psi}_\alpha\dot{\psi}_\alpha\Big]. \tag{3.11}$$

Taking into account the relations (2.9), (3.2), (3.3) and $(2.4)_7$, the equation (3.11) implies that

$$-G_{,\beta}\Big[M_{\beta\alpha}\dot{v}_\alpha + Q_\beta \dot{u} + 2h(\tau_{\beta 3}\dot{w} + \mu_{\beta\alpha}\dot{\psi}_\alpha)\Big] \leqslant G'_\delta U. \tag{3.12}$$

With the help of (3.9) and (3.12), we get

$$\int_0^t \int_\Sigma U\frac{\partial G}{\partial r} - G_{,\beta}\Big[M_{\beta\alpha}\dot{v}_\alpha + Q_\beta \dot{u} + 2h(\tau_{\beta 3}\dot{w} + \mu_{\beta\alpha}\dot{\psi}_\alpha)\Big]\, da\, dr \leqslant 0. \tag{3.13}$$

Thus, from (3.7) we have

(3.14) $$\int_\Sigma A(x,t)\,da \leqslant \int_\Sigma A(x,0)\,da + \int_0^t \int_\Sigma G\Big(H_\alpha \dot{v}_\alpha + P\dot{u} + 2hF\dot{w} + 2hK_\alpha \dot{\psi}_\alpha\Big)\,da\,dr + \int_0^t \int_L G\Big[M_{\beta\alpha}\dot{v}_\alpha + Q_\beta \dot{u} + 2h(\tau_{\beta 3}\dot{w} + \mu_{\beta\alpha}\dot{\psi}_\alpha)\Big] n_\beta\,ds\,dr.$$

We note that passage to the limit $\delta \to 0$ is permissible in the integrals in (3.14) by virtue of the Lebesgue dominated convergence theorem, since G tends boundedly to the characteristic function of the set Ω. If we take the limit in (3.14) as $\delta \to 0$, then we obtain (3.4). ■

The set $D(t)$ covers a domain of elastic disturbances produced by the data at time t; in fact, following [11, 12], we prove the

DOMAIN OF INFLUENCE THEOREM: *Let Q be a solution of the initial-boundary value problem (2.1) - (2.3) and (2.5) - (2.6), and $D(t)$ be domain of influence of its data at time t. Then,*

$$v_\alpha = 0, \quad w = 0, \quad \psi_\alpha = 0, \quad u = 0, \qquad \text{on } \Big(\bar{\Sigma} - D(t)\Big) \times [0,t].$$

P r o o f. If we put in the inequality (3.4) $t = \tau$ and $R = c(t - \tau)$, we obtain

(3.15) $$\int_{\Sigma(\mathbf{x}_0, c(t-\tau))} U(\mathbf{x},\tau)\,da \leqslant \int_{\Sigma(\mathbf{x}_0, ct)} U(\mathbf{x},0)\,da + \int_0^\tau \int_{\Sigma(\mathbf{x}_0, c(t-r))} \Big(H_\alpha \dot{v}_\alpha + P\dot{u} + 2hF\dot{w} + 2hK_\alpha \dot{\psi}_\alpha\Big)\,da\,dr + \int_0^\tau \int_{L(\mathbf{x}_0, c(t-r))} \Big[M_{\beta\alpha}\dot{v}_\alpha + Q_\beta \dot{u} + 2h(\tau_{\beta 3}\dot{w} + \mu_{\beta\alpha}\dot{\psi}_\alpha)\Big] n_\beta\,ds\,dr.$$

At fixed $(\mathbf{x}_0, \tau) \in \Big(\bar{\Sigma} - D(t)\Big) \times [0,t]$, we have

$$\begin{aligned} \bar{v}_\alpha(\mathbf{x},0) &= 0, & w(\mathbf{x},0) &= 0, & \psi_\alpha(\mathbf{x},0) &= 0, \\ u(\mathbf{x},0) &= 0, & v_{\alpha,\beta}(\mathbf{x},0) &= 0, & w_{,\beta}(\mathbf{x},0) &= 0, \\ \psi_{\alpha,\beta}(\mathbf{x},0) &= 0, & u_{,\beta}(\mathbf{x},0) &= 0, & \mathbf{x} &\in \Sigma(\mathbf{x}_0, ct), \end{aligned}$$

and

$$F(\mathbf{x},\tau) = 0, \qquad K_\alpha(\mathbf{x},\tau) = 0,$$

$$H_\alpha(\mathbf{x},\tau) = 0, \qquad P(\mathbf{x},\tau) = 0, \qquad (\mathbf{x},\tau) \in \Sigma(\mathbf{x}_0, ct) \times [0,t].$$

Thus, we get

$$\int\limits_{\Sigma(\mathbf{x}_0,ct)} U(\mathbf{x},0)da = 0.$$

(3.16)

$$\int\limits_0^\tau \int\limits_{\Sigma(\mathbf{x}_0,c(t-\tau))} \left(H_\alpha \dot{v}_\alpha + P\dot{u} + 2RF\dot{w} + 2RK_\alpha \dot{\psi}_\alpha\right) da\, d\tau = 0.$$

If we consider $\tau \leqslant t$ and $\Sigma(\mathbf{x}_0, ct) \subset \bar{\Sigma} - D(t)$, the last integral of Eq. (3.15) also vanishes. Then, the equation (3.15) becomes

$$\int\limits_{\Sigma(\mathbf{x}_0,c(t-\tau))} U(\mathbf{x},\tau)\, da \leqslant 0. \tag{3.17}$$

With the aid of the equations (3.3), (2.8) and (3.17), we conclude that

$$\dot{v}_\alpha(\mathbf{x}_0,\tau) = 0, \qquad \dot{w}(\mathbf{x}_0,\tau) = 0,$$

$$\dot{\psi}_\alpha(\mathbf{x}_0,\tau) = 0, \qquad \dot{u}(\mathbf{x}_0,\tau) = 0, \qquad (\mathbf{x}_0,\tau) \in (\bar{\Sigma} - D(t)) \times [0,t];$$

taking into account that

$$v_\alpha(\mathbf{x}_0,0) = 0, \qquad w(\mathbf{x}_0,0) = 0,$$

$$\psi_\alpha(\mathbf{x}_0,0) = 0, \qquad u(\mathbf{x}_0,0) = 0, \qquad \mathbf{x}_0 \in \bar{\Sigma} - D(t),$$

we obtain the desired result. ■

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Received April 14, 1998; revised version September 7, 1998.