

Gas filtration through porous coal medium Effect of the gas constrained in micropores

D. LYDZBA (WROCLAW) and J.L. AURIAULT (GRENOBLE)

GAS FILTRATION through the macropores in porous coal media, with diffusion of a gas constrained in micropores, is investigated by using the homogenization process for periodic structures. This technique leads to the macroscopic model of the considered phenomenon by starting from the description at the pore level. No prerequisite is imposed at the macroscopic scale. Three different macroscopic models are obtained. Their ranges of validity are defined by appropriate dimensionless numbers that describe the geometrical structure and the physico-chemical properties of the coal. In two of these models, the micropore diffusion is coupled to the filtration process by a source term in the macroscopic mass balance. Finally, we investigate a one-dimensional flow through a semi-infinite coal seam, when the coal is assumed to be composed of grains. This simple example demonstrates the strong influence of the characteristic sizes of the grains and of the macroscopic sample on the filtration process.

1. Introduction

ONE OF THE GREATEST DANGERS occurring in some underground coal mines are gas-coal outbursts. During this violent process, gas moving with a high velocity and crushed coal mixture endangers the health and lives of the miners. To reduce the hazard connected with such disastrous explosions, it is necessary to know their causes.

The mechanism of a coal outburst is investigated in several papers [e.g. 1–4]. Many factors are shown to be responsible for its occurrence. Large pressures, the kind of gas, the exploitation stresses, the physico-chemical and physico-mechanical properties of coal and internal structure of the coal porous medium play here the most important role. Many factors lead to the numerous formulae for an outburst danger. For instance, the influence of the geometrical structure on the outburst peril is represented by the following empirical relation [5]:

$$(1.1) \quad G = \frac{P_1 + 10P_2 + 100P_3}{P_1 + P_2 + P_3},$$

where P_1 is the mass of the grain fraction of a diameter greater than 4 mm, P_2 is the mass of the grains of a diameter within the range 0.5–4 mm, and P_3 is the mass of grains of a diameter smaller than 0.5 mm. All these values are obtained from the grain size distribution of a coal specimen that was primarily crushed according to definite prescription. When $G > 13$, the presence of an outburst danger is assumed.

However, a quantitative model describing such an instantaneous phenomenon is not available. We limit ourselves to the investigation of the early stage, before the explosion.

One of the most important factors is the gas seepage through the porous coal structure, representing a triple porosity system, with three different pore scales [6]:

- The scale of network sorption is characterized by capillaries with the pore radii up to 0.3–0.5 nm, in which the absorption process resembles the phenomenon of dissolution.
- The scale of micropores comprises capillaries with the radii up to 1.2–1.5 nm.
- The scale of macropores comprises pores with greater radii, where single- and multilayer adsorption takes place and where free gas is present.

Only a small part of the gas is in a free state. The main part of the gas is constrained at the two smaller scales, i.e., the scale of micropores and the scale of network sorption. Depending on the magnitude of its pressure, the free gas in the macropores may be or may not be in a thermodynamic equilibrium with the constrained gas. When the equilibrium is disturbed, the constrained gas acts on the gas filtration in the macropores by its emission through the internal surface of the coal. The intensity of gas emission through the internal surface directly depends on the geometrical structure and the physico-chemical properties of the skeleton [7]. Therefore it often results in a strong coupling between the gas filtration intensity and the parameters mentioned above.

The aim of this paper is to show the influence of the geometrical structure and the physico-chemical properties of the skeleton on the gas filtration process. The description of such complicated systems as porous media, with strong heterogeneities of high density, is practically possible at the macroscopic level only, where an equivalent continuous medium is defined. This can be obtained in the following two ways. The first way is the phenomenological approach. It was used in [3] to investigate the behaviour of the gas-coal system. The second way includes all the different averaging (homogenization) processes for investigating the passage from the local to the macroscopic level. The main characteristics of these processes can be found in [8].

Here we use the multiple scale asymptotic method. This technique has been already used in several papers to model porous materials. Some of them concern multiple porosity media. Deformable double porosity media saturated by an incompressible fluid are investigated in [9], by starting from the Navier–Stokes equations in the micropores and in the macropores. The analysis is extended to compressible fluids in [10]. In [11], the authors assume a rigid skeleton and a compressible fluid, with Darcy's law satisfied in the micropores and in the macropores. The analysis presented here is an extension of these works to the study of a porous coal medium.

In the Sec. 2, after introducing the local description of the gas-coal system, we briefly present the homogenization process. The flow in the macropores is de-

scribed by the Navier–Stokes equations for compressible fluids. Because of the small radii of micropore capillaries, we assume that the mass transport of the gas constrained in the micropores is a molecular diffusion process. For simplicity, the porous matrix is considered to be rigid. Since random and periodic microstructures lead to the same macroscopic description, [14], we assume a periodic porous matrix. Then, the homogenization process is applied to our problem and different macroscopic equivalent descriptions are obtained. The main result consists in the fact that the macroscopic gas filtration can be modelled by three different kinds of macroscopic descriptions. Their respective ranges of validity are defined by the values of appropriate dimensionless numbers. The reader who is not familiar with the mathematical approach used in the Sec. 2, can directly go over to the Sec. 3, where the results are summarized.

The quantitative influence of the gas constrained in the microporous part is illustrated in Sec. 4 of the paper. For this purpose, a one-dimensional flow through a semi-infinite coal seam is investigated, when the geometry of the internal structure of coal is assumed to be composed of spherical grains. In particular, we investigate the distribution of the gas pressure and its gradient near the long-wall head, depending on the grain radius. Determination of the small parameter of scale separation in each point of the seam enables us to show the domains of validity of the three descriptions.

2. The homogenization process

Let us introduce the physics at the different capillary and pore scales. We assume that these scales are well separated from the macroscopic scale. The local physics and the separation of scales represent the basic assumptions that lead to the macroscopic descriptions. The method of multiple scale developments does not introduce any prerequisite concerning the macroscopic scale.

2.1. Local description

Let us simplify the coal system to a single porosity medium composed of a solid part V_s and pores V_p . The solid part V_s comprises the porous matrix of coal and the capillaries of the two smaller scales. Pores V_p are the macropores introduced in Sec. 1. We assume that:

- a. Flow of the gas in the macropores (in V_p) is described by the Navier–Stokes equations of a barotropic liquid.
- b. Motion of the constrained gas (in V_s) obeys the Fick molecular diffusion law.
- c. The solid is undeformable.

With these assumptions, the local description (at the pore level) is given by:

- the Navier–Stokes equation:

$$(2.1) \quad \Delta \mathbf{v} + (\lambda + \mu) \mathbf{grad} (\operatorname{div} \mathbf{v}) - \mathbf{grad} p = \varrho \frac{\partial \mathbf{v}}{\partial t} + \varrho (\mathbf{v} \mathbf{grad}) \mathbf{v} \quad \text{in } V_p,$$

- the equation of mass conservation for free gas:

$$(2.2) \quad \frac{\partial \varrho}{\partial t} + \operatorname{div} \varrho \mathbf{v} = 0 \quad \text{in } V_p,$$

- the ideal gas law for isothermic processes:

$$(2.3) \quad \varrho = \frac{\varrho_a}{p_a} p \quad \text{in } V_p,$$

- the equation of mass conservation for molecular diffusion:

$$(2.4) \quad \frac{\partial C}{\partial t} - \operatorname{div} (\mathbf{D} \mathbf{grad} C) = 0 \quad \text{in } V_s.$$

Here \mathbf{v} is the velocity vector of the free gas in the macropores, p is the gas pressure, ϱ is the gas density, C is the overall concentration of constrained gas in the solid, \mathbf{D} is the effective micropore diffusion coefficient, p_a is the atmospheric pressure, ϱ_a is the gas density at atmospheric pressure, and μ and λ are the gas viscosities.

The set (2.1)–(2.4) is completed with the boundary conditions on the interface Γ between the solid and the macropores, i.e. continuity of the mass flux:

$$(2.5) \quad (\varrho \mathbf{v} + \mathbf{D} \mathbf{grad} C) \mathbf{n} = 0$$

and continuity of the gas pressure. Due to relation (2.3), it is reduced to the condition of continuity of the density. The overall gas concentration C in the solid part can be equated to the overall gas density $\phi_s \varrho$. Therefore, the condition of continuity of the gas pressure on Γ is written in the form

$$(2.6) \quad C = \phi_s \varrho.$$

The adhesion condition:

$$(2.7) \quad \mathbf{v} \eta = 0.$$

Here \mathbf{n} and η are unit vectors, normal and tangent to the common surface Γ , respectively. ϕ_s is the volume occupied by the gas constrained in the unit volume of the solid. In addition, we assume the thermodynamic equilibrium between the phases at the initial instant.

In many practical cases the bulk volume of the considered porous medium is very large compared to the size of the heterogeneities. Therefore a very good separation of scales exists which enables us to determine the equivalent continuous macroscopic description.

2.2. Homogenization principle

The separation of scales implies the existence of an elementary representative volume (ERV). In the very particular case of a periodic medium, the spatial period represents the ERV. If l is a characteristic length of the ERV and if L is a characteristic length of the sample of coal or of the phenomenon under consideration, we have

$$\varepsilon = \frac{l}{L} \ll 1.$$

If the order of magnitude of l is known for a given material, L is determined by the solution of the macroscopic boundary value problem (see Sec.4). Therefore the value of ε is known *a posteriori* only. It is generally assumed that $\varepsilon = 0.1$ is the limit for the separation of the scales to exist.

When the medium is random, the separation of scales implies a local asymptotic invariance. The volume averages of physical quantities in the ERV remain constant under a translation $O(l)$. When the medium is periodic, it results in the local periodicity of the physical quantities. However, independently of whether the medium is random or periodic, the structure of the macroscopic equivalent description remains unchanged [14]. Therefore it will be assumed that the medium is periodic, since in this case the process is much more powerful. Nevertheless, it must be mentioned that the determination of effective coefficients needs *a priori* different approaches for the two kinds of media considered. A periodic medium is shown in Fig. 1. Ω is the unit cell, Ω_s is the solid part of Ω , Ω_p is the porous part of Ω and Γ is the interface. The geometry of the pores inside the unit cell can be chosen arbitrarily. Variation of the geometry does not modify the structure of the macroscopic description, but only the effective coefficients appearing in it.

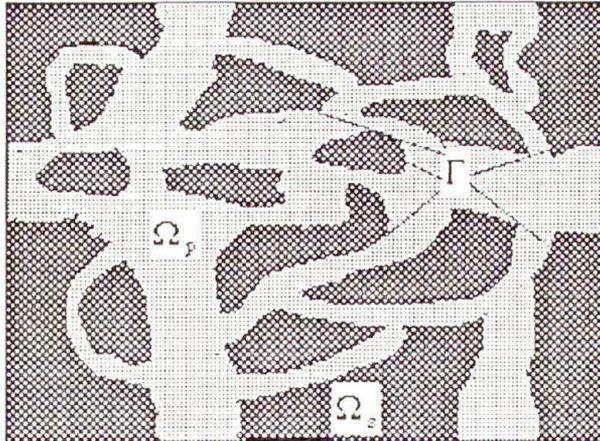


FIG. 1. Schematic view of the medium at the microscopic level: unit cell (2D case).

Two characteristic lengths l and L introduce two dimensionless space variables \mathbf{x} , \mathbf{y} and each physical quantity F is a function of these two variables and time t .

$$\mathbf{x} = \frac{\mathbf{X}}{L}, \quad \mathbf{y} = \frac{\mathbf{X}}{l}, \quad F = F(\mathbf{x}, \mathbf{y}, t).$$

Variable \mathbf{x} is the macroscopic space variable well suited to describe the macroscopic variations, while \mathbf{y} is the macroscopic space variable well suited for the local description.

The existence of two dimensionless space variables has to be taken into account in the expressions of the differential operators. Two equivalent descriptions are then possible. The first description corresponds to the microscopic point of view. We get:

$$(2.8) \quad \begin{aligned} \mathbf{grad} &= \frac{1}{l}(\varepsilon \mathbf{grad}_x + \mathbf{grad}_y), \\ \Delta &= \frac{1}{l^2} \left(\varepsilon^2 \Delta_x + 2\varepsilon \frac{\partial}{\partial x_j} \left(\frac{\partial}{\partial y_j} \right) + \Delta_y \right), \\ \mathbf{div} &= \frac{1}{l}(\varepsilon \mathbf{div}_x + \mathbf{div}_y). \end{aligned}$$

The second description corresponds to the macroscopic point of view:

$$(2.9) \quad \begin{aligned} \mathbf{grad} &= \frac{1}{L}(\mathbf{grad}_x + \varepsilon^{-1} \mathbf{grad}_y), \\ \Delta &= \frac{1}{L^2} \left(\Delta_x + 2\varepsilon^{-1} \frac{\partial}{\partial x_j} \left(\frac{\partial}{\partial y_j} \right) + \varepsilon^{-2} \Delta_y \right), \\ \mathbf{div} &= \frac{1}{L}(\mathbf{div}_x + \varepsilon^{-1} \mathbf{div}_y). \end{aligned}$$

Subscripts x and y denote partial derivatives with respect to x and y , respectively. By taking advantage of the small parameter ε , all the physical quantities are sought for in the form of asymptotic expansions

$$(2.10) \quad F(\mathbf{x}, \mathbf{y}, t) = F^{(0)}(\mathbf{x}, \mathbf{y}, t) + \varepsilon F^{(1)}(\mathbf{x}, \mathbf{y}, t) + \varepsilon^2 F^{(2)}(\mathbf{x}, \mathbf{y}, t) + \dots,$$

where $F^{(i)}$ is Ω -periodic in y .

The method consists in incorporating such expansions into the set of equations that describes the phenomenon at the local scale, and in identifying terms with the same powers of ε . Before that, it is necessary to normalize all equations of the local descriptions. This means that the local description is made dimensionless and the dimensionless numbers are evaluated according to the powers of ε . A quantity q is said to be $O(\varepsilon^p)$ if $\varepsilon^{p+1} \ll q \ll \varepsilon^{p-1}$.

The result of the homogenization process is a set of equations satisfied by the first terms of the asymptotic expansions, that represents the macroscopic description, within an approximation of the order of ε .

2.3. Estimations

Equations (2.1), (2.2), (2.4) and (2.5) introduce the following dimensionless numbers:

$$(2.11) \quad \begin{aligned} Q &= \frac{|\mathbf{grad} p|}{|\mu \Delta \mathbf{v}|}, & H &= \frac{|(\lambda + \mu) \mathbf{grad}(\operatorname{div} \mathbf{v})|}{|\mu \Delta \mathbf{v}|}, \\ R_{tl} &= \frac{\left| \frac{\partial \mathbf{v}}{\partial t} \right|}{|\mu \Delta \mathbf{v}|}, & R_e &= \frac{|\varrho(\mathbf{v} \mathbf{grad}) \mathbf{v}|}{|\mu \Delta \mathbf{v}|}, \\ S_t &= \frac{\left| \frac{\partial \varrho}{\partial t} \right|}{\operatorname{div} \varrho \mathbf{v}}, & M_t &= \frac{\left| \frac{\partial C}{\partial t} \right|}{|\operatorname{div}(\mathbf{D} \mathbf{grad} C)|}, \\ P_e &= \frac{|\varrho \mathbf{v}|}{|\mathbf{D} \mathbf{grad} C|}. \end{aligned}$$

Let us use the microscopic point of view. Therefore l is the characteristic length for estimating the dimensionless numbers (2.11). Using the characteristic values v_c , p_c , ϱ_c , C_c , t_c of the velocity, pressure, density, concentration and time, respectively, the dimensionless numbers (2.11) can be expressed by

$$(2.12) \quad \begin{aligned} Q_l &= \frac{p_c l}{\mu v_c}, & H_l &= \frac{\lambda + \mu}{\mu}, \\ R_{tl} &= \frac{\varrho_c l^2}{\mu t_c}, & R_{el} &= \frac{\varrho_c v_c l}{\mu}, \\ S_{tl} &= \frac{l}{t_c v_c}, & M_{tl} &= \frac{l^2}{D t_c}, \\ P_{el} &= \frac{\varrho_c v_c l}{D C_c}. \end{aligned}$$

We limit our study to the case when the gas flow in macropores is slow and quasi-permanent. It means that the Reynolds numbers R_{el} and R_{tl} are assumed to be small, i.e.,

$$R_{el} \ll O(\varepsilon) \quad \text{and} \quad R_{tl} \ll O(\varepsilon).$$

We assume that the gas viscosities λ and μ are of the same order of magnitude (with respect to ε). The dimensionless number H_l becomes

$$H_l = O(1).$$

The number Q_l can be estimated by physical considerations [15]. The gas flow is forced by a macroscopic gradient of pressure. Therefore,

$$|\mathbf{grad} p| = O\left(\frac{p_c}{L}\right).$$

Since the gas is flowing through pores of size l , the characteristic length in evaluating the viscous term is l :

$$|\mu \Delta \mathbf{v}| = O\left(\frac{\mu v_c}{l^2}\right).$$

For slow and permanent flows, the pressure term in Eq.(2.1) is equilibrated by the viscous term. It follows that

$$\frac{\mu v_c}{l^2} = O\left(\frac{p_c}{L}\right),$$

and the dimensionless number Q_l becomes

$$Q_l = O\left(\frac{p_c l}{\mu v_c}\right) = O(\varepsilon^{-1}).$$

Estimates of the dimensionless numbers S_{tl} and M_{tl} are obtained from the conditions for the homogenization to be possible. As it was shown in [16], number S_{tl} should fulfill the following inequality:

$$(2.13) \quad S_{tl} \leq O(\varepsilon).$$

In the same way it is easy to obtain a similar restriction on M_{tl} :

$$(2.14) \quad M_{tl} \leq O(1).$$

Now, by taking into account the definitions (2.12) of P_{el} , S_{tl} and M_{tl} , the following relation can be written:

$$P_{el} = \frac{M_{tl}}{S_{tl}} \frac{\rho_c}{C_c}.$$

Assuming that ρ_c and C_c are of the same order of magnitude, and assuming for the moment that

$$M_{tl} = O(\varepsilon^m) \quad \text{and} \quad S_{tl} = O(\varepsilon^s),$$

the following estimation of P_{el} is obtained:

$$P_{el} = O(\varepsilon^{m-s}),$$

where m and s are non-negative integers.

It is well known that the filtration coefficient is very much larger than the coefficient of the micropore diffusion, and that the main flux of the gas flow through the porous medium is due to the filtration process. Therefore we confine our study to the case

$$P_{el} \geq O(1).$$

This restriction, together with the above estimation of P_{el} , leads to the inequality for m and s :

$$m - s \leq 0.$$

In the following, numbers m and s will be used to distinguish between different types of the considered phenomenon.

On the other hand, it is interesting to introduce two characteristic times T_D and T_S of the fluid diffusion and fluid seepage, respectively:

$$T_D = \frac{l^2}{D}, \quad T_S = \frac{L}{v_c}.$$

Their ratio A can be put in the form

$$(2.16) \quad A = \frac{T_S}{T_D} = \varepsilon^{-1} \frac{D}{lv_c} = \varepsilon^{-1} \frac{S_{tl}}{M_{tl}} = \varepsilon^{s-m-1}.$$

Finally, by defining the dimensionless variables

$$(2.17) \quad v^* = \frac{v}{v_c}, \quad p^* = \frac{p}{p_c}, \quad \varrho^* = \frac{\varrho}{\varrho_c}, \quad C^* = \frac{C}{C_c},$$

and by taking into account the above estimates of the dimensionless numbers and the relations (2.8), we obtain the following dimensionless form of the local description:

$$(2.18) \quad \left(\varepsilon^2 \Delta_x + 2\varepsilon \frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial y_i} \right) + \Delta_y \right) \mathbf{v}^* + (\varepsilon \mathbf{grad}_x + \mathbf{grad}_y)(\varepsilon \operatorname{div}_x + \operatorname{div}_y) \mathbf{v}^* - (\mathbf{grad}_x + \varepsilon^{-1} \mathbf{grad}_y) p^* = \varepsilon \varrho^* \frac{\partial \mathbf{v}^*}{\partial t^*} + \varepsilon \varrho^* \mathbf{v}^* ((\varepsilon \mathbf{grad}_x + \mathbf{grad}_y) \mathbf{v}^*),$$

$$(2.19) \quad \varepsilon^s \frac{\partial \varrho^*}{\partial t^*} + (\varepsilon \operatorname{div}_x + \operatorname{div}_y)(\varrho^* \mathbf{v}^*) = 0,$$

$$(2.20) \quad \varrho^* = \frac{p_c}{\varrho_c} \frac{\varrho_a}{p_a} p^* \quad \text{in } \Omega_p,$$

$$(2.21) \quad \varepsilon^m \frac{\partial C^*}{\partial t^*} - (\varepsilon \operatorname{div}_x + \operatorname{div}_y) \mathbf{D}(\varepsilon \mathbf{grad}_x + \mathbf{grad}_y) C^* = 0 \quad \text{in } \Omega_s,$$

$$(2.22) \quad (\varepsilon^{m-s} \varrho^* \mathbf{v}^* + \mathbf{D}(\varepsilon \mathbf{grad}_x + \mathbf{grad}_y) C^*) \mathbf{n} = 0,$$

$$(2.23) \quad C^* = \frac{\varrho_c}{C_c} \phi_s \varrho^*,$$

$$(2.24) \quad \mathbf{v}^* \eta = 0 \quad \text{on } \Gamma.$$

At the initial instant of time, the thermodynamical equilibrium requires that

$$C^* = \frac{\varrho_c}{C_c} \phi_s \varrho^* \quad \text{everywhere.}$$

2.4. Macroscopic description

We introduce into the normalized set (2.18)–(2.24) asymptotic expansions (2.10) for v^* , p^* , ϱ^* and C^* . Grouping the terms with the same powers of ε , we get sets of equations to be satisfied by the consecutive terms of the asymptotic expansions. For the sake of simplicity, the asterisk marking the dimensionless variables is omitted in the following considerations.

From Eqs. (2.18), (2.20), (2.23) and (2.24) we obtain:

$$(2.25) \quad \begin{aligned} \mathbf{grad}_y p^{(0)} &= 0, \\ \Delta_y \mathbf{v}^{(0)} + \mathbf{grad}_y (\mathbf{div}_y) \mathbf{v}^{(0)} - \mathbf{grad}_x p^{(0)} - \mathbf{grad}_y p^{(1)} &= 0, \end{aligned}$$

$$(2.26) \quad \varrho^{(0)} = \frac{p_c}{\varrho_c} \frac{\varrho_a}{p_a} p^{(0)} \quad \text{in } \Omega_p,$$

$$(2.27) \quad C^{(0)} = \frac{\varrho_c}{C_c} \phi_s \varrho^{(0)},$$

$$(2.28) \quad \mathbf{v}^{(0)} \eta = 0, \quad \mathbf{v}^{(1)} \eta = 0 \quad \text{on } \Gamma.$$

Equations (2.19), (2.21) and (2.22) directly depend on the values of the parameters m and s . Therefore, to obtain the sequence of equations for the consecutive powers of ε , it is needed to assume the accurate values of m and s . Different values of m and s lead to different sets of equations and, as a consequence, to different equivalent macroscopic descriptions. Four cases of interest can be distinguished:

CASE I. Model I. Diffusion-filtration coupling with memory effects, $s = 1$ and $m = 0$, $A = O(1)$, $T_D = O(T_S)$.

CASE II. Model II. Classical diffusion-filtration coupling, $s = 1$ and $m = 1$, $A = O(\varepsilon^{-1})$, $T_D \gg O(T_S)$.

CASE III. Model III. Classical seepage law, $s \geq 2$ and $m \geq 0$, $A = O(\varepsilon)$, $T_D \ll O(T_S)$.

CASE IV. Non-homogenizable situation, $s = 0$ and $m = 0$, $A = O(\varepsilon^{-1})$. Clearly in this case the condition (2.13) of homogenizability is not fulfilled. Case IV leads to a non-homogenizable situation, i.e. a situation where an equivalent macroscopic description is not possible. A direct proof of that is presented in the Appendix.

Model I. Diffusion-filtration coupling with memory effects, $s = 1$, $m = 0$, $S_{tl} = O(\varepsilon)$, $M_{tl} = O(1)$, $P_{el} = O(\varepsilon^{-1})$, $T_D = O(T_S)$.

With this estimation we get from (2.19), (2.21) and (2.22) the following equations:

$$(2.29) \quad \begin{aligned} \operatorname{div}_y(\varrho^{(0)}\mathbf{v}^{(0)}) &= 0, \\ \frac{\partial \varrho^{(0)}}{\partial t} + \operatorname{div}_x(\varrho^{(0)}\mathbf{v}^{(0)}) + \operatorname{div}_y(\varrho^{(1)}\mathbf{v}^{(0)} + \varrho^{(0)}\mathbf{v}^{(1)}) &= 0 \quad \text{in } \Omega_p, \end{aligned}$$

$$(2.30) \quad \frac{\partial C^{(0)}}{\partial t} - \operatorname{div}_y(\mathbf{D} \operatorname{grad}_y C^{(0)}) = 0 \quad \text{in } \Omega_s,$$

$$(2.31) \quad \begin{aligned} \varrho^{(0)}\mathbf{v}^{(0)}\mathbf{n} &= 0, \\ (\varrho^{(0)}\mathbf{v}^{(1)} + \varrho^{(1)}\mathbf{v}^{(0)} + \mathbf{D} \operatorname{grad}_y C^{(0)})\mathbf{n} &= 0 \quad \text{on } \Gamma. \end{aligned}$$

Equations (2.25)–(2.31) give a sequence of boundary value problems for the first terms of the asymptotic expansions.

The first problem following from (2.25)₁, (2.26) leads to:

$$(2.32) \quad \begin{aligned} p^{(0)} &= p^{(0)}(\mathbf{x}, t), \\ \varrho^{(0)}(\mathbf{x}, t) &= \frac{p_c}{\varrho_c} \frac{\varrho_a}{p_a} p^{(0)}(\mathbf{x}, t). \end{aligned}$$

The first terms of the gas pressure and of the gas density are locally constant over the macropores Ω_p .

The second problem is given by (2.27) and (2.30). It is similar to that discussed in [17]. To solve it, the following substitution is applied:

$$U(\mathbf{x}, \mathbf{y}, t) = C^{(0)} - \frac{\varrho_c}{C_c} \phi_s \varrho^{(0)}(\mathbf{x}, t).$$

This leads to the set of equations

$$(2.33) \quad \begin{aligned} \frac{\partial U}{\partial t} - \operatorname{div}_y(\mathbf{D} \operatorname{grad}_y U) &= -\frac{\varrho_c}{C_c} \phi_s \frac{\partial \varrho^{(0)}}{\partial t} \quad \text{in } \Omega_s, \\ U(\mathbf{x}, \mathbf{y}, t) &= 0 \quad \text{on } \Gamma. \end{aligned}$$

The thermodynamic equilibrium at the initial time gives

$$U(\mathbf{x}, \mathbf{y}, 0) = 0.$$

By using the Laplace transform, we obtain

$$(2.34) \quad \begin{aligned} a\mathcal{L}(U) - \operatorname{div}_y(\mathbf{D} \operatorname{grad}_y \mathcal{L}(U)) &= -\frac{\varrho_c}{C_c} \phi_s \mathcal{L}\left(\frac{\partial \varrho^{(0)}}{\partial t}\right), \\ \mathcal{L}(U) &= 0 \quad \text{on } \Gamma, \end{aligned}$$

where a is the complex Laplace variable and

$$\mathcal{L}(U) = \int_0^\infty U e^{-at} dt.$$

The right-hand side of (2.34)₁ does not depend on the microscopic space variable y .

Therefore the solution of (2.34) is a linear function of this forcing term:

$$(2.35) \quad \mathcal{L}(U) = -\frac{\varrho_c}{C_c} \phi_s \mathcal{L}(G(y, t)) \mathcal{L}\left(\frac{\partial \varrho^{(0)}}{\partial t}\right),$$

where $\mathcal{L}(G(y, t))$ is the solution of (2.34), when the right-hand side of (2.34)₁ is equated to unity. We use now the volume average defined by the formula

$$\langle * \rangle = \frac{1}{\Omega} \int * d\Omega,$$

and we apply the inverse Laplace transform to (2.35). We obtain from the convolution theorem

$$(2.36) \quad \langle U \rangle = -\frac{\varrho_c}{C_c} \phi_s \int_0^t \frac{\partial \varrho^{(0)}}{\partial t} \langle G(t - \tau) \rangle d\tau.$$

Finally, introduction of the concentration gives the solution of the considered second boundary value problem in the form:

$$(2.37) \quad \langle C^{(0)} \rangle = \frac{\varrho_c}{C_c} \phi_s \left((1 - \phi) \varrho^{(0)} - \int_0^t \frac{\partial \varrho^{(0)}}{\partial t} \langle G(t - \tau) \rangle d\tau \right),$$

where ϕ is the porosity, $\phi = \Omega_p / \Omega$. The average is evaluated by assuming the concentration $C^{(0)}$ to be zero in Ω_p .

Relation (2.37) shows that the gas concentration depends on the history of the first time-derivative of the gas density. Function $G(t)$ represents a memory function.

The third problem to be solved is given by the equations (2.25)₂, (2.28)₁, (2.29)₁, (2.31)₁ and the condition of Ω -periodicity of $p^{(1)}$ and $\mathbf{v}^{(0)}$. By taking into account the relations (2.32), this set becomes

$$(2.38) \quad \begin{aligned} \Delta_y \mathbf{v}^{(0)} - \mathbf{grad}_x p^{(0)} - \mathbf{grad}_y p^{(1)} &= 0, \\ \text{div}_y (\mathbf{v}^{(0)}) &= 0, \quad \mathbf{v}^{(0)}|_\Gamma = 0. \end{aligned}$$

The system (2.38) represents the classical problem of flow of an incompressible fluid through a rigid porous medium. At this stage, $p^{(0)}$ is considered as a known function of \mathbf{x} . The unknowns $\mathbf{v}^{(0)}$ and $p^{(1)}$ are linear functions of the macroscopic gradient $\text{grad}_{\mathbf{x}} p^{(0)}$ (see for example [18, 19]). In what follows, only $\mathbf{v}^{(0)}$ is needed:

$$v_i^{(0)} = -k_{ij}(\mathbf{y}) \frac{\partial p^{(0)}}{\partial x_j} \quad \text{in } \Omega_p.$$

By taking the volume average of $\mathbf{v}^{(0)}$, we obtain the well-known Darcy law:

$$(2.39) \quad \langle v_i^{(0)} \rangle = -\langle k_{ij} \rangle \frac{\partial p^{(0)}}{\partial x_j}.$$

The fourth problem leads to the macroscopic mass conservation law and is given by (2.29)₂, (2.30) and (2.31)₂. By integrating (2.29)₂ with respect to \mathbf{y} on Ω_p and by using the divergence theorem, we obtain

$$\phi \frac{\partial \varrho^{(0)}}{\partial t} + \text{div}_{\mathbf{x}} \left(\varrho^{(0)} \langle \mathbf{v}^{(0)} \rangle \right) + \frac{1}{|\Omega|} \int_{d\Omega_p} \left(\varrho^{(1)} \mathbf{v}^{(0)} + \varrho^{(0)} \mathbf{v}^{(1)} \right) \mathbf{n} \, dS = 0.$$

By taking now into account (2.30) and (2.31)₂, the above equation leads to the following form of the macroscopic mass conservation law:

$$(2.40) \quad \phi \frac{\partial \varrho^{(0)}}{\partial t} + \text{div}_{\mathbf{x}} \left(\varrho^{(0)} \langle \mathbf{v}^{(0)} \rangle \right) + \frac{\partial \langle C^{(0)} \rangle}{\partial t} = 0.$$

The last term in the mass balance equation (2.40) represents a source term due to the diffusion process in the micropores.

Equations (2.32), (2.37), (2.39) and (2.40) represent the macroscopic description. Returning to the physical variables, they assume the form

$$(2.41) \quad \begin{aligned} p^{(0)} &= p^{(0)}(\mathbf{X}, t), \\ \varrho^{(0)}(\mathbf{X}, t) &= \frac{\varrho_a}{p_a} p^{(0)}(\mathbf{X}, t), \\ \langle C^{(0)} \rangle &= \phi_s \left((1 - \phi) \varrho^{(0)} - \int_0^t \frac{\partial \varrho^{(0)}}{\partial t} \langle G(t - \tau) \rangle d\tau \right), \\ \langle v_i^{(0)} \rangle &= -\frac{\langle k_{ij} \rangle}{\mu} l^2 \frac{\partial p^{(0)}}{\partial X_j}, \\ \phi \frac{\partial \varrho^{(0)}}{\partial t} + \text{div}_{\mathbf{X}} \left(\varrho^{(0)} \langle \mathbf{v}^{(0)} \rangle \right) + \frac{\partial \langle C^{(0)} \rangle}{\partial t} &= 0. \end{aligned}$$

The set (2.41) exhibits the memory effects, similarly to [9, 10] or [11].

Model II. Classical diffusion-filtration couplings, $s = 1$, $m = 1$, $S_{tl} = O(\varepsilon)$, $M_{tl} = O(\varepsilon)$, $P_{cl} = O(1)$, $T_D = O(\varepsilon^{-1}T_S)$.

In this case we get from (2.20), (2.22) and (2.23) the following sequence of equations.

$$(2.42) \quad \begin{aligned} \operatorname{div}_y(\varrho^{(0)}\mathbf{v}^{(0)}) &= 0, \\ \frac{\partial \varrho^{(0)}}{\partial t} + \operatorname{div}_x(\varrho^{(0)}\mathbf{v}^{(0)}) + \operatorname{div}_y(\varrho^{(1)}\mathbf{v}^{(0)} + \varrho^{(0)}\mathbf{v}^{(1)}) &= 0 \quad \text{in } \Omega_p, \end{aligned}$$

$$(2.43) \quad \begin{aligned} \operatorname{div}_y(\mathbf{D} \operatorname{grad}_y C^{(0)}) &= 0, \\ \frac{\partial C^{(0)}}{\partial t} - \operatorname{div}_x(\mathbf{D} \operatorname{grad}_y C^{(0)}) - \operatorname{div}_y(\mathbf{D} \operatorname{grad}_x C^{(0)} + \mathbf{D} \operatorname{grad}_y C^{(1)}) &= 0 \quad \text{in } \Omega_s, \end{aligned}$$

$$(2.44) \quad \begin{aligned} (\varrho^{(0)}\mathbf{v}^{(0)} + \mathbf{D} \operatorname{grad}_y C^{(0)})\mathbf{n} &= 0, \\ (\varrho^{(0)}\mathbf{v}^{(1)} + \varrho^{(1)}\mathbf{v}^{(0)} + \mathbf{D} \operatorname{grad}_x C^{(0)} + \mathbf{D} \operatorname{grad}_y C^{(1)})\mathbf{n} &= 0 \quad \text{on } \Gamma. \end{aligned}$$

Case II is described by the above system, together with Eqs. (2.25)–(2.28).

As before in the Case I, the first boundary value problem to be investigated is given by (2.25)₁ and (2.26), and it leads to the relations (2.32).

Equations (2.43)₁ and (2.27) constitute the second boundary value problem. By using an equivalent variational formulation, [17], and by taking into account the equation (2.31)₂, we obtain

$$(2.45) \quad C^{(0)} = \frac{\varrho_c}{C_c} \phi_s \varrho^{(0)}(\mathbf{x}, t) \quad \text{in } \Omega_s.$$

The third problem is described by (2.25)₂, (2.28)₁, (2.42)₁ and (2.44)₁. The above result (2.45) transforms Eq. (2.44)₁ into the relation (2.31)₁, and the set under consideration becomes equivalent to the corresponding one investigated in the Case I. Therefore the Darcy law (2.39) is valid in this case too.

The macroscopic mass conservation law follows from the fourth boundary value problem. It is given by the set (2.42)₂, (2.43)₂ and (2.44)₂. Using the above results, the considered system can be rewritten in a simpler form:

$$(2.46) \quad \begin{aligned} \frac{\partial \varrho^{(0)}}{\partial t} + \operatorname{div}_x(\varrho^{(0)}\mathbf{v}^{(0)}) + \operatorname{div}_y(\varrho^{(1)}\mathbf{v}^{(0)} + \varrho^{(0)}\mathbf{v}^{(1)}) &= 0, \\ \frac{\partial C^{(0)}}{\partial t} - \operatorname{div}_y(\mathbf{D} \operatorname{grad}_x C^{(0)} + \mathbf{D} \operatorname{grad}_y C^{(1)}) &= 0, \\ (\varrho^{(0)}\mathbf{v}^{(1)} + \varrho^{(1)}\mathbf{v}^{(0)} + \mathbf{D} \operatorname{grad}_x C^{(0)} + \mathbf{D} \operatorname{grad}_y C^{(1)})\mathbf{n} &= 0. \end{aligned}$$

By applying the same method as in the Case I, the set (2.42), (2.46) yields the macroscopic mass conservation law:

$$(2.47) \quad \phi \frac{\partial \varrho^{(0)}}{\partial t} + \operatorname{div}_x \left(\varrho^{(0)} \langle \mathbf{v}^{(0)} \rangle \right) + (1 - \phi) \frac{\partial C^{(0)}}{\partial t} = 0.$$

As in the Case I, the last term occurring in the above equation is a source term due to the diffusion process. Therefore, as in Case I, the gas constrained in the micropores interacts with the filtrating gas. However, the coupling is now clasical, and it does not introduce the memory effects.

The macroscopic equivalent description is given by Eqs. (2.32), (2.39), (2.45) and (2.47). When they are expressed in terms of physical variables, they have the following form:

$$\begin{aligned}
 (2.48) \quad & p^{(0)} = p^{(0)}(\mathbf{X}, t), \\
 & \varrho^{(0)}(\mathbf{X}, t) = \frac{\varrho_a}{p_a} p^{(0)}(\mathbf{X}, t), \\
 & C^{(0)} = \phi_s \varrho^{(0)}(\mathbf{X}, t), \\
 & \langle v_i^{(0)} \rangle = -\frac{\langle k_{ij} \rangle}{\mu} l^2 \frac{\partial p^{(0)}}{\partial X_j}, \\
 & \phi \frac{\partial \varrho^{(0)}}{\partial t} + \operatorname{div}_X \left(\varrho^{(0)} \langle \mathbf{v}^{(0)} \rangle \right) + (1 - \phi) \frac{\partial C^{(0)}}{\partial t} = 0.
 \end{aligned}$$

Model III. Classical seepage law, $s \geq 2$ and $m \geq 0$, $S_{tl} \leq O(\varepsilon^2)$, $M_{tl} \leq O(1)$, $P_{el} \geq O(1)$, $T_D = O(\varepsilon T_S)$.

For simplicity, we do not present here the homogenization process. The procedure is very similar to that of the Cases I and II. It results in a macroscopic description similar to (2.48), without the time derivatives.

The Case III describes, at the macroscopic level, the stationary gas filtration in the micropores, without any influence of the diffusion. The macroscopic equivalent description is given by the following set:

$$\begin{aligned}
 (2.49) \quad & \langle v_i^{(0)} \rangle = -\frac{\langle k_{ij} \rangle}{\mu} l^2 \frac{\partial p^{(0)}}{\partial X_j}, \\
 & \operatorname{div}_X \left(\varrho^{(0)} \langle \mathbf{v}^{(0)} \rangle \right) = 0.
 \end{aligned}$$

Moreover, the gas concentration in the solid is given at the first order of magnitude by

for $M_{tl} \leq O(\varepsilon)$

$$C^{(0)} = \phi_s \varrho^{(0)}(\mathbf{X}, t);$$

for $M_{tl} = O(1)$

$$\langle C^{(0)} \rangle = \phi_s \left((1 - \phi) \varrho^{(0)} - \int_0^t \frac{\partial \varrho^{(0)}}{\partial t} \langle G(t - \tau) \rangle d\tau \right),$$

where

$$p^{(0)} = p^{(0)}(\mathbf{X}, t), \quad \varrho^{(0)}(\mathbf{X}, t) = \frac{\varrho_a}{p_a} p^{(0)}(\mathbf{X}, t).$$

3. Remarks on the macroscopic behaviour

The passage from the pore scale to the macroscopic scale shows three different equivalent macroscopic descriptions, depending on the value of the dimensionless numbers:

CASE I. Diffusion-filtration coupling with memory effects

$$(3.1) \quad \begin{aligned} \phi \frac{\partial p^{(0)}}{\partial t} - \operatorname{div} \left(\frac{\langle k_{ij} \rangle l^2}{2\mu} \mathbf{grad} (p^{(0)})^2 \right) \\ + \phi_s (1 - \phi) \frac{\partial p^{(0)}}{\partial t} - \phi_s \frac{\partial}{\partial t} \left(\int_0^t \frac{\partial p^{(0)}}{\partial t} \langle G(t - \tau) \rangle d\tau \right) = 0, \\ \langle C^{(0)} \rangle = \phi_s \left((1 - \phi) \varrho^{(0)} - \int_0^t \frac{\partial \varrho^{(0)}}{\partial t} \langle G(t - \tau) \rangle d\tau \right). \end{aligned}$$

CASE II. Classical diffusion-filtration coupling

$$(3.2) \quad \begin{aligned} \phi \frac{\partial p^{(0)}}{\partial t} - \operatorname{div} \left(\frac{\langle k_{ij} \rangle l^2}{2\mu} \mathbf{grad} (p^{(0)})^2 \right) + \phi_s (1 - \phi) \frac{\partial p^{(0)}}{\partial t} = 0, \\ C^{(0)} = \phi_s \varrho^{(0)}(\mathbf{x}, t). \end{aligned}$$

The coupling is represented here by the term $\phi_s (1 - \phi) \frac{\partial p^{(0)}}{\partial t}$. As in the Case I, the coupling term disappears when $\phi_s = 0$.

CASE III. Classical seepage law

$$(3.3) \quad \operatorname{div} \left(\frac{\langle k_{ij} \rangle l^2}{\mu} \mathbf{grad} (p^{(0)})^2 \right) = 0,$$

and, additionally,

for $M_{tl} \leq O(\varepsilon)$

$$C^{(0)} = \phi_s \varrho^{(0)}(\mathbf{x}, t);$$

for $M_{tl} = O(1)$

$$\langle C^{(0)} \rangle = \phi_s \left((1 - \phi) \varrho^{(0)} - \int_0^t \frac{\partial \varrho^{(0)}}{\partial t} \langle G(t - \tau) \rangle d\tau \right).$$

The above equations have to be supplemented by suitable initial and boundary conditions for $p^{(0)}$.

We remark here that the physical meanings of the macroscopic quantities $p^{(0)}$, $\varrho^{(0)}$ and $\langle C^{(0)} \rangle$ do not pose any problems since they are equal or proportional to the corresponding local quantities. Relations (3.1)₂ and (3.2)₂ represent the constitutive equations of the gas. They give the concentration in the micropores as a function of the gas density in the macropores. The gas filtration is governed by the classical Darcy law. The macroscopic mass balance is represented by (3.1)₁ or (3.2)₁ or (3.3)₁. Their respective ranges of validity are obtained from the values of the dimensionless numbers. However, the description I is the most powerful since it comprises the descriptions II and III as particular behaviours. The descriptions II and III are obtained in the limit from description I for slow and rapid transient excitations, respectively.

Let us now study the total mass flux of the gas. It is the sum of the filtration flux and the diffusion flux,

$$\mathbf{F} = \varrho \langle \mathbf{v} \rangle - \langle \mathbf{D} \mathbf{grad} C \rangle.$$

To determine the contribution of filtration and diffusion in the total flux, we use again dimensionless variables. For the sake of clarity of the description, we do not omit now the asterisk which denotes the dimensionless variables. Within the approximation of $O(\varepsilon)$, the above relation becomes:

$$\mathbf{F}^{(0)} = \varrho_c v_c \left[\varrho^{*(0)} \langle \mathbf{v}^{*(0)} \rangle - \frac{\mathbf{D} C_c}{\varrho_c v_c l} \left(\varepsilon \mathbf{grad}_x \langle C^{*(0)} \rangle + \langle \mathbf{grad}_y C^{*(0)} \rangle \right) \right].$$

Now, from the definition (2.12) of the surface Peclet number

$$\frac{DC_c}{\varrho_c v_c l} = P_{el}^{-1},$$

we have

$$\mathbf{F}^{(0)} = \varrho_c v_c \left[\varrho^{*(0)} \langle \mathbf{v}^{*(0)} \rangle - P_{el}^{-1} \left(\varepsilon \mathbf{grad}_x \langle C^{*(0)} \rangle + \langle \mathbf{grad}_y C^{*(0)} \rangle \right) \right].$$

By using the estimations presented in the Sec. 2, it becomes in all cases

$$\left| \mathbf{F}^{(0)} - \varrho_c v_c \varrho^{*(0)} \langle \mathbf{v}^{*(0)} \rangle \right| \leq O(\varepsilon).$$

The total mass flux is equal to the pore filtrating flux within the approximation $O(\varepsilon)$.

4. One-dimensional problem

To emphasize the influence of the gas diffusion, let us consider the one-dimensional macroscopic boundary value problem. Consider the gas filtration through a horizontal and semi-infinite coal seam. In addition, we assume that:

- the coal stratum is an isotropic and homogeneous porous medium of constant thickness,
- the roof and the floor are impermeable to the gas,

- the mine opening is maintained at the atmosphere pressure p_a ,
- the initial pressure p_i in the coal seam is constant,
- the long-wall head moves with a constant velocity ω .

With the above assumptions it is possible to change the problem to a steady state problem. We introduce the moving system of coordinates (ξ_1, ξ_2, ξ_3) , Fig. 2, with $\xi_1 = x_1 - \omega t$ and $\xi_1 = 0$ on the long-wall head. The derivatives are transformed into the form:

$$(4.1) \quad \frac{\partial}{\partial x_1} = \frac{\partial}{\partial \xi_1}, \quad \frac{\partial}{\partial t} = -\omega \frac{\partial}{\partial \xi_1}.$$

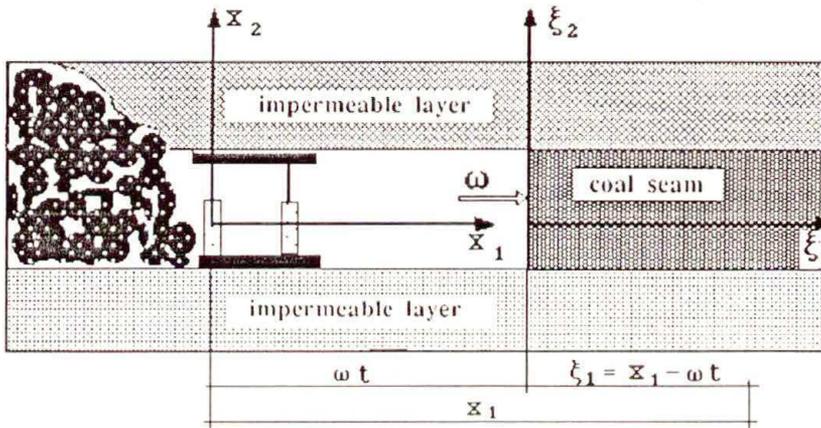


FIG. 2. Geometrical scheme of the one-dimensional problem.

We investigate three boundary value problems where one of the three descriptions is assumed to be valid everywhere throughout the seam:

I. Gas filtration with diffusion in the solid part and with memory effects (the model (3.1), Case I).

II. Gas filtration with gas diffusion in the solid part and without memory effect (3.2), Case II.

III. Gas filtration without any gas diffusion in the solid part (the classical model described by (3.3), Case III).

The solution of the Problem III can be obtained by direct integration of the differential equation describing this case. Taking into account the boundary conditions

$$\begin{aligned} \frac{\partial p}{\partial \xi_1} = 0, \quad p = p_i \quad \text{at } \xi_1 \rightarrow \infty, \\ p = p_a \quad \text{at } \xi_1 = 0, \end{aligned}$$

gives the gas pressure distribution and its gradient in the form [4]:

$$(4.2) \quad \xi_1 = \frac{\langle k \rangle l^2}{\phi \mu \omega} \left[p_a - p + p_i \ln \left(\frac{p_i - p_a}{p_i - p} \right) \right],$$

$$(4.3) \quad \frac{\partial p}{\partial \xi_1} = \frac{\phi \omega \mu}{\langle k \rangle l^2} \left[\frac{p_i}{p} - 1 \right].$$

Consider now the Problem II. It is easy to conclude that its solution can be obtained by changing ϕ into $\phi + \phi_s(1 - \phi)$:

$$(4.4) \quad \xi_1 = \frac{\langle k \rangle l^2}{(\phi + \phi_s[1 - \phi]) \mu \omega} \left[p_a - p + p_i \ln \left(\frac{p_i - p_a}{p_i - p} \right) \right],$$

$$(4.5) \quad \frac{\partial p}{\partial \xi_1} = \frac{[\phi + \phi_s(1 - \phi)] \omega \mu}{\langle k \rangle l^2} \left[\frac{p_i}{p} - 1 \right].$$

Solution of the Problem I necessitates the memory function $\langle G(t) \rangle$. It is defined from the set (2.34), where the right-hand side of (2.34)₁ is equal to unity. In order to present a closed analytical (not numerical) form of the memory function, we confine our study to a very simple model of the periodic cell. We assume spherical grains with radii R . The spatial structure of the grain packing is shown in Fig. 3. The grains are assumed to constitute of a homogeneous and isotropic microporous medium.

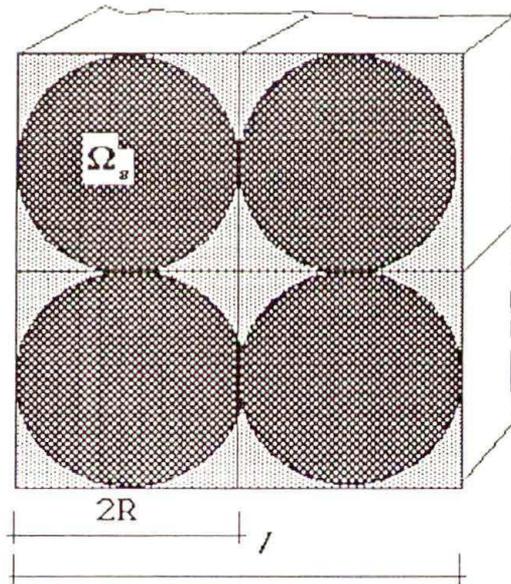


FIG. 3. Micro-geometry of the porous coal medium.

By using spherical coordinates and by putting $H(r, t) = G(r, t) \cdot r$, the set (2.33) can be written in the form:

$$(4.6) \quad a \mathcal{L}(H(r, t)) - D \frac{d^2 \mathcal{L}(H(r, t))}{dr^2} = r,$$

$$\mathcal{L}(H(r, t)) = 0 \quad \text{for} \quad r = 0 \quad \text{and} \quad r = R,$$

where r represents the radial coordinate.

The eigenvalues and eigenfunctions associated with the set (4.6) are:

$$\lambda_m = D \left(\frac{m\pi}{R} \right)^2, \quad \varphi_m = \sqrt{\frac{2}{R}} \sin \left(\frac{m\pi}{R} r \right).$$

By looking for $\mathcal{L}(H(r, t))$ in the form:

$$\mathcal{L}(H(r, t)) = \sum_{m=1}^{\infty} d_m \varphi_m,$$

we obtain

$$d_m = -\frac{1}{a + \lambda_m} \frac{\sqrt{2R^3}}{m\pi} \cos(m\pi),$$

and

$$\mathcal{L}(H(r, t)) = -2R \sum_{m=1}^{\infty} \frac{1}{a + \lambda_m} \frac{\cos(m\pi)}{m\pi} \sin \left(\frac{m\pi}{R} r \right).$$

The Laplace transform of the function $G(r, t)$ is

$$\mathcal{L}(G(r, t)) = -\frac{2R}{r} \sum_{m=1}^{\infty} \frac{1}{a + \lambda_m} \frac{\cos(m\pi)}{m\pi} \sin \left(\frac{m\pi}{R} r \right).$$

Finally, by taking the volume average of the above equation and applying the inverse Laplace transform, we obtain $\langle G(t) \rangle$ in the form

$$(4.7) \quad \langle G(t) \rangle = \sum_{m=1}^{\infty} \frac{1}{m^2 \pi} e^{-D(m\pi/R)^2 t}.$$

Let us return to the Problem I. The memory effect in Eq. (3.1) is given by the convolution product of the memory function by the time derivative of the pressure. By integration by parts, this product can be presented in the following equivalent form:

$$(4.8) \quad \int_0^t \frac{\partial p}{\partial \tau} \langle G(t - \tau) \rangle d\tau = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-1)^{n+1} \frac{1}{m^2 \pi} \left[\frac{1}{D} \left(\frac{R}{m\pi} \right)^{2n} \left\{ \left[\frac{\partial^n p}{\partial \tau^n} \right]_{\tau=t} - \left[\frac{\partial^n p}{\partial \tau^n} \right]_{\tau=0} e^{-D(k\pi/R)^2 t} \right\} \right].$$

By using the transformation rules (4.1) and by taking into account the above relation, we reduce the mathematical model of the Problem I to

$$(4.9) \quad -\phi \omega \frac{\partial p}{\partial \xi_1} - \frac{\langle k_{ij} \rangle l^2}{2\mu} \frac{\partial^2 p^2}{\partial \xi_1^2} - \phi_s (1 - \phi) \omega \frac{\partial p}{\partial \xi_1} - \phi_s \sum_{n=1}^{\infty} \omega^{n+1} \frac{R^{2n}}{D^n \pi^{2n+1}} d_n \frac{\partial^{n+1} p}{\partial \xi_1^{n+1}} = 0,$$

where

$$d_n = \sum_{m=1}^{\infty} \left(\frac{1}{m^2} \right)^{n+1}.$$

Clearly, Eq. (4.9) is too complicated for analytical solution. Therefore, a numerical iteration procedure is introduced to obtain an approximate solution. It gives the distributions of the gas pressure and its gradient. The results are plotted in Fig. 4 and 5, together with the results of III and II. The gas is carbon dioxide. The following typical values have been used in the calculations:

- macropore porosity: $\phi = 0.05$,
- micropore porosity: $\phi_S = 0.11$,
- coefficient of filtration: $\frac{\langle k \rangle l^2}{\mu} = 10^{-4} \frac{\text{m}^4}{\text{MN}\cdot\text{s}}$,
- diffusion coefficient in the micropores: $D = 10^{-11} \frac{\text{m}^2}{\text{s}}$,
- radius of grain (three cases): $R_1 = 10^{-3} \text{ m}$, $R_2 = 2 \times 10^{-3} \text{ m}$, $R_3 = 4 \times 10^{-3} \text{ m}$,
- initial gas pressure in the coal seam: $p_i = 4 \text{ MPa}$,
- velocity of the long-wall head: $\omega = 8 \times 10^{-5} \text{ m/s}$.

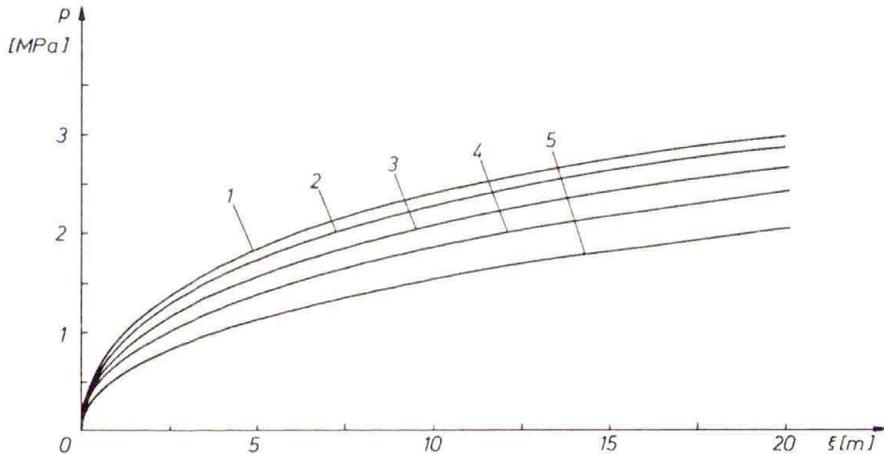


FIG. 4. Distribution of the gas pressure in the coal seam: 1 – Solution II, 2 – Solution I for $R = 1 \text{ mm}$, 3 – Solution I for $R = 2 \text{ mm}$, 4 – Solution I for $R = 4 \text{ mm}$, 5 – Solution III.

Figure 4 and Fig. 5 show that II yields larger values of the gas pressure and of its gradient, whereas III gives lower values. The solutions III and II can be considered as bounds for the solution I. When there is no available information about the geometrical structure of the coal, they can be used as rough approximations of the pressure and its gradient. Note, however, the large difference between the two solutions III and II, in particular between the pressure gradients at the long-wall head.

The most important factor responsible for the occurrence of a gas-coal outburst is the gradient of the gas pressure at the long-wall head [4]. It is shown in

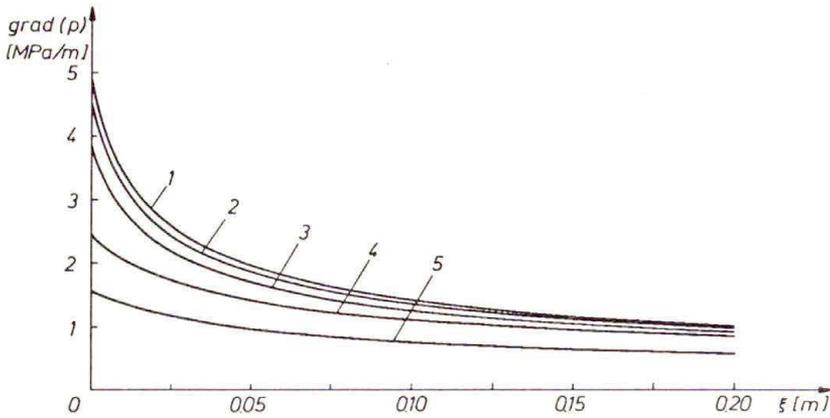


FIG. 5. Distribution of the gradient of the gas pressure in the vicinity of the long-wall head: 1 – Solution II, 2 – Solution I for $R = 1$ mm, 3 – Solution I for $R = 2$ mm, 4 – Solution I for $R = 4$ mm, 5 – Solution III.

Fig. 6 as a function of the grain radius. We conclude that the solution I converges to the solution II when the radius of the grain becomes smaller and smaller, and converges to the solution III when the radius becomes larger and larger. The curve in Fig. 6 shows also that a smaller radius yields a larger value of the gas pressure gradient at the long-wall head. We can immediately see the important role played by the grain radius or, more generally, the geometrical structure of coal. Our results agree with the empirical relation (1.1).

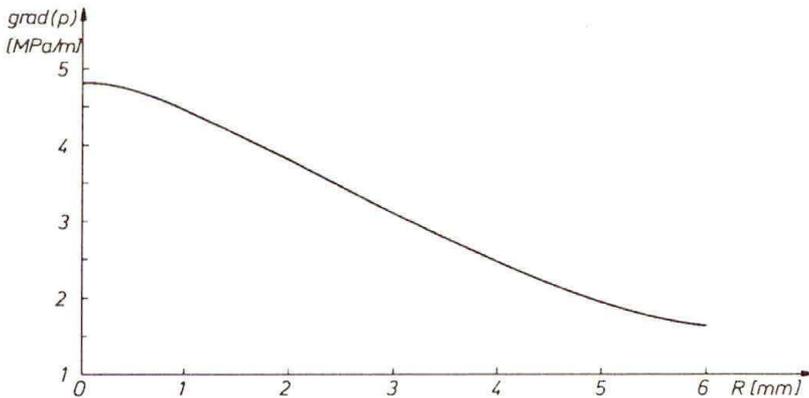


FIG. 6. The gradient of the gas pressure at the long-wall head versus the grain radius.

It is interesting to investigate the domain of validity of each description in the seam. It is now possible to estimate the macroscopic characteristic length $L(\xi)$ in each point of the seam, by using

$$L = \frac{\partial p}{\partial \xi_1} / \frac{\partial^2 p}{\partial \xi_1^2}.$$

The solutions I, II and III give approximately the same result. The resulting parameter ε is shown in the Fig. 7. It is seen that ε is small everywhere, except in a thin layer at the long-wall head where it goes to infinity. In this region there is no separation of scale and, consequently, there is no macroscopic description. The solutions I, II and III remain valid outside this boundary layer, i.e., approximately where $\varepsilon \geq 0.1$. The results in Fig. 6 are nevertheless valid because of the momentum balance applied to the boundary layer.

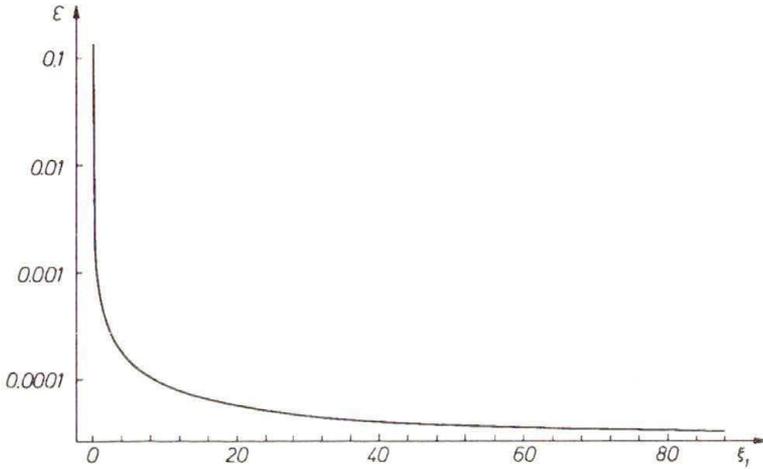


FIG. 7. Distribution of the parameter of scale separation ε in the coal seam.

The domain of validity of each description can be investigated by using the dimensionless number:

$$A = \frac{T_D}{T_S} = \frac{\varepsilon^{-1} S_{ll}}{M_{ll}} = \frac{DL}{l^2 v_c},$$

where v_c is given by

$$v_c = \frac{k}{\mu} l^2 \frac{\partial p}{\partial \xi_1}.$$

We have $A = O(1)$, $O(\varepsilon^{-1})$ and $O(\varepsilon)$ in the Case I, II and III, respectively. A , ε and ε^{-1} are plotted for comparison in the Fig.8. The figure shows four regions:

$$\xi_1 < 0.01 \text{ m}, \quad \text{i.e.} \quad \varepsilon > 0.1,$$

corresponds to the boundary layer where no macroscopic description is possible.

$$0.01 \text{ m} < \xi_1 < 0.3 \text{ m},$$

near the boundary layer, $A = O(\varepsilon)$, $T_D = O(\varepsilon T_S)$, and the classical description III can be applied.

$$0.3 \text{ m} < \xi_1 < 20 \text{ m}, \quad A = O(1), \quad T_D = O(T_S),$$

and the description I, with memory effects has to be considered.

$$\xi_1 > 20 \text{ m}, \quad A = O(\varepsilon^{-1}), \quad T_D = O(\varepsilon^{-1}T_S),$$

and the description II, classical coupling, is valid.

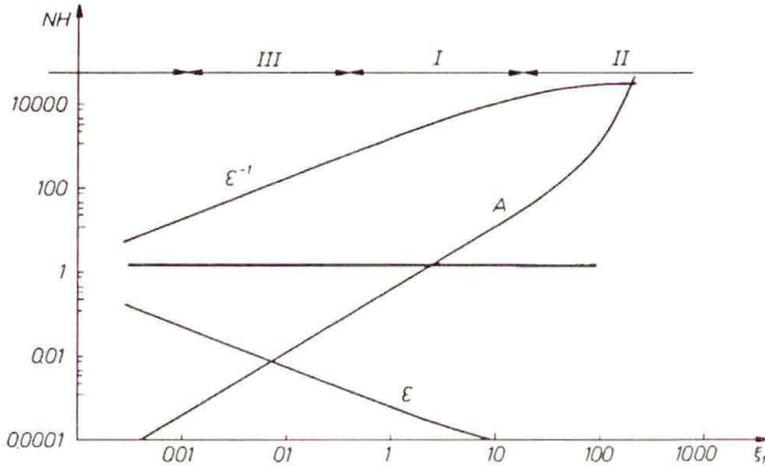


FIG. 8. Domains of validity of the three models. I: $A = O(1)$, Model I. II $A = O(\varepsilon^{-1})$ Model II. III $A = O(\varepsilon)$, Model III. NH: non-homogenizable.

5. Conclusions

The above study shows that the influence of the diffusion process in the micropores on the gas filtration in the macropores depends on a source term in the macroscopic equation of mass conservation. The filtration and the simultaneous diffusion of the gas are modelled by three different macroscopic descriptions. Appropriate dimensionless numbers, related to the physico-chemical properties and the geometrical structure of the coal, determine the model to be used. In particular, it is shown that the gas concentration exhibits memory effects if A , the ratio of the diffusion to the convection characteristic times, is of $O(1)$. When A decreases to $A = O(\varepsilon)$, the memory effects disappear and the model converges to the classical filtration model. The diffusion in the solid part is ignored. When A increases to $A = O(\varepsilon^{-1})$, the memory effects disappear too, and the model converges to a filtration-like model. The behaviour is described by an equation similar to the classical filtration process, but where the porosity of the macropores is replaced by the total porosity of the micropores and the macropores. The two last behaviours, i.e., the filtration without any diffusion and the filtration with the classical diffusion process, give bounds for the solution of the filtration with memory effects.

Appendix

Non-homogenization situation: $s = 0, m = 0$ (Case IV)

From (2.19), (2.21) and (2.22) we get

$$(A.1) \quad \frac{\partial \varrho^{(0)}}{\partial t} + \text{div}_y(\varrho^{(0)}\mathbf{v}^{(0)}) = 0 \quad \text{in } \Omega_p,$$

$$(A.2) \quad \frac{\partial C^{(0)}}{\partial t} - \text{div}_y(\mathbf{D} \text{grad}_y C^{(0)}) = 0 \quad \text{in } \Omega_s,$$

$$(A.3) \quad (\varrho^{(0)}\mathbf{v}^{(0)} + \mathbf{D} \text{grad}_y C^{(0)})\mathbf{n} = 0 \quad \text{on } \Gamma.$$

The above set, together with Eqs. (2.25)–(2.28), yields the sequence of the boundary value problems to be solved.

The first one is described by (2.25)₁, (2.26) and leads again to the relation (2.32).

Equations (A.2) and (2.27) determine the second problem. They are equivalent to the corresponding ones in the Case I. Therefore the first term of the gas concentration satisfies the relation (2.37).

Now we solve the fourth boundary value problem described by (A.1), (A.2) and (A.3). Taking the volume average and using the divergence theorem, Eq.(A.1) takes the form:

$$\phi \frac{\partial \varrho^{(0)}}{\partial t} + \frac{1}{|\Omega|} \int_{d\Omega_p} (\varrho^{(0)}\mathbf{v}^{(0)})\mathbf{n} dS = 0.$$

The condition (A.3) transforms the above equation into:

$$\phi \frac{\partial \varrho^{(0)}}{\partial t} - \frac{1}{|\Omega|} \int_{d\Omega_s} (\mathbf{D} \text{grad}_y C^{(0)})\mathbf{n} dS = 0.$$

Now, by using (A.2) and again the divergence theorem, we obtain the following relation:

$$\phi \frac{\partial \varrho^{(0)}}{\partial t} - \frac{\partial \langle C^{(0)} \rangle}{\partial t} = 0.$$

Substitution of (2.37) leads to

$$\phi \frac{\partial \varrho^{(0)}}{\partial t} - \frac{\varrho_c}{C_c} \phi_s \left((1 - \phi) \frac{\partial \varrho^{(0)}}{\partial t} - \frac{\partial}{\partial t} \left[\int_0^t \frac{\partial \varrho^{(0)}}{\partial \tau} \langle G(t - \tau) \rangle d\tau \right] \right) = 0.$$

Application of the Laplace transform and the convolution theorem leads to the equation

$$\mathcal{L} \left(\frac{\partial \varrho^{(0)}}{\partial t} \right) \left[\phi - \frac{\varrho_c}{C_c} \phi_s (1 - \phi) + a \frac{\varrho_c}{C_c} \phi_s \mathcal{L}(\langle G(t) \rangle) \right] = 0,$$

where a is the complex Laplace variable.

The above relation must be valid for any values of a and for any geometry of the period Ω . Therefore, it is clear that

$$\mathcal{L} \left(\frac{\partial \varrho^{(0)}}{\partial t} \right) = 0,$$

and then

$$\frac{\partial \varrho^{(0)}}{\partial t} = 0.$$

This condition leads to the rescaling of the dimensionless number S_{II} . This one becomes of the order of magnitude $O(\varepsilon)$, that is in a contradiction with our initial assumption $S_{II} = O(1)$. Remark that $S_{II} = O(1)$ does not satisfy the condition (2.13). We conclude that the case under consideration is not homogenizable.

Acknowledgement

D. Lydzba wishes to thank Ministère Français de la Recherche et de l'Espace for the support by the grant No: 080350B 1 2215E.

References

1. W. BURGERT and H. LIPPMANN, *Models of translatory rock bursting in coal*, Int. J. Rock Mech. Min. Sci. and Geomech. Abstr., **18**, 285–294, 1981.
2. J. LITWINISZYN, *A model for the initiation of coal gas outbursts*, Int. J. Rock Mech. Min. Sci. and Geomech. Abstr., **22**, 39–46, 1985.
3. D. LYDZBA, *State of strength and stress in a gas-coal system near a wall head* [in Polish], Ph.D. Thesis, Inst. of Geotechn. of Technological University of Wrocław, 1990.
4. L. PATERSON, *A model for outbursts in coal*, Int. J. Rock Mech. Min. Sci. and Geomech. Abstr., **23**, 327–332, 1986.
5. M. LASON, *Specific surface of porous materials*, Arch. Min. Sci., **33**, 475–497, 1988.
6. H. GIL and A. SWIDZINSKI, *Rock and gas outbursts in underground mines* [in Polish], Silesian University of Technology, 1982.
7. J. LITWINISZYN, *Gas emission from a crushed rock medium during its sudden outburst*, Arch. Min. Sci., **32**, 155–168, 1987.
8. J.-L. AURIAULT and D. CAILLERIE, *Quelques remarques sur les méthodes d'homogénéisation*, Rev. Fr. Géotech., **49**, 43–50, 1989.
9. J.-L. AURIAULT and C. BOUTIN, *Deformable porous media with double porosity. Quasi-statics. I. Coupling effects*, TiPM, **7**, 63–82, 1992; *Quasi-statics. II. Memory effects*, TiPM, **10**, 153–169, 1993; *III. Acoustics*, TiPM, **14**, 143–162, 1994.
10. J.-L. AURIAULT and P. ROYER, *Écoulement d'un fluide très compressible dans un milieu poreux à double porosité*, CRAS, **317**, II, 431–436, 1993.
11. T. ARBOGAST, J. DOUGLAS and U. HORNING, *Derivation of the double porosity model of single phase flow via homogenization theory*, SIAM J. Math. Anal., **21**, 4, 823–836, 1990.
12. A. BENSOUSSAN, J.-L. LIONS and PAPANICOLAOU, *Asymptotic analysis for periodic structures*, North-Holland, Amsterdam 1978.
13. E. SANCHEZ-PALENCIA, *Non-homogeneous media and vibration theory*, Springer Verlag, New York 1980.
14. J.-L. AURIAULT, *Heterogeneous media. Is an equivalent macroscopic description possible?* Int. J. Engng. Sci., **29**, 785–795, 1991.

15. C.C. MEI and J.-L. AURIAULT, *The effect of weak inertia on the flow through a porous medium*, J. Fluid Mech., **222**, 647–663, 1991.
16. J.-L. AURIAULT, T. STRZELECKI, J. BAUER and S. HE, *Porous deformable media saturated by a very compressible fluid: quasi-statics*, Eur. J. Mech., A/Solids, **9**, 373–392, 1990.
17. J.-L. AURIAULT, *Effective macroscopic description for heat conduction in periodic composites*, Int. J. Heat Mass Transfer., **26**, 861–869, 1983.
18. J.-L. AURIAULT and T. STRZELECKI, *On electro-osmotic flow in a saturated porous medium*, Int. J. Engng. Sci., **19**, 915–928, 1981.
19. J.-L. AURIAULT, *Non-saturated deformable porous media: quasi-statics*, TiPM, **2**, 45–64, 1987.

WROCLAW UNIVERSITY OF TECHNOLOGY
INSTITUTE OF GEOMECHANICS AND HYDROTECHNICS, WROCLAW
and
UNIVERSITÉ JOSEPH FOURIER
INSTITUT NATIONAL POLYTECHNIQUE DE GRENOBLE, FRANCE.

Received March 30, 1995.