Arch. Mech., **51**, 2, pp. 167–179, Warszawa 1999

Dusty plasma solitons in Vlasov plasmas

A. J. TURSKI, B. ATAMANIUK and K. ŻUCHOWSKI
Institute of Fundamental Technological Research
Polish Academy of Sciences, Warsaw, Poland

Vlasov-Gauss/Ampère equations for dust-ion-electron plasmas are considered. Dispersion relations are derived for longitudinal dusty plasma waves by use of linearized Vlasov equations. Assuming suitable equilibrium distributions for cold and hot charged particle/grain components, due to negligence of wave-particle interactions and assuming traveling the wave "far field" solutions, fully nonlinear ODE is obtained for SAGDEEV potentials. Examples of Sagdeev potentials and negative solitary waves associated with fully nonlinear dust-acoustic waves (DAW) and dust-ion acoustic waves (DIAW) are computed.

1. Introduction

Dust is a common component of many space plasmas. Waves in dusty plasmas have recently been considered when the dust grains are negatively charged. Dusty plasma physics is of great importance with a number of applications. Recently, attention has been focused on planetary rings in which heavy micron-sized dust grains are charged to high degree voltage. In particular, in the F-ring of Saturn there is an extreme anomalous situation where the number density of free electrons is much smaller than the number density of ions. This could happen because the charged dust grains collect electrons from the background medium and the number density of free electrons is anomalously small. Recent Voyager 1 and 2 observations of Saturnian ring systems have started research interest in physics of dusty plasmas. Dust is rather ubiquitous in space; some well-known systems where the presence of dust has been established, are inter- and circumstellar clouds, solar system interplanetary dust, earth's magnetosphere, comets and planetary rings. Also, dusty plasmas appear in laboratory system, e.g. rf plasmas used in the microelectronic processing industry, and it may also be present in the limiter regions of fusion plasmas due to the sputtering of carbon by energetic particles. We shall mainly discuss space dusty plasmas although many of the conclusions are valid for the laboratory plasmas as well. Grains are dielectric (ices, silicates...) or metallic (graphite, magnetite...), see [1]. We use the term "dusty plasma" when the number of grains in Debye sphere is greater than one, and "dust in a plasma" when the number density of grains is less than one. Usually, we have $\lambda_D > Ro \gg a$ (dusty plasma), where λ_D is a plasma

Debye length, Ro is an average separation ($Ro = (3/4N_{0d})^{1/3}$, and "a" is the grain radius.

After [1] and [2], we note the following typical data for dusty plasmas in:

i) interstellar clouds, where $a/\lambda_D \approx 10^{-8}$ and $Ro/\lambda_D \approx 10^{-1}$,

ii) ionosphere (80km), where $a/\lambda_D \approx 10^{-6}$ and $Ro/\lambda_D \approx 10^{-2}$,

iii) rings, where $a/\lambda_D \approx 10^{-6}$ and $Ro/\lambda_D \approx 10^{-3}$,

iv) comets, where $a/\lambda_D \approx 10^{-6}$ and $Ro/\lambda_D \approx 10 - 100$,

v) magnetosphere, where $a/\lambda_D \approx 10^{-6}$ and $Ro/\lambda_D \approx 10 - 100$.

The mechanism of dust grain charging due to plasma current, photoelectron and secondary emission currents can be found in [1] and [2]. Recently, see [3], it has been proved that *Vlasov description of dusty plasmas is valid not only in the usual weakly coupled plasma regime but also in the strong-coupling limit* for dusty plasmas. Deviations from both limits are to be expected for the intermediate range of coupling when Coulomb crystallization occurs.

The main objective of the paper is to determine asymptotic solutions to the initial-value conditions for Vlasov-Ampère equations, that is to find the "far field" solutions. Next, we determine dispersion relation for longitudinal waves (DAW and DIAW) by use of the linearized Vlasov equations. In case of simplified equilibrium velocity distributions but for fully nonlinear plasmas, we determine velocity distributions $f_\alpha(u, \xi)$ where $\xi = x - Ut$ as well as the Sagdeev potential equations and we compute the solitary waves for a set of dusty plasma parameters.

2. Statement of the problem

We investigate the Vlasov-Ampère/Gauss system of equations for multi-species plasmas, that is

(2.1) $$\left[\partial_t + u\partial_x + \frac{q_\alpha}{m_\alpha} E\partial_u\right] f_\alpha(u, x, t) = 0\,, \quad \partial_u \equiv \frac{\partial}{\partial u} \quad \text{(Vlasov)}\,,$$

(2.2) $$\epsilon_0\, \partial_t E + \sum_\alpha q_\alpha \int u f_\alpha du = 0 \quad \text{(Ampère)}\,,$$

(2.3) $$\epsilon_0\, \partial_x E = \int\limits_{-\infty}^{\infty} f_\alpha du \equiv \sum_\alpha \rho_\alpha\,, \quad E = -\partial_x \phi \quad \text{(Gauss)}\,,$$

where x, u and t are space, velocity and time variables, respectively. $E(x,t)$, $\phi(x,t)$, $f_\alpha(u,x,t)$, q_α and m_α are the electric field, potential, function of velocity distribution, charge and mass of α-particles, respectively. Let us assume

$$f_\alpha(u,x,t) = N_0^\alpha f_{0\alpha}(u) + \sum_{n=1}^{\infty} f_{n\alpha}(u,x,t)\,, \tag{2.4}$$

where $N_0^\alpha, f_{0\alpha}(u)$ are the equilibrium particle concentration and the velocity distribution for $E = 0$, and $f_{n\alpha}$ is of the order of E. Substituting (2.4) into (2.1), we derive the well-known hierarchy of linear equations, see [4],

$$\begin{gathered}(\partial_t + u\partial_x)\, f_{1\alpha} = -\frac{N_0^\alpha q_\alpha}{m_\alpha} E\partial_u f_{1\alpha}\,, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ (\partial_t + u\partial_x)\, f_{n\alpha} = -\frac{N_0^\alpha q_\alpha}{m_\alpha} E\partial_u f_{n-1,\alpha}\end{gathered} \tag{2.5}$$

and we search for a solution of a given initial-value problem. According to [4], we note, that if the solution $f_\alpha(u,x,t)$ exists for a given equilibrium $f_{0\alpha}(u)$, then it takes the following form:

$$f_\alpha(u,x,t) = N_0^\alpha\, f_{0\alpha}\left(u + W_\alpha(u,x,t)\right)\,, \tag{2.6}$$

where $W_\alpha(u,x,t)$ satisfies the following equation

$$\left[\partial_t + u\,\partial_x + \frac{q_\alpha}{m_\alpha} E(x,t)\partial_u\right] W_\alpha(u,x,t) = -\frac{q_\alpha}{m_\alpha} E(x,t)\,. \tag{2.7}$$

The relation (2.6) exhibits an equilibrium distribution memory of Vlasov plasmas. The plasma response to an initial disturbance of plasma equilibrium as well as "far field " solutions cannot be composed of the arbitrary stationary solutions. In particular, if one assumes the Maxwellian equilibrium distribution for "hot electrons" and a proper equilibrium distribution for "cold electrons", then the "far field" solution does not exist (no solution) for any initial disturbances, see [4]. It appears that (2.4) is divergent due to wave-particle interactions, that is due to nonlinear Landau instabilities. We note that assuming the Dirac delta equilibrium for cold plasma species, that is

$$f_{oc}(u) = \delta(u)\,,$$

a stationary "far field" solutions evolves in the form

$$f_c(u,\xi) = \delta\left(u + W_c(u,\xi)\right)\,,$$

where $\xi = x - U\,t$,

$$W_c(u,\xi) = (u-U)\left[(1+r)^{\frac{1}{2}} - 1\right],$$

where

$$r = \frac{2q_c\phi(\xi)}{m_c(u-U)^2} \qquad \text{and} \qquad E(\xi) = -\partial_\xi\,\phi(\xi)\,.$$

We note that W_c satisfies Eq. (2.7). The well-known "cold particle" number density can be obtained

$$n_c(\xi) = N_{0c}\int_{-\infty}^{\infty}\delta\left(u + W_c(u,\xi)\right)du = N_{0c}/\left[1 - \frac{2q_c\phi(\xi)}{m_cU^2}\right]^{\frac{1}{2}}. \tag{2.8}$$

In the case of "hot particles", we accepted "square" equilibrium distribution

$$f_{oh}(u) = \frac{1}{2a_h}\left[H(u+a_h) - H(u-a_h)\right], \tag{2.9}$$

where $H(\cdot)$ is a Heaviside function and

$$f_h(u,\xi) = \frac{1}{2a_h}\left[H\left(u + a_h + W_h(u,\xi)\right) - H\left(u - a_h + W_h(u,\xi)\right)\right],$$

where $\xi = x - Ut$ and (2.4) is convergent.

The hot particle number density takes the following form:

$$n_h(\xi) = N_{0h}\int_{-\infty}^{\infty} f_h(u,\xi)du = N_{0h}\frac{a_h + U}{2a_h}\left[1 - \frac{2q_h\phi(\xi)}{m_h(a_h+U)^2}\right]^{\frac{1}{2}} + N_{0h}\frac{a_h - U}{2a_h}\left[1 - \frac{2q_h\phi(\xi)}{m_h(a_h-U)^2}\right]^{\frac{1}{2}}. \tag{2.10}$$

Assuming that $U/a_h \ll 1$, we have

$$n_h(\xi) \simeq N_{0h}\sqrt{1 - \frac{2\,q_h\,\phi(\xi)}{m_h\,a_h^2}}\,. \tag{2.11}$$

By use of $n_c(\xi)$ and $n_h(\xi)$, we can determine the Sagdeev potentials and then calculate the dust-ion-sound solitary waves for fully nonlinear plasmas. We note that the results of "far field" solutions derived here, which are the asymptotic solutions to the initial value-problem, are strictly related to the results which can be obtained by use of the "water-bag" model, see [5], [6] and [7].

3. Dispersion relation for dusty plasmas

Assuming sufficiently small disturbances of plasma equilibrium, the solution of the linearized Vlasov-Ampère equations takes the form, see [4],

$$E(x,t) = E_0(x,t) + \int_0^t dt_1 \int_{-\infty}^{\infty} E(x-x_1, t-t_1) K(x_1,t_1) dx_1 , \tag{3.1}$$

where

$$K(x,t) = -\sum_{\alpha} \omega_{\alpha}^2 \, f_{o\alpha}\left(\frac{x}{t}\right) .$$

Let us consider three components of a dusty plasma having the following equilibrium distributions:

$$\begin{aligned} f_{0d}(u) &= \delta(u) && \text{(dust)} , \\ f_{0e}(u) &= \frac{1}{2a_e}\left[H(u+a_e) - H(u-a_e)\right] && \text{(hot electrons)} , \\ f_{0i}(u) &= \frac{1}{2a_i}\left[H(u+a_i) - H(u-a_i)\right] && \text{(hot ions)} , \end{aligned} \tag{3.2}$$

$\omega_{\alpha}^2 = N_0^{\alpha}\, q_{\alpha}^2/\varepsilon_0\, m_{\alpha}$ is the plasma α-component frequency and a_{α} is the thermal velocity. We take Fourier–Laplace transform of Eq. (3.1) to obtain:

$$E(k,s) = \frac{E_0(k,s)}{D(k,s)} ,$$

where $D(k,s) \equiv 1 - K^e(k,s) - K^i(k,s) - K^d(k,s)$ and the dispersion relation for longitudinal plasma waves takes the form:

$$\omega^2 = \frac{\omega_e^2}{1 - \dfrac{k^2 a_e^2}{\omega^2}} + \frac{\omega_i^2}{1 - \dfrac{k^2 a_i^2}{\omega^2}} + \omega_d^2 , \tag{3.3}$$

where $s = -i\omega$. Following the papers [8] and [9], we assume rather cold ions, that is $\dfrac{k^2 a_i^2}{\omega^2} \ll 1$ and hot electrons $\dfrac{k^2 a_e^2}{\omega^2} \gg 1$ to obtain the following dispersion relation for the dust-ion-acoustic waves (DIAW)

$$\omega^2 \simeq \frac{k^2 \lambda_{De}^2 (\omega_{oi}^2 + \omega_d^2)}{1 + k^2 \lambda_{De}^2} , \qquad \text{where} \quad \lambda_{De} = \frac{a_e}{\omega_{0e}} .$$

If $k^2\lambda_{De}^2 \ll 1$, (long wave approximation) and $\omega_d^2 \ll \omega_{oi}^2$, we have the dust-modified ion-acoustic speed

$$C_s^2 = \frac{\omega^2}{k^2} \simeq \frac{N_{0i}}{N_e} v_s^2 \quad , \quad \text{where} \quad v_s = a_e \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \quad \text{and} \quad q_d = Z_d e \,. \tag{3.4}$$

The relation (3.4) is similar to the usual ion-sound wave spectrum for non-isothermal plasmas, that is $m_i a_i^2 \ll m_e a_e^2$. However in dusty plasmas, we usually have $T_e \simeq T_i$. The existence of new dust-acoustic waves, which occur as $m_i a_i^2 \simeq m_e a_e^2$, requires the presence of a very small amount of electrons in the background of the collisionless dusty plasma.

In virtue of (3.3), we have

$$\omega^2 \simeq \frac{\omega_d^2 k^2 \lambda_{De}}{1 + k^2\lambda_{De} + \dfrac{N_{0i}}{N_{0e}}} \simeq \omega_d^2 k^2 \lambda_{De} \frac{N_{0e}}{N_{0i}} ,$$

since

$$\omega^2 \ll k^2 a_i^2 \ll k^2 a_e^2$$

and

$$\frac{\lambda_{De}^2}{\lambda_{Di}^2} = \frac{N_{0i}}{N_{0e}} \gg 1 + k^2\lambda_{De}^2 \,.$$

The frequency ω of the dust-acoustic wave is very low and the dust-acoustic speed is

$$C_d \simeq \omega_d \lambda_{De} \sqrt{\frac{N_{0e}}{N_{0i}}} \,.$$

It is interesting to note, in the virtue of the conditions: $a_i \ll \omega/k \ll a_e$ for DIAW as well as in view of the condition $\omega/k \ll a_i \ll a_e$ for DAW, that the waves are subjected to insignificant electron and ion Landau damping. These waves should have some relevance to the low-frequency noise in the F-ring of Saturn.

4. Calculation of Sagdeev potentials and solitons

In the view of (2.8), (2.10) and (2.11), we derive the following expressions for dust, ion and electron charge number densities:

$$\frac{n_d(\xi)}{N_{0i}} = \frac{S_d}{\left(1+\frac{2y}{\mathrm{M}^2}\right)^{\frac{1}{2}}},$$

$$\frac{n_{i(\xi)}}{N_{0i}} = \frac{1+\nu_i}{2}\left(1-\frac{2y}{(1+\nu_i)^2}\right)^{\frac{1}{2}} + \frac{1-\nu_i}{2}\left(1-\frac{2y}{(1-\nu_i)^2}\right)^{\frac{1}{2}}$$

(4.1)
$$\simeq (1-2y)^{\frac{1}{2}}, \qquad \text{if} \quad \nu_i = \frac{U}{a_i} \ll 1,$$

$$\frac{n_{e(\xi)}}{N_{0e}} = S_e\left[\frac{1+\nu_e}{2}\left(1+\frac{2yR}{(1+\nu_e)^2}\right)^{\frac{1}{2}} + \frac{1-\nu_e}{2}\left(1+\frac{2yR}{(1-\nu_e)^2}\right)^{\frac{1}{2}}\right]$$

$$\simeq S_e(1+2yR)^{\frac{1}{2}}, \qquad \text{if} \quad \nu_e = \frac{U}{a_e} \ll 1,$$

where $y = y(\xi) = q\phi(y)/m_i\alpha_i^2, \xi$ is normalized with respect to $\lambda_{Di} = \dfrac{a_i}{\omega_{0i}}$ – is the ion Debye length and, M$= \dfrac{U}{a_i}\sqrt{\dfrac{m_d}{Z_d m_i}} = \dfrac{U}{c_s}$ – is the Mach number, $c_s = a_i\sqrt{\dfrac{Z_d m_i}{m_d}}$ – is the dust-ion-acoustic speed and $R = \dfrac{m_i a_i^2}{m_e a_e^2} = \dfrac{T_i}{T_e}$. We assume global charge neutrality $N_{0i} = Z_d N_{0d} + N_{0e}$ and $S_d = S = \dfrac{Z_d N_{0d}}{N_{0i}}, S_e = S - 1 = \dfrac{N_{0e}}{N_{0i}}$. We note that $m_d \gg m_i \gg m_e, 0 \leqslant S \leqslant 1$.

In view of Eq.(2.3), we have

(4.2)
$$\frac{\partial^2 y}{\partial \xi^2} + \frac{1}{\varepsilon_0}\sum_\alpha \rho_\alpha = 0 \equiv \quad \text{and} \quad V'(y) = \frac{1}{\varepsilon_0}\sum_\alpha \rho_\alpha\,, \alpha = e, i, d,$$

where $V(y)$ is the *Sagdeev potential* [10]. The energy integral is

(4.3)
$$\frac{1}{2}\left(\frac{\partial y}{\partial \xi}\right)^2 + V(y) = 0,$$

where

$$V(y, \mathrm{M}, S, R) = V_d(y, \mathrm{M}, S) + V_i(y) + V_e(y, R)$$

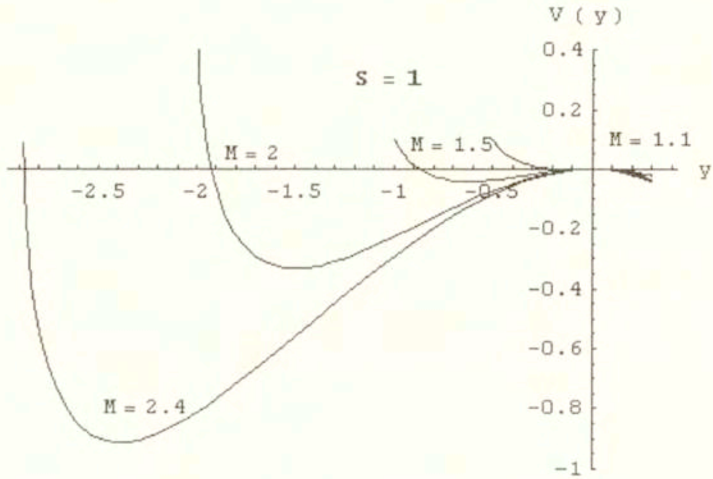

FIG. 1. Sagdeev potentials $V(y)$ versus y for Mach numbers M= 1.1, 1.5, 2.0 and 2.4 as $S = 1$ (no electrons). Soliton amplitudes y_{amp} increase with increasing M.

and

$$V_d(y, \mathrm{M}, S) = \mathrm{M}^2 S \left(1 - \left(1 + \frac{2y}{\mathrm{M}^2}\right)^{\frac{1}{2}}\right) > 0 \quad \text{for} \quad y < 0,$$

$$V_i(y) = \frac{1}{3}\left(1 - (1 - 2y)^{3/2}\right) < 0 \quad \text{for} \quad y < 0,$$

$$V_e(y, S, R) = \frac{1 - S}{3R}\left(1 - (1 + 2yR)^{3/2}\right) > 0 \quad \text{for} \quad y < 0.$$

The inertial dust term for $y < 0$ delivers the restoring force while thermal ions and electrons deliver wave pressures. We can expect negative potential solitons (rarefactive solitons, also called antisolitons). The case $S = 1$ ($S_e = 0$) represents the plasma where all the electrons are attached to the dust grains to form the two-component plasma. Whereas the case $S = 0 (S_e = 1)$ is an electron-ion plasma.

The figures 1 to 4 show a number of Sagdeev potentials $V(y, \mathrm{M}, S, R)$ for a given set of parameters: M, S, R. The exhibited shape of $V(y, \cdot)$ secures the existence of solitons.

The first figure depicts $V(y, \cdot)$ for two-component dust-ion plasmas ($S = 1$), when all electrons are collected by dust grains and the solitons exist in the anomalously large range of Mach number $1 < \mathrm{M} \leqslant 2.4$. Soliton amplitudes increase with increasing M up to $y_{\mathrm{amp}} = 2.9$.

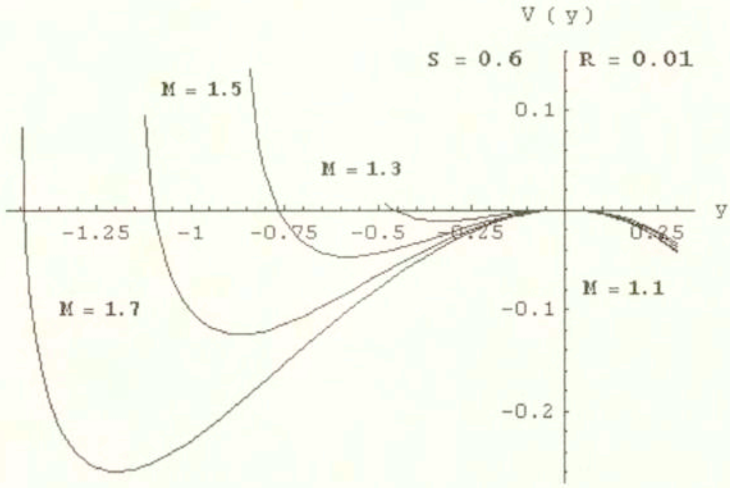

FIG. 2. Sagdeev potentials $V(y)$ versus y for Mach numbers M= 1.1, 1.3 and 1.7 as $S = 0.6$. Non-isothermal plasmas; hot electrons and cold ions $R = T_i/T_e = 0.01$ (DIAW).

The case of three-component dusty plasmas with electron number $S_e = 1 - S$, where $S = Z_d N_d / N_{0i} = 0.6$ and the law ion-electron temperature ratio $R = T_i/T_e = 0.01$, see Fig. 2, exhibits dust-ion-acoustic-wave (DIAW) solitons. The solitons exist in the Mach number range 1 <M$\leqslant$ 1.7.

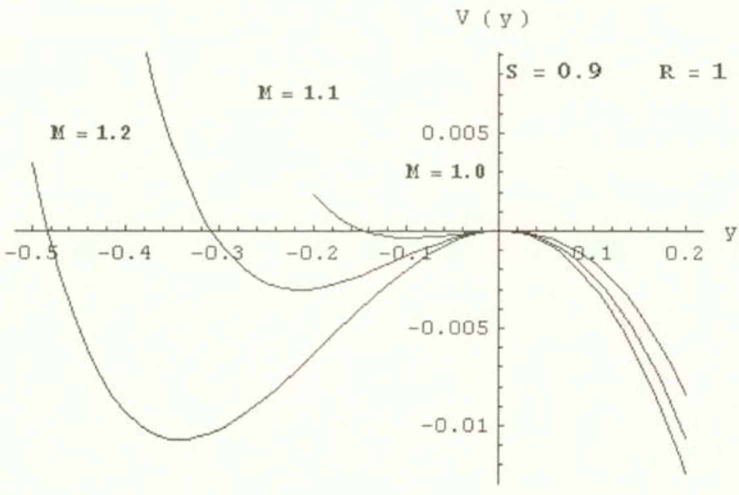

FIG. 3. Sagdeev potentials $V(y)$ versus y for Mach numbers M= 1.0, 1.1 and 1.2 as $S = 0.9$, $S_e = 0.1$. Isothermal plasmas $R = T_i/T_e = 1$ (DAW).

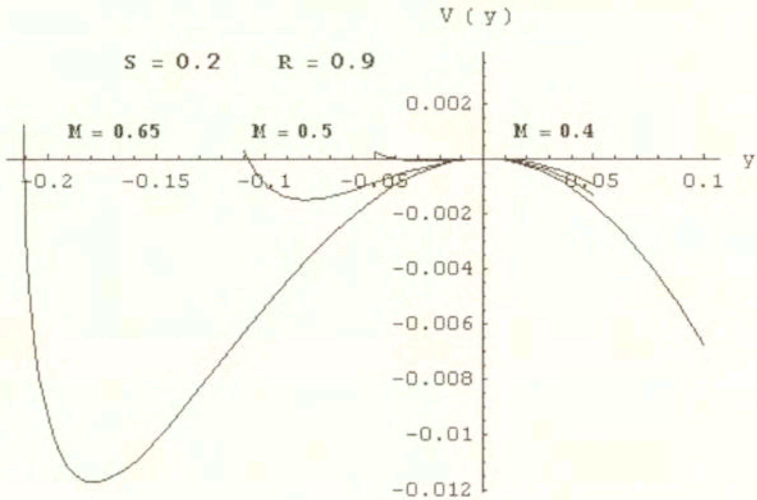

FIG. 4. Sagdeev potentials $V(y)$ versus y for Mach numbers M= 0.4, 0.5 and 0.65 as $S = 0.2$. Almost isothermal plasmas $R = T_i/T_e = 0.9$ (DAW).

Dust-acoustic-wave (DAW) solitons for isothermal ions and electrons are shown in Fig. 3. The Mach number range of soliton existence is the smallest one and it amounts to M$= 1 \div 1.2, S = 0.9, R = 1$.

It is interesting to note that, if there are dusty plasmas with almost isothermal ions and electrons $R = 0.9$ and the number density of dust is small enough $S = S_d = 0.2$, then solitons still exist but of smaller amplitudes and for M< 1, see Fig. 4. For such plasmas the Mach number is to be redefined. We conclude that even a small number density of dust $Z_d N_d$ gives enough inertia to support negative solitons.

Figure 5 depicts the $V(y)$-insert, and the respective soliton $y(\xi)$ in the case of three-component, non-isothermal plasma: M$= 1.5$, $S = 0.6$ and $R = 0.01$ (DIAW). This soliton is computed on the basis of one of the Sagdeev potentials presented in Fig. 2. We note that if $Z_d = 1000$ then for 3 dust grains, we would have 5000 ions and 2000 electrons — on the average. We can conclude that a small number of dust grains gives enough inertia to support negative solitons like in the case of isothermal plasmas.

Figure 6 exhibits the case of the rarefactive soliton for three-component plasma which is nearly isothermal $R = 0.9$ (DAW), $S = 0.2$ and M$= 0.65$. This soliton is computed on the basis of one of the Sagdeev potentials presented in Fig. 4. We note that if $Z_d = 1000$ then for 1 dust grain, we would have 5000 ions and 4000 electrons — on the average.

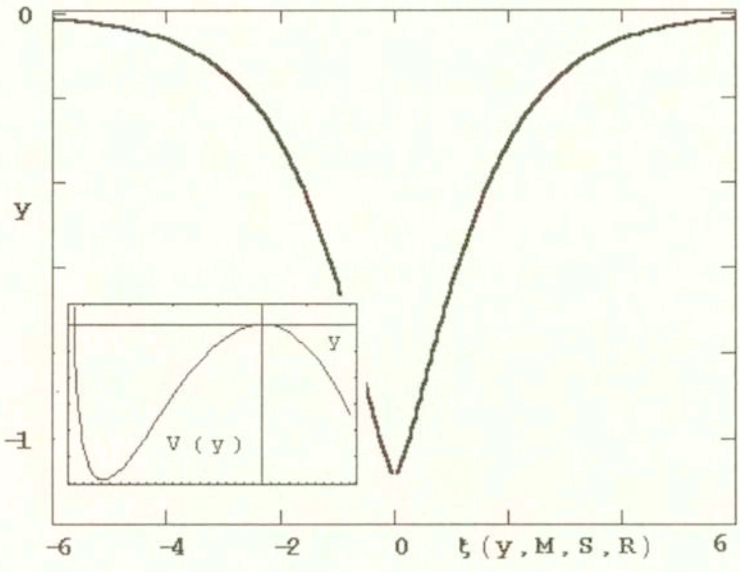

FIG. 5. Sagdeev potentials $V(y)$-insert and the respective soliton $y(\xi)$. Three-component plasma; M= 1.5, $S = 0.6$ and $R = 0.01$ (DIAW).

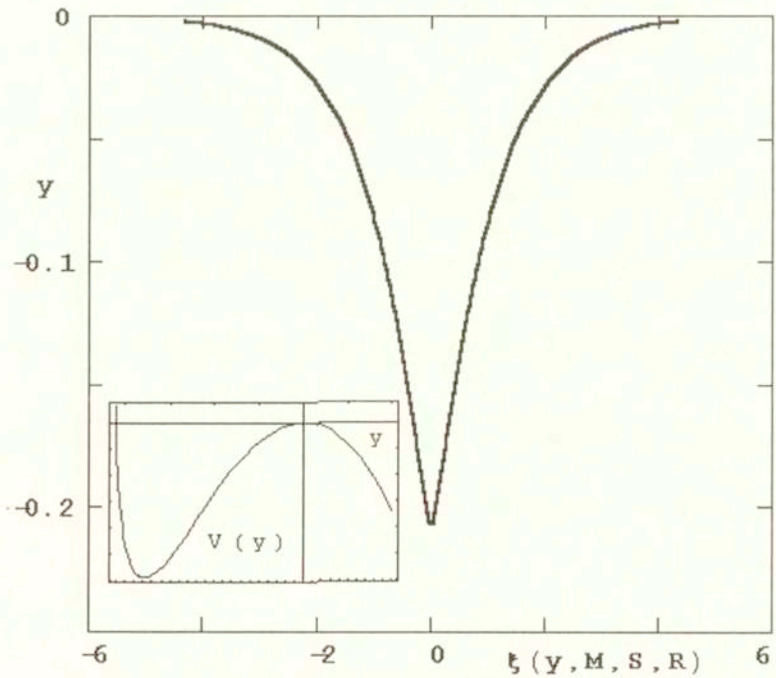

FIG. 6. Sagdeev potentials $V(y)$-insert and the respective soliton $y(\xi)$. Three-component plasma; M= 0.65, $S = 0.2$ and $R = 0.9$ (DIAW).

5. Conclusions

The initial-value-boundary problem was solved for "far field", asymptotic solutions, solitons in the case of the fully nonlinear dusty plasma. Landau damping/wave-particle interactions have been excluded by choosing artificial e-quilibrium distributions. Velocity distributions can be evaluated and the calculation of all moments is possible, e.g. energy flow as well as energy of traveling solitary waves. The dispersion relations here obtained are the same as those in the fluid description of dusty plasmas.

Due to the presence of negatively charged dust grains:

i) Only negative potential solitons (rarefactive solitons) can exist.

ii) Linear waves and solitons can exist in dust-ion-acoustic-wave plasmas (DIAW $R \ll 1$, non-isothermal ions and electrons) as well as in dust-acoustic-wave plasmas (DAW $R \simeq 1$, isothermal ions and electrons).

iii) The existence of DAW solitons in the case of small amount of dust grains in plasmas ($S = S_d = 0.2$) is secured for the soliton speeds less than the dust-ion-acoustic speed c_s, which was defined in Sec. 4. The presence of the electron component ($S < 1$) lowers the soliton amplitude as R is fixed and reduces the range of M, in which solitons can exist.

For small solitons ($|y| \ll 1$ and $S = 1$) there is a full resemblance between fluid, see [11], and kinetic solitons revealed in this paper. But in the case of fully nonlinear two-component plasmas, $S = 1$, the soliton shape, amplitudes and the soliton existence range of M are different in these two descriptions. The respective results for fluid description in case of three-component dusty plasmas, $S < 1$, are not known to the authors. We note that for DIAW and DAW, we have $\alpha_i \ll U \ll \alpha_e$ and $U \ll a_i \ll a_e$, respectively, and hence the solitons considered here are subjected to insignificant electron and ion Landau damping. It justifies the Dirac delta accepted here and the Heaviside functions as the suitable equilibrium velocity distributions.

Acknowledgments

The paper was partly supported by the State Committee for Scientific Research (KBN) through the grant No 2P03C01210.

References

1. U. DE ANGELIS, *The physics of dusty plasmas*, Physica Scripta, **45**, 465–472, 1992.
2. C. K. GEORTZ, *Dusty plasmas in the solar system*, Reviews of Geophysics, **27**, 2, 271–292, May, 1989.
3. X. WANG and A. BHATTACHARJEE, *On a kinetic theory for strongly coupled dusty plasmas*, Phys. Plasmas, **3**, 4, 1189–1191, 1996.

4. A. J .TURSKI and B. ATAMANIUK, *Far field solutions of Vlasov-Maxwell equations and wave-particle interactions*, J. Techn. Phys., **30**, 2, 147, 1989.
5. R. C. DAVIDSON, *Methods in nonlinear plasma theory*, Academic Press, N. York, 1972.
6. E. INFELD and G. ROWLANDS, *Nonlinear waves, solitons and chaos*, Cambridge Univ. Press, Cambridge 1992.
7. H. L. BERK and K. V. ROBERTS, *Numerical study of Vlasov's equation for special class of distribution functions*, Phys. Fluids, **10**, 1595–1597, 1967.
8. P. K. SHUKLA and V. P. SILIN, *Dust ion-acoustic waves*, Physica Scripta, **45**, 508, 1992.
9. R. L. MERLINO, A. BARKAN, C. THOMPSON and N. D'ANGELO, *Laboratory studies of waves and instabilities in dusty plasmas*, Phys. Plasmas, **5**, 5, 1607–1614, 1996.
10. R. Z. SAGDEEV and M. N. ROZENBLUTH, *Foundations of plasma physics* [in Russian], Vol.1, 2, Suppl., Moskwa, Energo-Atomizdat 1984.
11. A. MAMUN, R. CRAINS and P. SHUKLA, *Solitary potentials in dusty plasmas,* Phys. Plasmas, **3**, 2, 702, 1996.

Received November 30, 1998.