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From material discrete memory patterns to the study of demagnetisation-like processes

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Dedicated to the legacy of A.A.Ilyushin
as a token of scientific gratitude

THE PRESENT PAPER is an attempt to study the links between the constitutive pattern of the idealized behaviour of pure hysteresis and the general theory of demagnetisation-like processes of real materials. Modelling based on this discrete-memory pattern has been previously introduced, and used to describe rather successfully the puzzling properties of various deformable materials ranging from granular media to shape memory alloys and deformable ferromagnets or ferroelectrics. However, one of the five basic ingredients of the pattern has not been, up to now, studied extensively: it has been introduced rather as an assumption, namely that of the general possibility to obtain recurrent recovery of a unique thermomechanical initial neutral state. Therefore the aim of this paper is threefold: firstly, to give some tensorial mechanical examples suggesting that the fifth ingredient of the pattern is relevant with respect to the others; secondly, to suggest the importance of a general study of demagnetisation-like processes, purely mechanical or not; thirdly, to suggest that the fifth ingredient is not only relevant on the theoretical grounds and of importance at the level of physical principles, but may be also justified owing to data from now on available.

1. Introduction

i. RELIANCE on physical justifications of the material discrete-memory concept [1] is now rather soundly established in the field of nonlinear continuum mechanics: mainly based on results obtained from transmission electron microscopy, this reliance is fully compatible with the fact that the strain notion is essentially of discrete-memory form [1, 2]. The discrete-memory concept also proved to be rather effective regarding the modelling of the deformable media

[1] Owing to experimental evidences, this notion is frequently implemented in the numerical simulations performed in the field of multiaxial cyclic plasticity. The notion may be sketched through the path 01213 of Fig. 1: in order to describe this rate-independent behaviour with perfectly closed cycle, it is well known that it is necessary to take into account the following (discrete) sets of memorised states: state {0} along the path 01; states {0, 1} along the path 12; states {0, 1, 2} along 21; state {0} along 13.

which are, at the "micro"-scale, structured and with defects [3, 4, 5, 6]. However, such a pattern is still more like an incipient program rather than a theory already expanded for the benefit of engineers. This feature arises from the fact that two fundamental problems are still insufficiently clarified or entirely open, namely the demagnetisation-like problem and the preferred reference frame problem.

ii. Resolute endeavour is needed regarding the former, because it calls the whole pattern in question. In contrast, the choice among the possible solutions of the preferred reference frame problem is "only" decisive at the level of the definitions of the discrete-memory condition and of the thermomechanical rates, and also taking into account the material anisotropy or the case of large deformations and strains (obtained through monotonic loading or through cumulative second order effects of ratchet type). Moreover, regarding the second problem, the (unpublished) results obtained so far are not only encouraging but they are also an incentive to pursue in the direction already chosen [3], whereas no tensorial results were available, at the time, in the field of demagnetisation-like processes.

iii. Dilemma is therefore removed as to the hierarchy of research priorities, but the demagnetisation-like problem has many facets, the one-dimensional intuitions may act as ominous lures and any first approach, chosen tentatively, cannot rely on or refer to previous endeavours. To maintain the bearing of the results, the basic assumptions are chosen so as to focus the analysis on a special but not too restrictive case. Basically, the analysis is performed in the field of multiaxial plasticity (2): the aim is therefore to study the recovery of a relevant state under cyclic stress paths. The material under consideration is a rather idealised one: it does not undergo ageing, fatigue, damage, rate-independent hardening or viscous effects [5]. The main property of the material is that of elastic-plastic hysteresis regarding only the deviatoric part of the behaviour. Moreover, there are no coupling effects between deviatoric and isotropic part of the behaviour [6]. It must be also emphasised that there are no coupling effects between the mechanical behaviour and another behaviour such as that of ferrohysteresis: *the study may be considered as purely thermomechanical and the word demagnetization is not used in order to be reminiscent of coupling effects* (3). *In fact this word is convenient to suggest briefly the process of recovery which is studied here.* Moreover, this word is actually heuristic through a rule which is proved to be effective in order to perform some numerical simulations ("first loading in reverse" rule, introduced in § 3.3 point ii).

To simplify the formal features of the study, the material is supposed to be isotropic and its limit surface to be of Huber – von Mises type. The constitutive definition is given in the frame of the irrotational triaxial kinematics associ-

(2) However, it is worth noting that, owing to the analysis of [3] resulting in (3.13) to (3.16), the mechanical case studied here may be considered as a special ferromagnetic case.

(3) The same remark holds as to the heuristic terminology ("polarization") introduced by Hill in a purely elastic study [37].

ated with the extensions K_1, K_2, K_3 (the usual associated components of the strain rate tensor $\mathbf{D}$ being then: $\dot{K}_n/(1+K_n)$, $n = 1, 2, 3$). Moreover, the stress paths are performed in the deviatoric plane, so that the vanishing of the first invariant I_σ of the Cauchy stress tensor $\boldsymbol{\sigma}$ (components σ_{11}, σ_{22}, σ_{33}) is always imposed. Consequently, the graphical sketches given for illustration are basically two-dimensional (in fact, the numerical simulations are obtained with a four-dimensional program, associated with the kinematics K_1, K_2, K_3, $2\tau_{23}$ and implemented by imposing the vanishing of I_σ and τ_{23}).

iv. As suggested by the abstract, the aim of the paper is threefold. Firstly, basic hints are introduced regarding the theoretical ground of the demagnetisation-like problem tackled with the help of the pure hysteresis pattern. Secondly, the study of a mechanical example is given for illustration: this study is an opportunity to suggest the interest of a general approach and to point out some puzzling features of the demagnetisation-like process. Thirdly, some remarks are introduced regarding the question of experimental evidence currently available in order to illuminate the enigma thereby appeared.

2. On the theoretical ground of the demagnetisation-like problem

2.1. Preliminaries on the fivefold structure of the pattern of pure hysteresis

i. Let us take as a starting-point the usual symbolic mechanical model consisting of an infinite number of springs and friction sliders [3, 4, 6]. More precisely, the model is an ordered infinite parallel succession of couples, each couple being defined by a spring and a friction slider associated in series. First of all, it is possible to perform, in the cyclic case, a careful analysis of this entirely well-defined model devoid of hidden variables (cf. [3], § 2.2.2 and 2.2.3, for example): the main result of this analysis is that the pattern is of discrete-memory form. Next, a thermodynamic analysis may be added (cf. [3], § 2.2.4, for example, or Appendix if necessary). Then, the analysis results in a radical departure from the classical point of view, because the reversible power Π is also of discrete-memory form. It is worth noting that accordingly, the associated intrinsic dissipation Φ and the disorder rate $\dot{I}$ are also of discrete memory form. Moreover the disorder rate plays, in a Gibbs-like balance equation, a role similar to that played by the entropy rate $T\dot{S}$ [7].

ii. The one-dimensional character of the model allows us to give easily a set of heuristic illustrations. Let t and t_R be the current time and the inversion time of reference because it is associated with the origin of the current branch of cycle. Then, along each current branch or arc of branch of cyclic evolution, the one-dimensional pattern is as follows:

1. Discrete memory existence expressed as the invariance of the current (dragged along from t_R to $t > t_R$) Almansi and Cauchy scalars (tensors) ${}^t_R\varepsilon$ and ${}^t_R\sigma$,

respectively:

$$\frac{\partial}{\partial t}\,{}^{t}_{R}\varepsilon \equiv \frac{\partial}{\partial t}\,{}^{t}_{R}\sigma \equiv 0. \tag{2.1}$$

2. Constitutive differential-difference equations of mechanical character (where the "Masing rule" is expressed through the piecewise constant similarity functional ω equal to 1 or 2), such as, for example:

$$(2.2)_1 \qquad \dot\sigma \equiv \frac{\partial}{\partial t}\Delta\,{}^{t}_{R}\sigma = G_0\left[1-\left(\frac{\Delta\,{}^{t}_{R}\sigma}{\omega S_0}\right)^{c}\right]\frac{\partial \Delta\,{}^{t}_{R}\varepsilon}{\partial t}, \qquad \Delta\,{}^{t}_{R}\varepsilon = \varepsilon - {}^{t}_{R}\varepsilon, \qquad \Delta\,{}^{t}_{R}\sigma = \sigma - {}^{t}_{R}\sigma,$$

the associated equations of thermodynamic character being:

$$(2.2)_2 \qquad \begin{aligned} \Pi &= -P_i - \Phi, & P_i &= -\sigma\Delta^{\dot{t}}_{R}\varepsilon, \\ \Phi &= \Delta\,{}^{t}_{R}\sigma\,\Delta^{\dot{t}}_{R}\varepsilon, & -\dot{Q}_{ii} &= \frac{\Phi - C}{\omega}, \\ C &= \Delta^{\dot{t}}_{R}\sigma\,\Delta\,{}^{t}_{R}\varepsilon, & \dot{I} &= \frac{(\omega-1)\Phi + C}{\omega}, \\ I(t_{R^+}) &= 0, & \dot{E} &= \frac{-P_i - \Phi + C}{\omega}, \end{aligned}$$

where the rates $\dot{E}$ and $-\dot{Q}_{ii}$ denote the current rates of internal energy (associated to the springs) and of internal intrinsic heat supply (due to the friction of the sliders), respectively (cf. Appendix, if necessary).

3. Inversion criterion giving the definition of the inversion time of reference t_R:

$$\text{if}\quad \delta W < 0 \quad \forall\, t \in\,]t, t+\delta t] \;\Rightarrow\; t = t_R, \quad \sigma(t) = {}^{t}_{R}\sigma, \quad W(t) = {}^{t}_{R}W \tag{2.3}$$

through the sign of a virtual variation (δW) because $(2.3)_1$ expresses the second principle of thermodynamics. In the form $(2.3)_1$ the criterion is expressed with the aid of the "help function" W:

$$W = \frac{2}{\omega^2}\int_{t_R}^{t}\Phi(t)\,dt, \qquad W(t_{R^+}) \equiv 0 \tag{2.4}$$

which is, here, of "ln cosh" form when $c = 2$. Other forms are introduced later (§ 3.3, point iii).

4. Algorithm A, also expressed with the aid of W, and giving ω, ${}^{t}_{R}\sigma$, and the set of still memorised variables $\{{}^{t}_{M}W\}$ and $\{{}^{t}_{M}\sigma\}$ (cf. [4] or [6], for example).

5. Existence of the *unique thermomechanical neutral* state defined by:

$$
(2.5)\qquad \begin{gathered} k=1, \qquad \alpha=1, \qquad n=1, \\ {}_{M}W_1(0)=\infty, \qquad W(0_+)=0, \qquad \omega(0)=1, \qquad \Delta_{\,0}^{0^+}\sigma=\Delta_{\,0}^{0^+}\varepsilon=0. \end{gathered}
$$

This basic state is restored by the rather well known "symmetrical" *slowly decreasing cyclic* path.

Under cyclic loading defined by a set of inversion times $\{t_I\}$ and by:

$$
(2.6)\qquad \begin{aligned} D^2=1, \qquad & \Delta_{\,R}^{\,t}\varepsilon=\varepsilon(t)-{}_{R}^{t}\varepsilon=\varepsilon(t)-\varepsilon(t_R)=\pm(t-t_R), \\ t>t_R>0, \qquad & \Delta_{\,0}^{\,t}\varepsilon=\textstyle\sum\Delta_{\,R}^{\,t}\varepsilon, \end{aligned}
$$

the one-dimensional behaviour of pure hysteresis is obtained (Fig. 1, obtained with $c = 0.8$, and suggesting the importance of the notion of sky-line, this curve being continuous only if symmetrical *slowly decreasing cyclic* paths are performed). Owing to the simple form of the stress rate definition, the useful integral forms are immediately obtained in the case $c = 2$:

$$
(2.7)\qquad \begin{gathered} (\sigma/S_0)^2=1-\exp(-G_0W/S_0^2) \Leftrightarrow (\Delta\sigma/\omega S_0)^2=1-\exp(-G_0W/S_0^2), \\ W=-(S_0^2/G_0)\ln\left[1-(\sigma/S_0)^2\right] \Leftrightarrow W=-(S_0^2/G_0)\ln\left[1-(\Delta\sigma/\omega S_0)^2\right], \\ W=(2S_0^2/G_0^2)\ln\cosh\left[G_0\varepsilon/S_0\right] \Leftrightarrow W=(2S_0^2/G_0^2)\ln\cosh\left[G_0\Delta\varepsilon/\omega S_0\right]. \end{gathered}
$$

2.2. From the notions of state and of U.T.N. state to the role of the fifth ingredient

i. In the field of classical thermodynamics it is always admitted that the state of a physical system can be described using a Duhem set of "normal" variables: in short, if $D = 0$ and if the work of the external action vanishes during a temperature change, then the set is normal. The theory of linear infinitesimal thermoelasticity is a well known heuristic example of application (4). In this section, the attention is devoted to the symbolic model and the study is restricted to the case of a unique constant temperature, a choice which does not mean that the study of temperature-dependent symbolic models cannot bring heuristic hints ([8], § IV. 2.3). Therefore, one must pay attention to one of the assumptions generally not recalled, although always implied: in the classical definition of the state, there is no interference of the internal history of the system. By contrast with this usual assumption, the behaviour of the model suggests that the relevant set of state variables must include the history. In order to define a state in the sense both effective and deterministic in the large (cf. [9, 10] and point iii, below), it is indeed necessary to know not only the current strain, but also the current values of the piecewise constant functionals ω, ${}_{R}^{t}\varepsilon$, ${}_{R}^{t}\sigma$, and the current sets of

(4) The aim of the section is neither to point out the special role of the temperature, nor to underline that the thermoelastic case is actually quite misleading, owing to the difficulties appearing in the thermohyperelastic case (cf. [8], § II.5).

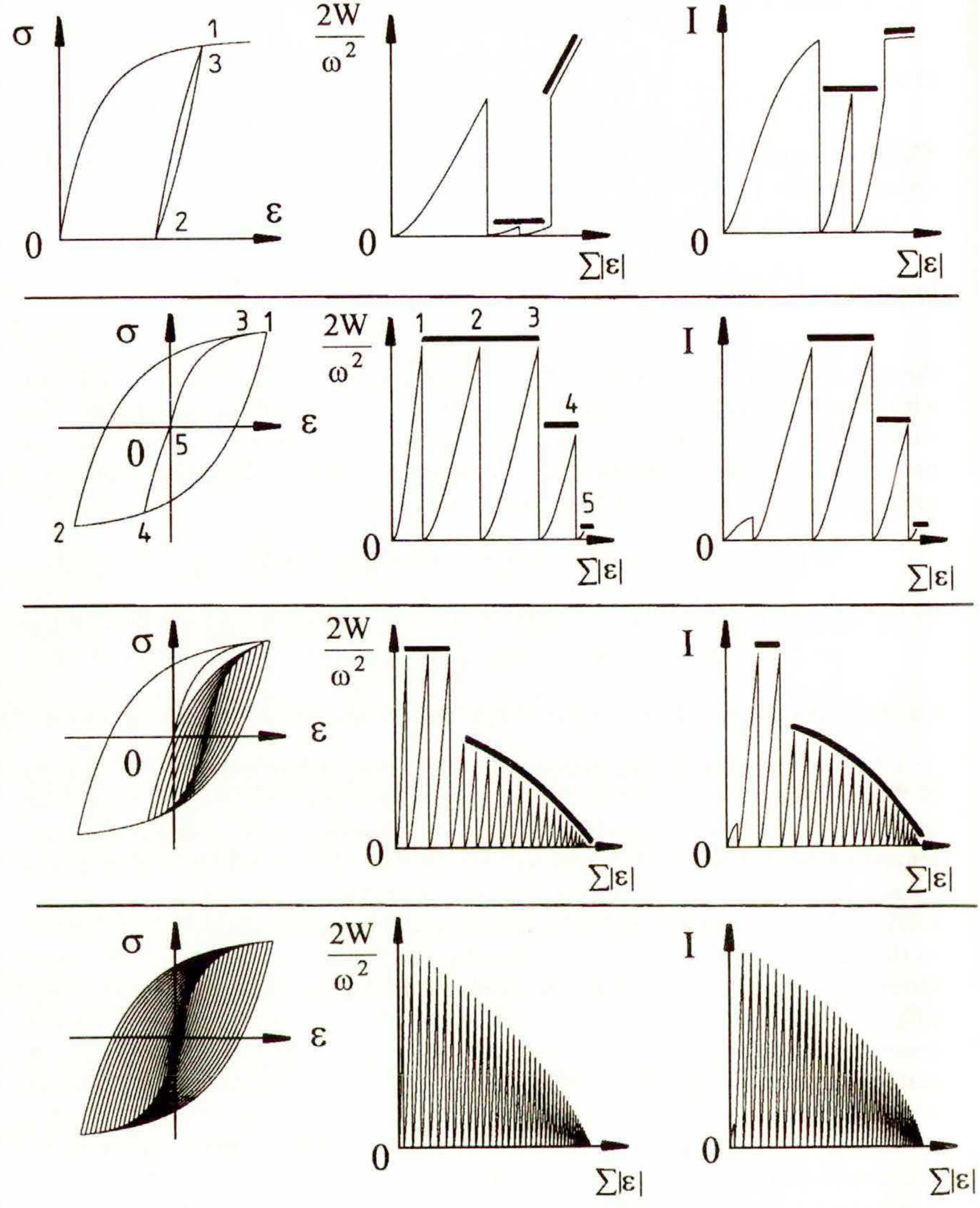

FIG. 1. Introduction of the sky-line notion through the one-dimensional form of the pattern.

the discrete and ordered series of the still memorised information $\{{}^t_M W\}$, $\{{}^t_M \sigma\}$. Taking as a starting point a well known, but often overlooked, graphical sketch, it is possible to stress the radical departure from the classical trend which is actually instilled by the discrete memory state notion.

ii. This well known graphical sketch gives the set of the individual states of spring strain of each couple for an imposed strain history (Fig. 2 a, where for sake of brevity S.S. is for *spring strain* reached at the current imposed strain; L.S. is for *limit strain* of a current couple of the model; S.E. is for *spring energy*; L.E. is for *limit energy*). Owing to (2.7), it is clear that, moreover, the strain variations $\Delta {}^t_R\varepsilon$ and the energy variations by intrinsic dissipation, W, are equivalent variables along each branch or arc of branch. Consequently, an equivalent graphical representation is possible if individual state of energy is associated with an imposed energy (Fig. 2 b, where the evolution of an ordered discrete set of *non-increasing* values of W may be conventionally displayed through the successive broken lines $\infty A0$, $\infty AB0$, $\infty ABC0$, ..., without introducing a physical definition of the $[0, \infty[$ axis of the integers). The specific "constitutive properties" (G_0, S_0, tanh function, ln cosh function) do not play any role, and the basic feature of the state appears: it is neither dimensional nor specific of a special reality (defined through parameters such as G_0 and functions such as tanh): it is basically generic and it involves an algorithm for the comparison of non-dimensional values.

The non-dimensional character is not restrictive, for it is sufficient in order to express the relative vicinity of the plastic limit of a couple, whatever may be the values of G_n and S_n: the interesting vicinity parameter of a given couple is the ratio S_n/G_n. The classical definition of the state is extensive-intensive in character ($S - T - \rho T dS$; $V - p - p dV/V$; $\varepsilon - \sigma - \sigma d\varepsilon$), and, in contrast, the new definition is basically topologic-algorithmic in character. In the pattern, this character is quite explicitly involved through (2.1), (2.3) and (2.4), but not clearly in (2.5). This point leads us to the question of the definition of the *unique thermomechanical neutral* state.

iii. Owing to the actual physical properties of the model, it is well known that after a fundamental *slowly decreasing cyclic* path, "symmetrical with respect to the origin", the demagnetized state is obtained: the graphical representation of this state is then close to the axis of abscissa (Figs. 2 c, 2 d, where for sake of brevity U.T.N. is for *unique thermomechanical neutral* state, I.H. is for *individual history*, F.L. is for *first loading*, E.L. is for *effective loading*, D.P. is for *demagnetisation process*, R.D.S. is for *remnant discrete set*). A similar state may be recurrently obtained, whatever may be the loading tentatively imposed in order to deviate from it (cf. [3], § 2.2.5). It cannot be identical with the axis itself, for the axis is the line of state for the incipient system, exclusively: this unique incipient state is always impossible to recover exactly, a property in accordance with the fact that the behaviour is always entirely irreversible. Returning

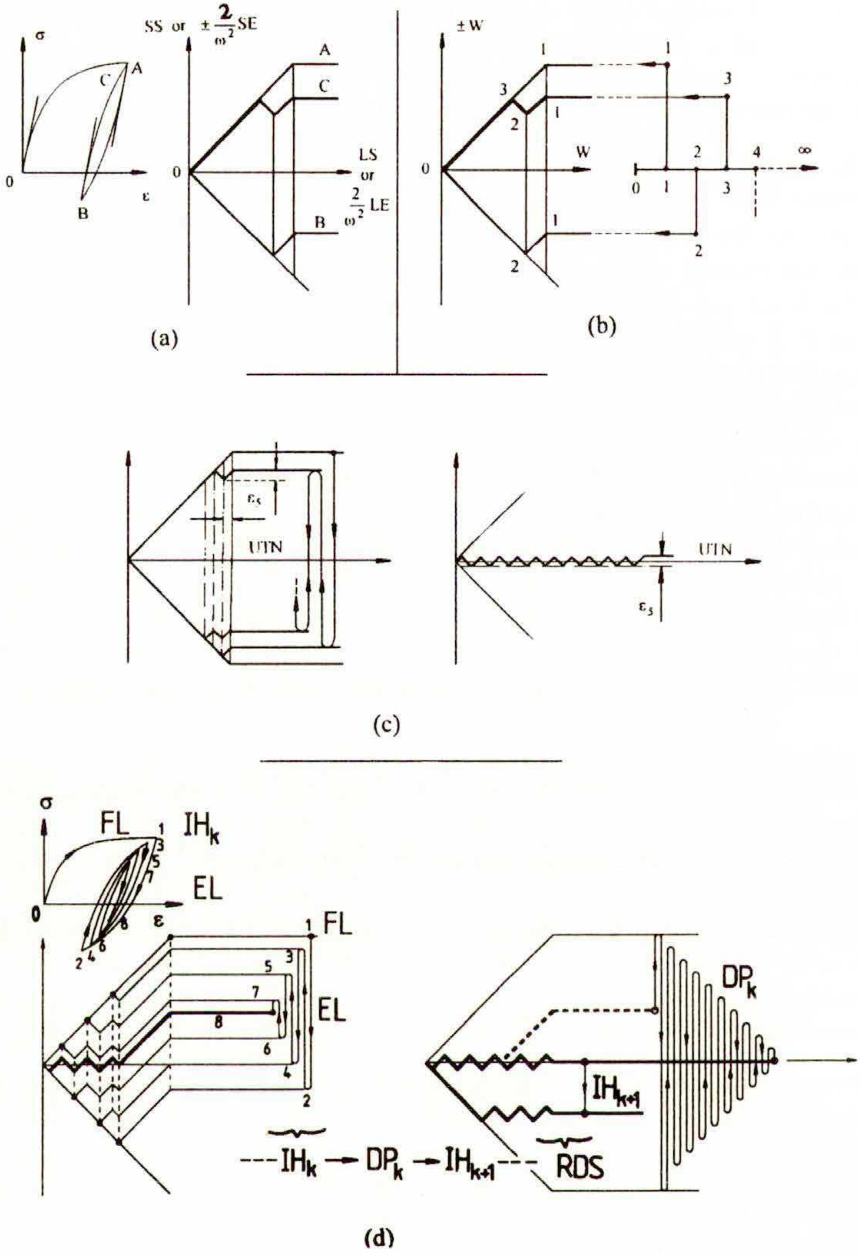

FIG. 2. Topological representation of the current states of the couples of the "springs-friction sliders" model or of the energetic state involved in the pattern.

to the physically relevant case, one notices that through a much more accurate demagnetisation-like process, the number of discrete information involved in the state is infinite: moreover, during any cyclic loading unable to fade the whole discrete set, the number of discrete information remains infinite (Fig. 2 d). Any sufficiently accurate demagnetisation-like process actually leads to a state which is so close to the (unique and unrecoverable) incipient state, that it is not necessary, practically, to discern between them. Hence, the set of all these states may be reduced to a unique state, recurrently recoverable. This *unique thermomechanical neutral* state may be defined with the aid of the unique approximation (2.5), provided that: 1) a sufficiently accurate demagnetised-like state exists, whatever the previous history may be; 2) a general strategy is defined in order to perform the indefinitely accurate demagnetisation. These conditions are fulfilled by the symbolic model and, obviously, by the one-dimensional pattern, built as close to the model as it is possible. In contrast, the fulfilment of these conditions is a puzzling problem in the three-dimensional, tensorial case. Consequently, the role of the fifth ingredient of the pattern is important at the level of physical principles [9, 10].

Indeed, it is well known that, when the principle of determinism is understood in the classical sense supported by Painlevé (1910), any pattern introducing the history to define the current state must be taken as a provisional pattern: anyway, it must be considered as devoid of sound significance in order to foster the improvement of knowledge, for: "La conception d'après laquelle, pour prdire l'avenir d'un système matériel, il faudrait connaître tout son passé, est la négation même de la science" [9]. In contrast, the principle of determinism was originally proposed with a more broad view, implying a continuous improvement of knowledge though involving possibly the history together with the causality principle [10]. In that spirit, the intrinsic properties of the symbolic model may, for example, suggest the following: if a system is restorable and if, for different times of recovery, the set of initial "generalised" causes of this isolated system are the same up to a space translation, the same phenomena occur in the system after these different times, up to this space translation. Restoration is not involved in the case introduced by Picard (1907). Following the fundamental analysis of Picard, where, for the first time, the distinction is introduced between the variables which are "visibles" and those which are "cachées", the general equations may be Galilean invariant "...mais il n'est pas possible néanmoins de remonter le cours du temps". Accordingly, "Il n'est donc pas impossible qu'un système irréversible puisse être conservatif et obéisse aux lois générales de la mécanique classique". Following Picard, the case where all the variables are "visibles" is fanciful, for it involves that the system is entirely reversible and that, consequently, it will be possible to stem the tide of time. The classical point of view is essentially an assumption ("une hypothése de non-hérédité"). The caution of Picard [5] may

[5] The caution of Picard echoes that exhibited by Gibbs to the point of the limit of validity of its fundamental equation.

be justified through the symbolic model. This model is indeed endowed with the following properties: 1) the behaviour is always entirely irreversible; 2) the definition of the state involves the past history since the last conventional U.T.N. state obtained with the aid of an accurate demagnetisation-like process; 3) the behaviour is not only nearly reversible "locally" (to the right of any inversion at time t_R) following the DE CARBON analysis [11], but also "globally reversible from one *unique thermomechanical neutral* state to the following one"; 4) from the very beginning of the first loading history, the exact incipient state is lost forever. Being restorable as accurately as required, the model is then in accordance with the determinism in the broad meaning of the term. In the sequel of this paper a fifth point is suggested: in the tensorial, three-dimensional case, it is possible to introduce a generalised pattern involving a theoretical property, especially interesting with respect to the Painlevé - Picard dilemma: if an appropriate set of variables is hidden on purpose, a basic classical feature of the behaviour (in short, an invariant form of the strain-stress behaviour) may appear as reversible in spite of the utter irreversibility. This puzzling behaviour happens during the generalised demagnetisation-like process necessarily associated with the pattern. Being entirely irreversible although apparently "reversible", the pattern is an example of theory where the history plays a basic role, "..., mais dans des conditions plus complexes encore", as anticipated by Picard.

2.3. From the multiaxial pattern to the study of the demagnetisation-like problem in general form

i. The sketch of the multiaxial pattern has been introduced previously (cf. [6] § 2.2 and 2.3 or [4] pp. 1186–1189, for example). Basically, the pattern is defined by introducing, in the stress space of Ilyushin type (cf. [38] or [3], § 3.5, for example), surfaces (or loops) along which the intrinsic dissipation rate Φ is annihilated and, therefore, W is constant [12, 6, 4]. Moreover, owing to (2.2), one considers the stress rate defined by the differential-difference equations:

$$\frac{\partial}{\partial t}\sigma^i_j = a_0\delta^i_j + a_1 D^i_j + a_2 \Delta\,{}^t_R\overline{\sigma}^i_j\,, \qquad \overline{\sigma}^i_j = \sigma^i_j - \frac{1}{3}I_\sigma\delta^i_j\,. \tag{2.8}$$

The definition in the preferred reference frame being not studied in the present paper, (2.8) is given in the frame of the irrotational kinematics under consideration. This constitutive form leads to a simple reversible isotropic part as soon as:

$$\begin{gathered} a_0 = \lambda I_D\,, \qquad a_1 = 2\mu, \qquad a_2 = \beta_4\overline{M}\,, \\ \beta_4 = -\mu/(\omega S_0)^2, \qquad \overline{M} = \mathrm{tr}\,(\Delta\,{}^t_R\overline{\sigma}\,\overline{\mathbf{D}}) \qquad I_D = \mathrm{tr}\,(\mathbf{D}). \end{gathered} \tag{2.9}$$

The deviatoric part and its associated invariant scalar form (similar to $(2.2)_1$) are then:

$$\frac{\partial}{\partial t}\overline{\sigma^i_j} = a_1\overline{D^i_j} + a_2\Delta^t_R\overline{\sigma^i_j}\,, \tag{2.10}$$

$$\dot{\overline{\Pi}}_{\Delta\overline{\sigma}} = 2\mu \left(1 - \frac{2\overline{\Pi}_{\Delta\overline{\sigma}}}{(\omega S_0)^2}\right) \overline{M}, \qquad 2\overline{\Pi}_{\Delta\overline{\sigma}} = \operatorname{tr}(\Delta\overline{\sigma}\,\Delta\overline{\sigma}) = Q_\sigma^2 . \tag{2.11}$$

If $Q_0 = \sqrt{2}S_0$ is the radius of the Huber–von Mises circle, the above invariant form yields:

$$\begin{aligned} Q_\Delta^2 &= (\omega Q_0)^2 \left[1 - \exp(-\mu W/S_0^2)\right], \\ W &= -(S_0^2/\mu) \ln\left[1 - (Q_\Delta/\omega Q_0)^2\right]. \end{aligned} \tag{2.12}$$

These forms are similar to (2.7). The generalization of (2.9), (2.10) is obtained substituting to $\overline{M}$ the expression (3.6) and substituting to ωS_0 a scalar functional taking into account two requirements. The first one is that of "orientation" in the stress space of Ilyushin: the relative vicinity of asymptotic states must be defined for any current point of an unloading stress path [12, 6, 4]. The second requirement is that of "compatibility": during the first loading the neutral paths are circles centred at the origin, suggesting therefore the use of a general basic assumption consisting of the similarity of neutral paths with respect to the measured or *a priori* specified yield limit [12, 6, 4]. The formal consequences of this similarity hypothesis of the neutral loci is expressed through the identity between the mechanical definition and the thermodynamic definition of the neutral paths, whatever the W level may be. For the given (circular) shape of the (von Mises) yield limit, this simple identification allows, once for all, to obtain the scalar functional:

$$\gamma_4 = \frac{Q_{\Delta\overline{\sigma}}}{2\mu} A, \qquad A = 2\tan(\varphi_{\Delta\overline{\sigma}} - \varphi_R), \tag{2.13}$$

and the required tensorial generalisation of $(2.2)_1$, (2.9), (2.10) is:

$$\frac{\partial}{\partial t}\overline{\sigma_j^i} = 2\mu\overline{D_j^i} + \beta_4 \left(\overline{M} + \gamma_4 \dot{\varphi}_{\Delta\overline{\sigma}}\right) \Delta_R^t \overline{\sigma_j^i} . \tag{2.14}$$

A first consequence of this generalization is that the onset of the first unloading gives rise to a discontinuous process obtained through the sliding of a relevant set of neutral loci and giving, to the right of an inversion event, a family of loci without intersections (Fig. 3, where it is worth to note two points: firstly, the strains associated with R_1, IR and C are large with respect to S_0/μ; secondly, IR is an unfavourable initial state because the family of neutral loci has jumped). Secondly, it is useful to notice that the discrete memory process remains founded on information sets regarding the current and the previous reference states [12, 4]. As a consequence, the generalisation of the algorithm of the pattern may be easily obtained making use of the set $\{{}_M\varphi_R\}$ and of the associated "previous set":

$$\left\{{}_M\varphi_{\rho R}\right\}_n = \{0,\ \varphi(t_1),\ \dots\}_n$$

and introducing the possibility of coincidence of the W levels without closing a cycle [12, 4].

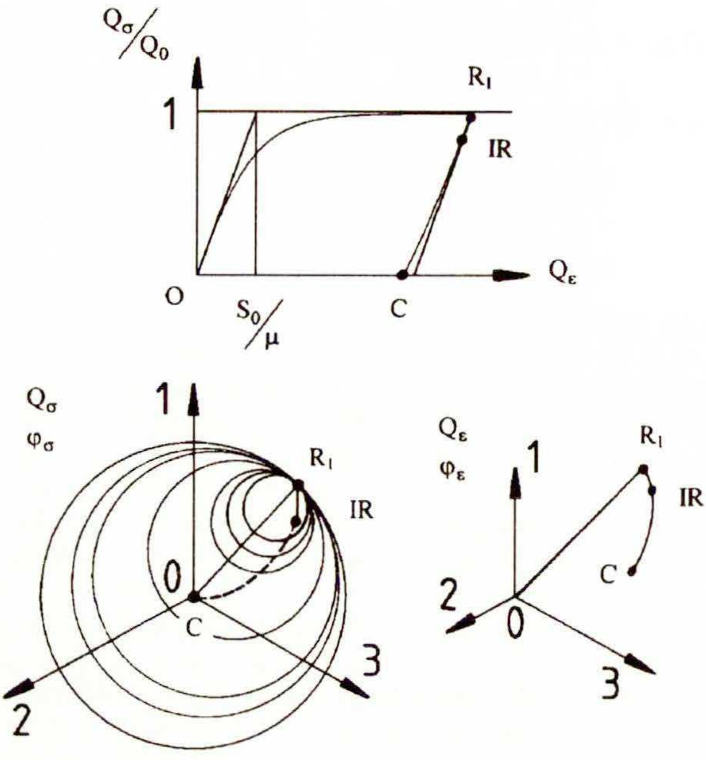

FIG. 3. Sketch suggesting the unfavourable initial residual states implemented in the study.

ii. The third consequence of the above process of generalisation is to give rise to a sophisticated feedback between the family of loci and the imposed cyclic loading. Accordingly, it becomes quite impossible to follow intuitively the global or local features of the functional correspondence between the stress and strain paths in the associated Ilyushin spaces (cf. Fig. 5 a, for example): neither the existence of the *unique thermomechanical neutral* state is *a priori* warranted, nor some intuitive set of strategies may be suggested (on the basis of some energetic or mathematical argument) as probably successful to recover it. The birth of the demagnetisation-like problem proceeds from this theoretical difficulty, necessarily involved in the definition of the pattern.

Moreover, it is now possible to give some hints regarding the notion of "unfavourable" remnant states. In short, such states are associated with strong irreversibility and weak symmetry of the family of iso-W loci. The first condition implies that large values of W are implemented. The second one implies that it is not necessary to start from a null field (stress) classical residual state: one may start the study of the demagnetisation-like process from the state IR (Fig. 3) close to R_1, the "historical field" (in the sense of CORB [13]).

3. A mechanical example suggesting the interest of a general study of the demagnetisation-like strategies

3.1. Towards a definition of the strategy

The main qualitative features of a strategy are, for example: the vicinity of the resulting demagnetised state to the idealised unique thermomechanical neutral state; the insensitivity or the weak dependence of this vicinity with respect to the loading; the question whether preliminary constitutive tests are required or not; the type of control. The main quantitative features are, basically, those associated with the accuracy of the recurrent recovery of a series of *unique thermomechanical neutral* states, lost as a result of successive loading paths which yield a series of successive associated residual states (Fig. 4, where the notations are those previously introduced in Fig. 2). If $\varepsilon_5 \ll 1$ denotes the vicinity parameter, the recovery of the idealised *unique thermomechanical neutral* state may be considered as obtained when

$$Q_\sigma/Q_0 < \varepsilon_5 \qquad \text{or} \qquad \min[W/(S_0^2/\mu)] < \varepsilon_5 . \tag{3.1}$$

If the insensitivity noticed above is warranted, a simple example is that of spiral-like type: the strategy is independent of preliminary constitutive tests, only stress-controlled, and the stress path is continuous up to any order. Then, the quantitative feature is the "pitch" of the spiral-like path; more precisely, the set of parameters involved in the definition of the family of paths. This case is

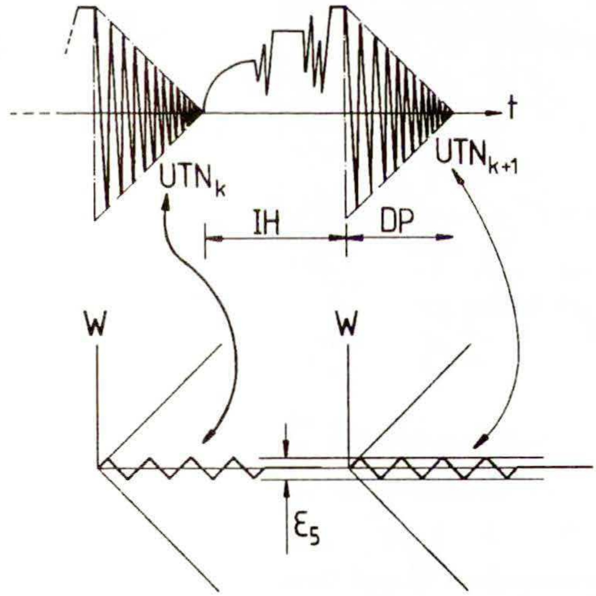

FIG. 4. Sketch suggesting the role of the fifth ingredient of the pattern.

studied below (§ 3.3). At this stage of the analysis one may limit oneself to a provisional definition which does not specify further the role of the variables of the state and of the constraints and costs: a strategy is then a family of stress paths allowing to obtain a series of accurate approximations of the idealised *unique thermomechanical neutral* state defined by the fifth ingredient of the pattern (Fig. 4). This vague definition will be commented upon later (§ 3.3, point viii).

3.2. Towards the choice of heuristic slowly decreasing cyclic stress paths

The aim of this paragraph is fourfold. Owing to the basic assumptions previously introduced (§ 1, point iii), the constitutive pattern may appear as rather theoretical and/or incomplete. It may be useful to counterbalance this (plausible) idea by showing that the model is nevertheless able to give results similar to those of the experiments. Secondly, it is useful to underline that intuitive strategies may prove to be immediately efficient as well as plainly false. Finally, it is useful to compare, even briefly, the piecewise continuous and the continuous slowly decreasing cyclic loading paths.

i. If one considers only simple classical mechanical paths (avoiding to implement neutral paths, for example), the properties of the pattern may appear as rather theoretical. However, if the first monotonous radial loading is followed by a constant pitch spiral first unloading path, then the properties of the pattern appear surprisingly unusual (Fig. 5 a, where Q denotes an invariant of the devia-

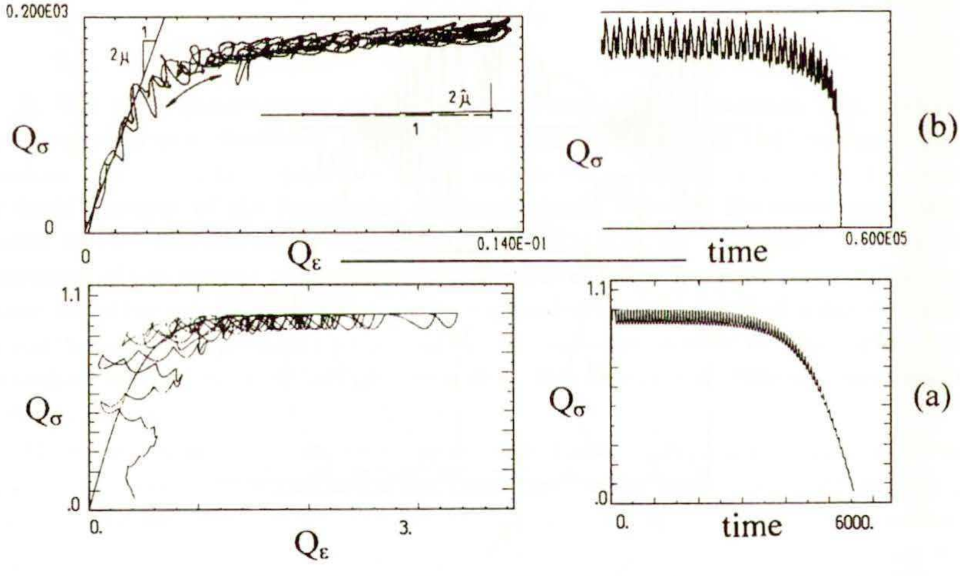

FIG. 5. Qualitative comparison between theoretical (a) and experimental (b) figures (stress-strain invariant diagrams are at left and stress second invariant – "time" diagrams are at right).

toric part of a tensor – cf. (2.11) for example) and the interesting point is that a similar figure is experimentally observed in the case of traction-torsion tests on thin tubes (Fig. 5 b, regarding the case of copper).

ii. It is easy to illustrate that intuitive simple strategies may yield rather encouraging results as well as grossly irrelevant ones. Typical examples of irrelevant strategies are obtained with the aid of unloading paths of one-dimensional type and of constant pitch spiral type: the trick consists of the neutral path which is included in the first loading process (Fig. 6, where the conventions of representation in the deviatoric planes of stress and strain are based on Cartesian axes XS and YS for the stress and XE and YE for the strain: these conventions allow to define the values of Q_σ/Q_0 and $Q_\varepsilon/S_0/\mu$).

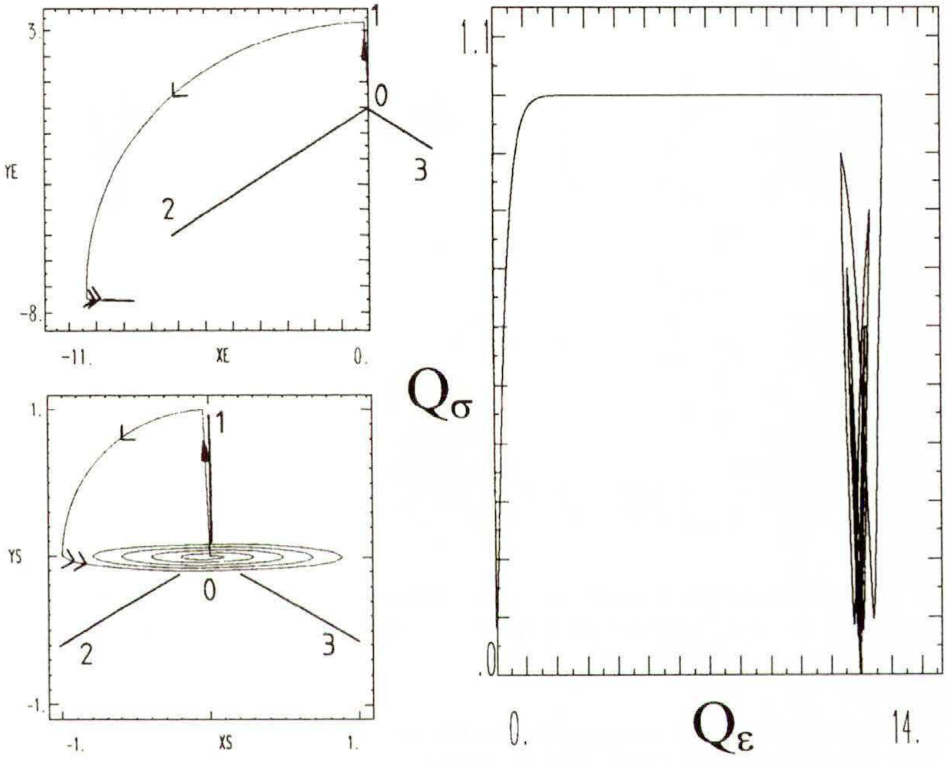

FIG. 6. An example of intuitive but unsuccessful continuous process of "demagnetisation".

iii. These examples are sufficient to suggest the interest of the simultaneous evolution of the intensity Q_σ and of the phase φ_σ of the stress deviator. Consequently, it is interesting to compare at least two types of S.D.C. stress paths, those which are piecewise radial and those which are continuous. The result may be summarised in mechanical form (Fig. 7, where the convention of the display is that of Fig. 6). The interest of sophisticated paths is not obvious. The simplest type (the spiral-like one) of continuous path being actually cyclic, it must be, from now on, studied further.

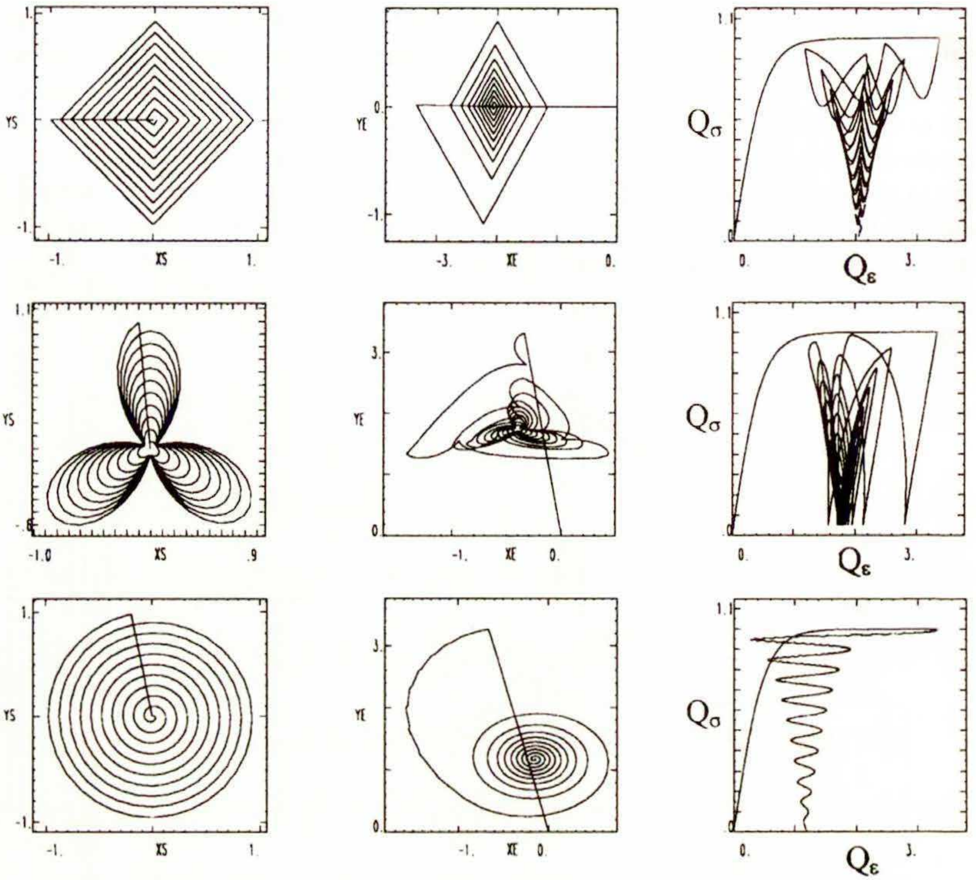

FIG. 7. Comparison between piecewise continuous and continuous processes: the 10 turns, constant pitch spiral-like case is relatively encouraging.

3.3. Towards the study of demagnetisation-like process in the special case of the spiral-like S.D.C. stress paths of slow-fast type

The numerical integrations are performed restricting to $n = 10$ the number of turns of the demagnetisation-like paths. Each turn of any spiral-like stress path is a 32-sides regular polyhedron, each side being described by 200 steps. From one point to the following one, the integration is obtained with the aid of a Runge–Kutta–Merson algorithm allowing an imposed accuracy provided that the number of subdivisions of a step remains smaller than 20. Owing to these numerical conditions, the integration can be performed "in a relevant way" even when the state of stress is "very near" the Huber - von Mises limit surface, that is to say such as the difference $1 - Q_\sigma/Q_0$ is as small as 10^{-7}.

i. The parameter of the unfavourable residual state is defined as follows. Firstly, a radial first loading up to Q_σ sufficiently close to Q_0 to warrant a dissipated energy to be very much larger than S_0^2/μ ($W = nw \cdot S_0^2/\mu$, $nw \gg 1$), is performed. The parameter nw may be used to compare the subsequent numerical simulation imposing similar initial state of strain. Secondly, the radial first loading is followed by the small unloading of the first step of the integration, setting the iso-W circles in the most unfavourable non-symmetric state (point IR of Fig. 3).

ii. The simple strategy under consideration is spiral-like and of the slow-fast type to be understood as follows: the imposed variation of the deviatoric radius Q is defined by following the course of the first loading in reverse. The rate of the phase is defined by introducing a scalar parameter $n\varphi$. The associated formulas are given below (point iii).

iii. As suggested by (2.11) and $(2.2)_1$, the relative stress intensity and the reverse rule are obtained, with $\omega = 1$, starting from

$$
\begin{aligned}
Q_\Delta\, dQ_\Delta &= 2\mu\left(1 - \left(\frac{Q}{Q_0}\right)^c\right)\overline{M}\, dt, \\
\overline{M}\, dt &= Q_\Delta Q_D\, dt = Q_\Delta\, dQ_\varepsilon\,, \\
\omega = 1, \qquad dW &= \frac{2\overline{M}}{\omega^2}\,.
\end{aligned}
\tag{3.2}
$$

Convenient values of c are 2/3, 1, 2. Owing to the reverse rule, the associated imposed S.D.C. stress paths are defined as follows. Let us introduce two parameters c_r and ξ, c_r playing the role of the constitutive parameter c and ξ being a "vicinity" parameter of the order of 10^{-7}. For $c_r = c_2$, and if HFR is for the historical field of Corb used as reference state, the associated relative stress x_R and the S.D.C. loading path definition are:

$$
\begin{aligned}
x_R &= \frac{Q_{\mathrm{HFR}}}{Q_0} = (1-\xi)\left(1 - \exp\left(\frac{-\mu n w c_r}{Q_0^2}\right)\right)^{1/c_r}, \\
Q_{\mathrm{spiral}}(\varphi_\sigma) &= Q_0 \tanh(k_s(2\pi n - \varphi_\sigma)), \\
2\pi n k_s &= \frac{1}{2}\ln\left(\frac{1+x_R}{1-x_R}\right) = \tanh^{-1}(x_R)
\end{aligned}
\tag{3.3}
$$

and for $c_r = c_1$ $(= 1)$, one obtains the same form regarding x_R, but the definition of Q_{spiral} is:

$$
Q_{\mathrm{spiral}}(\varphi_\sigma) = Q_0\left(1 - \exp\left[k_s(2\pi n - \varphi_\sigma)\right]\right), \qquad 2\pi n k_s = \ln(1 - x_R). \tag{3.4}
$$

In the case of $c_r = c_0$ $(= 2/3)$, only an approximate definition of the S.D.C. stress path is available. It is obtained by neglecting the terms which are not of the $\ln(1-Q/Q_0)$ form. For Q close to Q_0 this approximation results in the same form of the definition of x_R, because a factor $5-4(Q/Q_0)^{2/3}$ is close to 1. Hence:

$$x_R = (1-\xi)\left(1-\exp\left(\frac{-3nw\mu}{S_0^2}\right)^{3/2}\right).$$

A simple form may be used to define a function Q_{spiral} which is close to the exact solution of:

$$d\left(\frac{Q_\sigma}{Q_0}\right) \Big/ \left[1-\left(\frac{Q_\sigma}{Q_0}\right)^{2/3}\right] = \left(\frac{2\mu}{Q_0}\right) dQ_\varepsilon$$

when Q_σ is close to Q_0. The form:

$$\text{(3.5)} \qquad Q_{\text{spiral}}(\varphi) = Q_0\left(1-\exp\left((2\pi n-\varphi)k_s\right)^3\right), \qquad 2\pi n k_s = \ln(1-x_R^{1/3}),$$

is simple and similar to that of the previous "exponential" case defined by (3.4).

iv. The numerical integration is performed with the following set of parameters: $S_0 = 200\,\text{MPa}$, $\mu = 75\,\text{GPa}$, $c = c_i$ $(i = 0,1,2,\ c_i = 2/3,1,2$ for example). A typical result is given under the usual mechanical form (Fig. 8, where the convention of the display is that of Fig. 6) and also under thermodynamic form (Fig. 9, where the discontinuities of the sky-lines are pointed out by the arrows, and where the strain measure $Q_\varepsilon^{\text{rad}}$ – cf. [4], equations (44) to (49) – is obtained by integration of a rate – Φ/Q_σ – which is of radial type in the Ilyushin space). One notices the following points: first, the number of inversion points is much greater than in the cases sketched in Fig. 7; secondly, the continuity and convexity of the W and I sky-lines are made conspicuous; thirdly, the spiral-like response in the strain space is of almost constant pitch type; fourthly, the unfavourable remnant state is not actually highly unfavourable, for the initial remnant strain (of the order of $5 10^{-3}$) remains of the order of magnitude of $S_o/2\mu$.

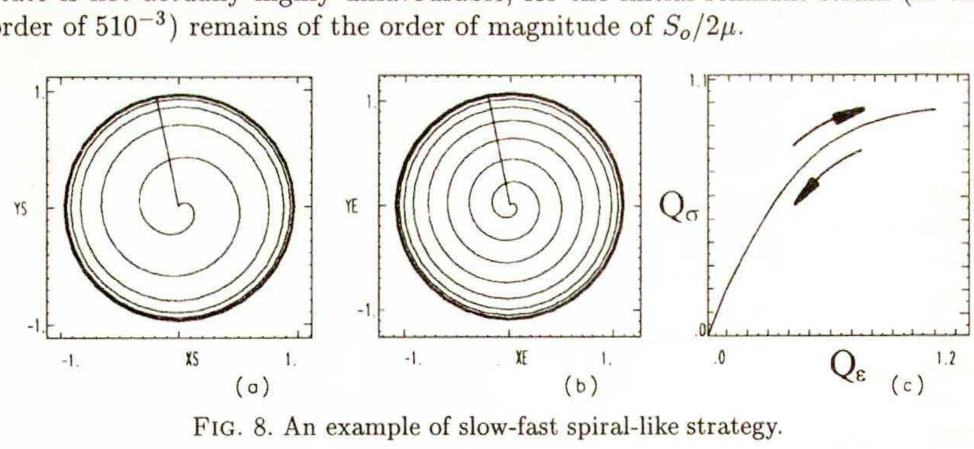

FIG. 8. An example of slow-fast spiral-like strategy.

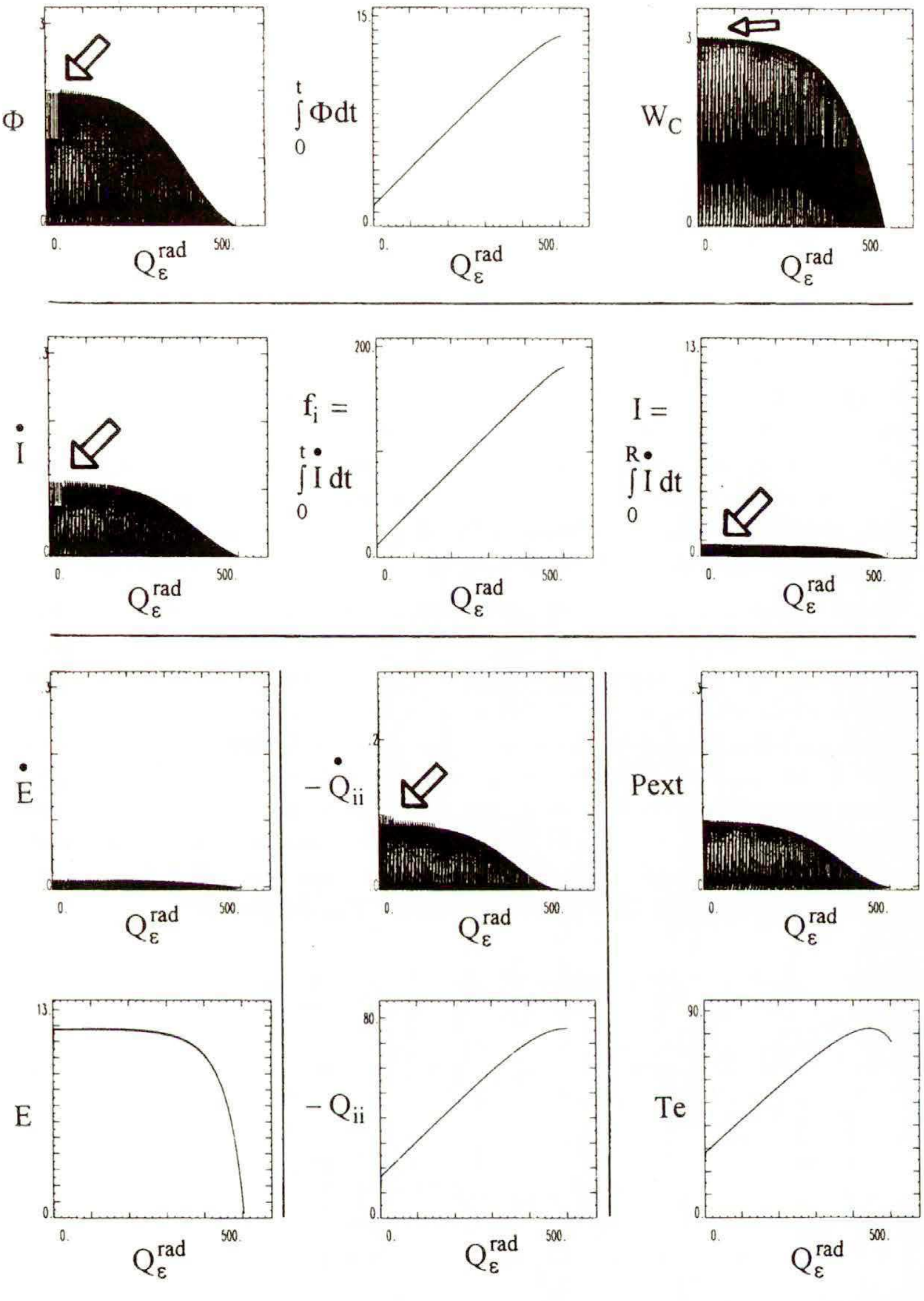

FIG. 9. Thermomechanics associated with the cyclic evolution introduced through Fig. 8.

v. A set of numerical simulations has been performed in order to study the role of increasingly large remnant strains with respect to the accuracy of the recovery of the U.T.N. state. The values of nw was: 8, 12, 16 for $c = c_2$; 12, 24, 32 for $c = c_1$; 24, 32, 48 for $c = c_0$. The last set is introduced (Fig. 10, where $Q\varepsilon/4$ is of the order of the elongation K in percent, and where it is possible to verify that the behaviour is similar to that of Fig. 5 a through the enlargement of the location pointed out by the symbol *). One notices the following points: firstly, the recovering of the remnant strain is more difficult to obtain for large c (convenient in the case of "hard" ferromagnetic materials) as well as for large residual strains; the strain response is coarsely similar to a constant pitch spiral when the demagnetisation-like process is not too bad, and the first turn is of importance for it is not well centred if the imposed stress state gets away too quickly from the limit Huber – von Mises circle; the W sky-line is strongly discontinuous when the recovery is only roughly performed; the lack of recovery is not associated with the breaking through the transition range; even when the relative recovery is very weak because the absolute recovery is constant, the final strain state remains on the first loading path obtained in the strain space.

vi. The purpose of the present paragraph is temporarily neglected in order to suggest without delay that the special results obtained above (as encouraging as they may be) do not necessarily warrant further success: the generalisation to the five (six)-dimensional case is still an open problem. Even if one relays on the previous analysis, it is indeed once more impossible to foresee the existence of the *unique thermomechanical neutral* state and the feature of a simple strategy: the origin of the difficulty is not associated with the appearance of a cumbersome formalism. Let σ_n $(n = 1, ..., 6)$ denote the components of the Cauchy stress tensor σ in the preferred reference frame. The associated Ilyushin deviatoric representation $(Q, \varphi^d, \theta_1, \theta_2, \theta_3)$ of the deviator $\overline{\sigma}$ is defined by:

$$Q^2 = q^2 + 2\sum_4^6 \sigma_n^2, \qquad q^2 = \sum_1^3 \overline{\sigma}_n^2,$$

$$\overline{\sigma}_1 = q\left(\frac{2}{3}\right)^{1/2} \cos\varphi^d, \qquad \overline{\sigma}_2 = q\left(\frac{2}{3}\right)^{1/2} \cos\left(\varphi^d - \frac{2\pi}{3}\right),$$

$$\overline{\sigma}_3 = q\left(\frac{2}{3}\right)^{1/2} \cos\left(\varphi^d + \frac{2\pi}{3}\right),$$

$$\sigma_{n+3} = \frac{Q}{\sqrt{2}}\cos\theta_n, \qquad n = 1, 2, 3.$$

Owing to the Huber – von Mises assumption, the yield surface is an hypersphere of the Ilyushin space and, by hypothesis, the neutral surfaces are also hyperspheres. This is the origin of the difficulty regarding the demagnetisation-like problem: the discontinuous feedback process between the "sliding of the neutral spheres"

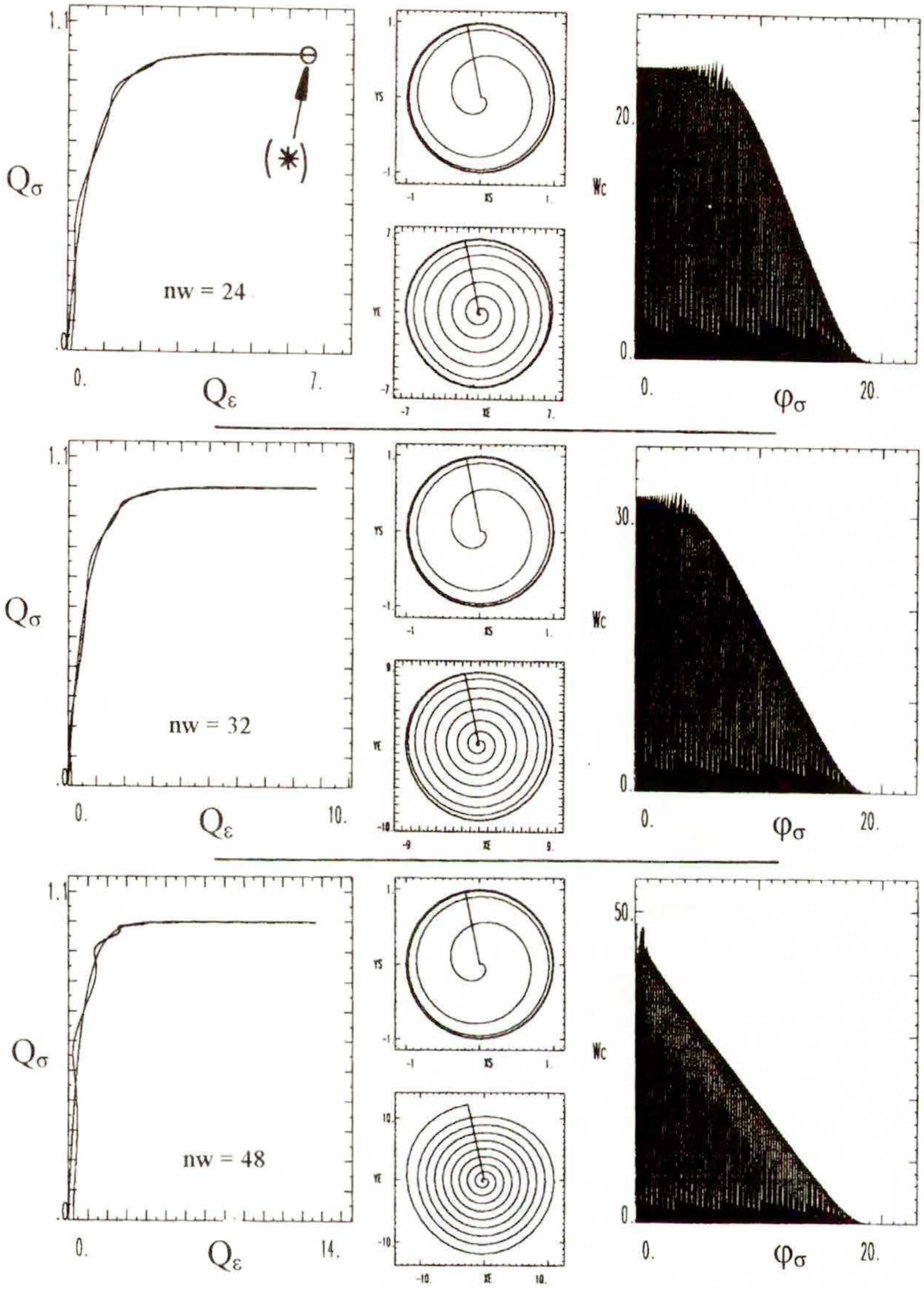

FIG. 10. Trying for demagnetisation of increasingly large residual strains ($c = c_0 = c_r$, $\xi = 10^{-7}$).

and the imposed cyclic loading may be, *a priori*, infinitely more sophisticated than in the simplest deviatoric situation studied above. In this simple situation, the ratio of the circumferencial path to the radial path was large in order to go along the yield circle "for a long time": the question is now to warrant that a four-dimensional path is able to go along "for a long 4-time" and in a relevant way. The point is more puzzling than the cumbersome character of the formal features, already briefly suggested elsewhere [3].

It is worth noting that one of the main interests of the present digression is also to point out the puzzling problem of the physical interpretation of the Cauchy stress tensor, a problem which is made conspicuous through the intrinsic dissipation form:

$$\overline{\Phi} = \Gamma \dot{p} + \left[(\overline{M} + \gamma_4 \dot{\varphi}^d)_{\mathrm{spiral}} + (\delta_{41}\dot{\theta}_1 + \delta_{42}\dot{\theta}_2 + \delta_{43}\dot{\theta}_3)_{\mathrm{cyclic}} \right] \tag{3.6}$$

of discrete memory type through the scalar functionals Γ, γ_4, δ_{4n} $(n = 1, 2, 3)$. The first term of the right-hand is related to the possible isotropic-deviatoric coupling effects, and the bracket suggests the distinction "$(Q, \varphi)_{\mathrm{spiral}}$" versus "$(\theta)_{\mathrm{cyclic}}$", and the special interest of shear tests and of approaches similar to that introduced by LODGE regarding shear flows [39]. Consequently, a strategy *a priori* interesting is that of (θ-cyclic)-(φ-spiral-like) wrapping type (Fig. 11 a, where the deviatoric plane of Ilyushin is equatorial and where the initial point 0 plays a role

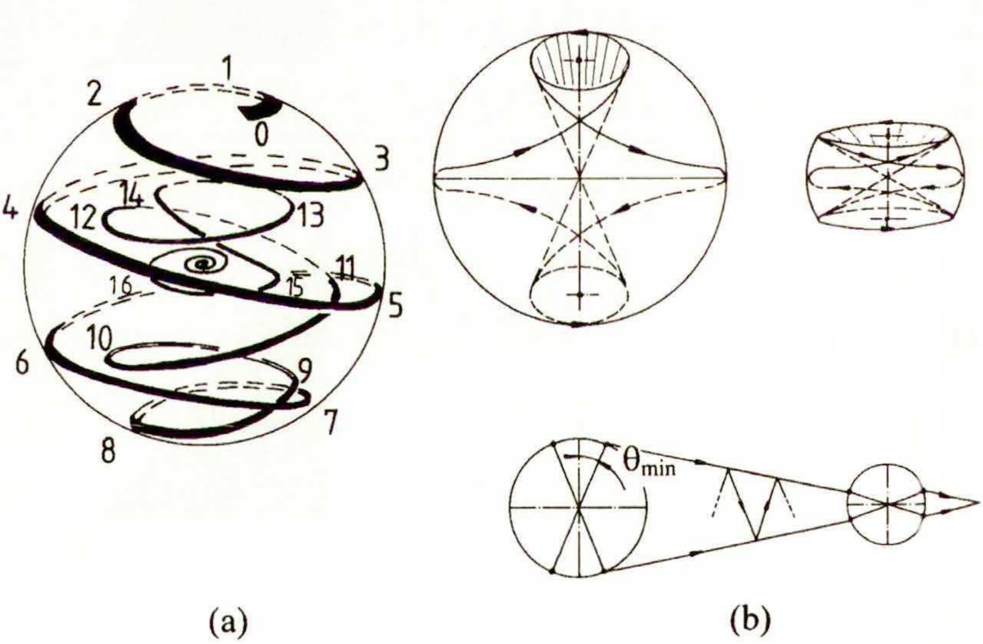

FIG. 11. Sketch of the multiaxial demagnetisation through the $(\theta_{\mathrm{cyclic}} - \varphi_{\mathrm{spiral}})$ wrapping.

similar to that of IR in Fig. 3). It can be generalized under the "3-wrapping" form (curves of the five-dimensional Ilyushin space yielded through three simultaneous wrappings of symmetric *slowly decreasing cyclic* type, as sketched in Fig. 11 b). This strategy is interesting as soon as the preferred reference frame problem is considered because a cyclic process must be imposed on the shear stress components through a symmetric *slowly decreasing cyclic* path starting from the interval $[\theta_{\min},\ \pi - \theta_{\min}]$ and leading to the $\pi/2$ value, so as to recover this frame [3].

vii. Let us now return to the problem of the Unique Thermomechanical Neutral state, underlining the relationship between the state of disorder and the state of dissipated energy.

The effective *evolution* $E(S_0, \mu, c)$ is to be understood as a series of couples of individual history (demagnetisation-like) processes through a relevant *slowly decreasing cyclic* paths. Each individual history is a sequence of two paths, the first loading and the effective loading path, and each demagnetisation leads to an approximate *unique thermomechanical neutral* state (Fig. 4). In the simple tensorial case studied above, the situation is not so clear as in the one-dimensional case. When σ tends towards zero, each successive disorder-order outburst tends also towards zero by definition of the pattern, and the external control does not introduce variations of disorder during the evolution from one individual history to the next one. In spite of the fact that no mathematical proof has been provided, the result obtained above leads to the conclusion that a necessary condition to recover a fading strain $\Delta^{\,t}_{R}\varepsilon$ is the continuity of the W sky-line during a demagnetisation-like process. However, the *unique thermomechanical neutral* state is no more defined through the simple scalar form of differences $W_k - W_{k+1}$: the evolution of the centres of the iso-W circles (Fig. 3) is now associated with the W sky-line continuity. For very large W never reached, the location of the centers is the axis. For the largest values actually reached and involved at the beginning of the demagnetisation-like process, the jumps of the centers are small and the short shaking leads quickly to the final positions of the centres: by contrast, the jumps of the centers are large and the required shaking lengthened for the small values of W leading also, but with "delay" (not viscous delay), to the final positions of the associated centres. The approximate *unique thermomechanical neutral* state is associated with the set of the final positions of the centres of iso-W circles. The geometry of this set is not studied. Even in the spiral-like simple case, the numerical investigation is cumbersome: to study, for example, a conjecture of helix-like geometry with envelope of fading radius, it must involve a very large number of inversion points.

viii. Among the results obtained through numerical evidences it is worth to note the following points: the approximate *unique thermomechanical neutral* state may be obtained through a spiral-like *slowly decreasing cyclic* loading path in the stress (field) Ilyushin space (reduced to a plane in the case under consideration) and the pattern is therefore deterministic in the broad sense of the term; during a *slowly decreasing cyclic* loading path of demagnetisation, the behaviour exhibits a mechanical feature which looks like that of the reversible type in spite of the fact that the process is entirely irreversible (cf. point v); the first loading behaviour and the feature of the demagnetisation-like process are associated (cf. point v) in a way which is a generalisation of the one-dimensional property, namely: the apexes of the symmetric cycles are on the first loading curve; the first loading appears as the generic form of the basic demagnetisation-like feature (cf. point ii) which is necessary in order to obtain the recurrent recovery of the line of the *evolution* $E(S_0, \mu, c)$: in short, ontogenesis recurs philogenesis; regarding the dissipated energy, the main part of an evolution consists of the demagnetisation-like processes. By contrast, regarding the total variation of internal energy, the main part of the evolution consists of the individual histories. Moreover, cyclic disorder-order outbursts are permanent.

4. Remarks on experimental evidence supporting the notion of idealised systems entirely irreversible although rejuvenating

4.1. Plastic hysteresis and the mesoscale problem

i. Both at the elementary microscale (dislocation pinning effect, Frank - Read source effect) and at the usual macroscale (push-pull, traction-torsion and shear tests at constant temperature and low "constant" strain rate) it is now clear that the behaviour of polycrystal-like materials is of pure hysteresis type when the rates of all "hardening" or "softening" effects simultaneously fade temporarily or finally [14, 15, 16]. It is also worth noting that the genuine nature of granular media behaviour is directly taken into account with the aid of the pattern of pure hysteresis extended to the case of isotropic and deviatoric coupling effects [6].

ii. However, the situation may appear as quite puzzling at the scale of the mesoscale substructures of the material point (walls, veins and microbands of various types, and labyrinths). This scale is that of the "natural intermediate level of milli-structure" introduced by KRUMHANSL [40]. For nearly 30 years it has been possible to observe such mesostructures, especially in the case of fatigue tests [17, 18, 19, 20]. The microanalysis has been generally performed in the null stress, residual strain state. Accordingly, given the current stage of results, to introduce a basic assumption of pure hysteresis type, namely the actual possibility of cyclic steady annihilation and creation of the mesostructures periodically converted one into another, is neither intuitive nor evident, in spite of the impressive number of available references [21]. Moreover, the consequences of a demagnetisation-like process have never been studied, involving possibly a basic gap in the experimental approach.

iii. The question of mesoscale structures may be provisionally revisited bringing together, from now on, the fields of plastic hysteresis and the field of ferrohysteresis. Such an attempt follows the analysis given by Friedel in his preface to the summer school at YRAVALS [22]: "Si ces dislocations sont retenues dans les cristaux, c'est par une friction solide qui les y bloque et qu'il faut surmonter pour les propager et les multiplier: sauf dans le cas extrême du fluage de Nabarro pur, la plasticité des cristaux a nécessairement ce caractère hystérétique violent qui la rapproche des phénomènes d'aimantation des corps ferromagntiques; tout indique qu'il en est de même de la plasticité à froid des amorphes."

4.2. Ferrohysteresis as the current genuine heuristic case

i. It is well known that, at macroscale, the behaviour of ferromagnets is compatible with the pure hysteresis pattern and that the accuracy and the repeatability of the demagnetisation-like process is of basic technological importance [23]: a similar situation exists regarding micro-devices implying high information densities [24].

ii. It is also possible to underline a rather relevant compatibility at the microscale level (cf. for example, [3, 25, 26, 24]). However, a comprehensive study is both out of the range of this paper and beyond the competence of the authors. The aim is "only" to underline an important experimental result, extremely difficult to obtain in the arena of mechanical tests: during a cyclic loading of fatigue type, field-controlled on the interval $[0, H]$, the microstructural pattern is cyclic, periodically restored [25, 26], as recalled by the sketch (Fig. 12, where the interpretation of the imperfect recovery attracts our attention to the analysis of RAYLEIGH [27]).

iii. However, the afore-mentioned restoration is not perfect. In spite of the "structural stability" of the sample (thin layer of a single crystal endowed with an initial lattice of bubbles), a "loss of memory" [25] is observed when many field cycles are applied. One notices also that no demagnetisation-like process is implemented in order to study the possible restoration of the initial lattice of bubbles. Moreover, the result is obtained by implementing a process which is basically one-dimensional and infinitely more simple than the processes which may be involved in three-dimensional mechanical samples submitted to multiaxial loading. One may also add that the coupled fields effects are not encompassed in the investigation [28, 29, 30]. Nevertheless, the special result under consideration may be remembered as sufficient in order to clarify the enigma of "restoration at all relevant scales" and associated appearance of reversibility yielded by an entirely irreversible system. Invariant material defect allows material discrete memory, Néel relationship between reversibility and irreversibility, and exchange of various forms of energy, including, at the ordinary point of a branch of cycle, that associated with disorder.

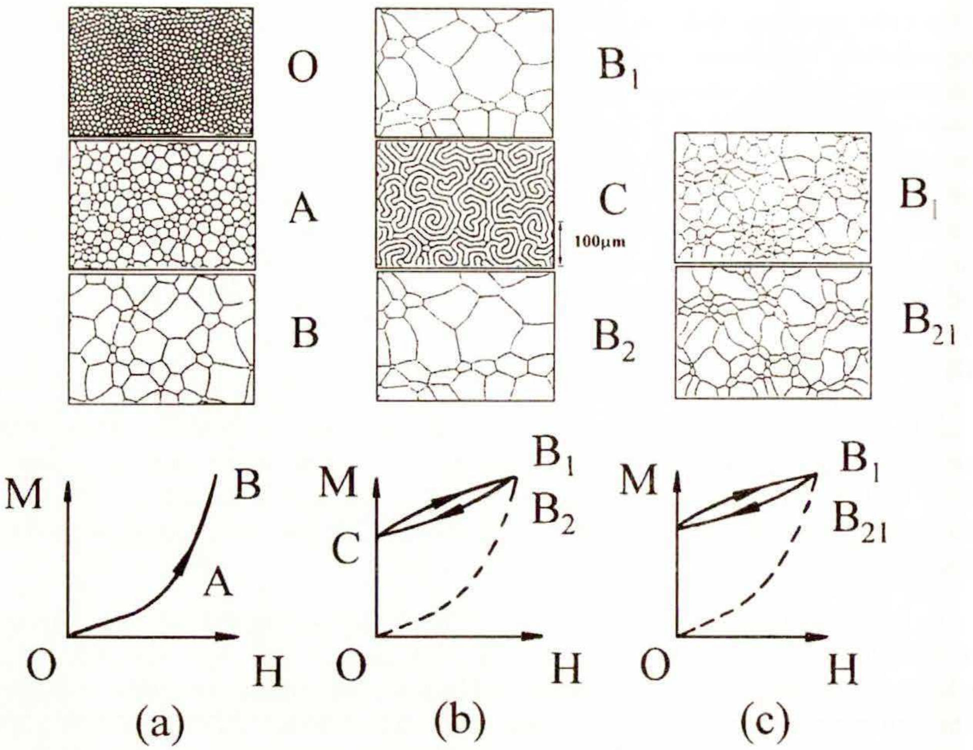

FIG. 12. Evolution of a bubble lattice (from Figs. 1, 2 and 3 of [26]): a) first loading; b) fatigue-like effective loading with almost recovered labyrinth and cellular patterns associated with null field and maximum field, respectively; c) however, the cellular pattern is not perfectly recoverable whatever may be the number of cycles.

5. Concluding remarks

i. One encouraging result has been obtained (§ 3.3 point v), suggesting what may be, perhaps, the simplest strategy available to tackle the demagnetisation-like problem. However, neither the isotropic six-dimensional "general" case, nor the role of the preferred reference frame, nor even the problem of the anisotropic cases have been studied comprehensively enough. Neither were they exemplified through any special results: the general pattern of pure hysteresis remains insufficiently investigated regarding these basic theoretical features. Accordingly, the study of the features which are of outstanding interest for engineers, such as hardening, "viscous" effects [32] or coupled fields effects, is still rather untimely. But it is worth noting that the implementation of the current pure hysteresis pattern in finite element approaches is from now on, relevant in order to prepare for the numerical simulation of the six-dimensional pattern and of the general kinematics in the coming years [33, 34, 35].

ii. In its current state, the pattern of pure hysteresis is, from now on, heuristic when some particular insight is needed at the level of fundamental microscopic processes involving the notion of dislocation [3, 14, 15, 16]. Accordingly, if a general phenomenological theory may be introduced under the five (or six)-dimensional form (§ 3.3, point vi), most likely it will not be entirely incomplete in the sense of BUNGE [31], especially regarding the problem of the (basically three-dimensional) second order effects, of the generalised ratchet type.

iii. The present provisional study of demagnetisation-like mechanical processes may be transposed to the case of ferrohysteresis (cf. Footnote 2). It is worth noting that the discrete memory pattern is then useful in order to avoid, from time to time, the drawbacks initiated through the implementation of basic classical concepts. For example, the "anomalous" effect introduced in [36] may be immediately recognised as normal in the frame of the discrete memory pattern: the increase of magnetisation along the path going from H_1 to H_2 (Fig. 1 of [36] is indeed a standard hysteresis behaviour [3].

Appendix. Two basic rates – those of internal energy and of internal intrinsic heat supply-involved by the symbolic model

i. If $g''(e)\,de$ denotes the rigidity coefficient for the couples having their limit strain values between e and $e+de$, and if ε is the current external strain imposed during the first monotonic loading, then the resulting stress for the couples whose limit value is not reached is:

$$\sigma_1 = \int_{\varepsilon}^{\infty} \varepsilon g''(e)\,de.$$

For the other couples for which the friction slider move, the resulting stress is:

$$\sigma_2 = \int_{\varepsilon}^{\infty} e g''(e)\,de.$$

The total stress is:

$$\sigma = G_0\varepsilon - g(\varepsilon) = S(\varepsilon), \qquad G_0 = \int_{0}^{\infty} g''(e)\,de = g'(\infty), \qquad g'' = -\sigma'' \geq 0.$$

The internal power is then:

$$P_i = -\sigma_1\dot{\varepsilon} - \sigma_2\dot{\varepsilon} = -\sigma\dot{\varepsilon} = -(G_0\varepsilon - g(\varepsilon))\dot{\varepsilon} = -S(\varepsilon)\dot{\varepsilon}\,.$$

The mechanical behaviour is described in terms of the function $g(e)$ such as:

$$g(0) = 0, \qquad g'(0) = g'(\infty) - G_0 = 0, \qquad g''(0) = g''(\infty) = 0.$$

The curvature rule is: g'' is not negative on R^+, the interval of definition of g.

ii. By definition of s_1 and s_2, the basic rates are:

$$\dot{E} = \varepsilon\dot{\sigma}(\varepsilon) = \sigma\dot{\varepsilon} - (\sigma\dot{\varepsilon} - \varepsilon\dot{\sigma}) = -P_i + \dot{Q}_{ii},$$
$$-\dot{Q}_{ii}(\varepsilon) \equiv \sigma_2\dot{\varepsilon} = \sigma(\varepsilon)\dot{\varepsilon} - \varepsilon\dot{\sigma}(\varepsilon)$$

in the first loading case and, in the cyclic case:

$$\Delta\, {}^t_R\sigma = \int\limits_{\Delta\varepsilon/2}^{\infty} \Delta\varepsilon g''(e)\, de + \int\limits_{0}^{\Delta\varepsilon/2} 2eg''(e)\, de$$
$$= 2\left[G_0\frac{\Delta\varepsilon}{2} - g\left(\frac{\Delta\varepsilon}{2}\right)\right] = 2S\left(\frac{\Delta\varepsilon}{2}\right),$$
$$-2\dot{Q}_{ii}(\Delta\, {}^t_R\varepsilon) = \Delta\, {}^t_R\sigma\frac{\partial}{\partial t}\Delta\, {}^t_R\varepsilon - \Delta\, {}^t_R\varepsilon\frac{\partial}{\partial t}\Delta\, {}^t_R\sigma,$$
$$2\dot{E}(\Delta\, {}^t_R\varepsilon) = (\sigma + {}^t_R\sigma)\frac{\partial}{\partial t}\Delta\, {}^t_R\varepsilon + \Delta\, {}^t_R\varepsilon\frac{\partial}{\partial t}\Delta\, {}^t_R\sigma.$$

The generic description of the cyclic properties is then $(2.2)_2$.

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